

**A METHODOLOGY FOR THE DEVELOPMENT OF MODELS FOR  
THE SIMULATION OF NON-OBSERVABLE SYSTEMS**

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Andrew J. Turner

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**A METHODOLOGY FOR THE DEVELOPMENT OF MODELS FOR  
THE SIMULATION OF NON-OBSERVABLE SYSTEMS**

Approved by:

Dimitri Mavris, Advisor  
School of Aerospace Engineering  
*Georgia Institute of Technology*

Daniel Schrage  
School of Aerospace Engineering  
*Georgia Institute of Technology*

Charles Domercant  
School of Aerospace Engineering  
*Georgia Institute of Technology*

David Goldman  
School of Industrial and Systems  
Engineering  
*Georgia Institute of Technology*

Eugene Paulo  
School of Engineering and Applied  
Sciences  
*Naval Postgraduate School*

Date Approved: May 2014

*For my brother, Josh*

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## SUMMARY

The use and application of modeling and simulation (M&S) is pervasive in today's world. A key component in the application of models is to conduct appropriate verification and validation (V&V). V&V is conducted to make sure the model represents reality to the appropriate level of detail based on the questions posed. V&V techniques are well documented within the literature for observable systems, i.e. required data can be collected from the operations of the real system for comparison with the simulation results; however, V&V techniques for non-observable systems are limited to subjective validation. This subjective validation can be applied to the simulation outputs, operational validation, or towards the model development, conceptual validation. Oftentimes subjective operational validation of the simulation is the primary source of validation efforts. It is shown in this thesis that the sole reliance on subjective operational validation of the simulation can easily lead to the inaccurate acceptance of a model.

In order to improve M&S practices for the representation of non-observable systems, models must be developed in a methodological manner that provides a traceable and defensible argument behind the models representation of reality. Though there is growing discussion within the recent literature, few methods exist on proper conceptual model development and validation. The research objective of this thesis is to identify a methodology to develop a model in a traceable and defensible manner for a system or system of systems that is non-observable. To address this research objective the proposal will address eight aspects of model development.

The first is to define a set of terms that are common vernacular in the field of M&S. This is followed by the assessment of what defines a good model and how to determine if the model is good or not. This leads to a review of V&V and the observation that subjective validation in isolation is not sufficient for model validation. Next, a review of model development procedures is conducted and analyzed against a set of criteria. A selection is made using

the Analytic Hierarchy Process (AHP). A procedure developed by Balci in 1986 is selected for the use in development of models for non-observable systems. Specific steps within Balci's 1986 procedure are investigated further to determine appropriate techniques that should be used when developing models of non-observable systems. These steps are system and objective definition, conceptual model, communicative model, and experimental models and results

Five techniques are identified in the literature that can be applied to system and objective definition: Soft Systems Methodology, Requirements Engineering, Unified Modeling Language, Systems Modeling Language, and Department of Defense Architecture Framework. These techniques are reviewed and selection is made using AHP. The System Modeling Language (SysML) is selected as the best technique to perform System an Objective Definition.

Significant resources are devoted to the study of conceptual model development. Proposed in this thesis is a process to decompose the impacts of the system and apply subjective weightings in order to identify aspects of the system with significant importance. This approach enables the modeling of the system in question to the appropriate level of fidelity based on the identified importance of the system impacts. Additionally, this process provides traceability and defensibility of the final model form.

Communicative model development is rarely addressed in the literature; however, many of the techniques used in system and objective definition can be applied to developing a communicative model. A similar study to the system and objective definition, AHP was utilized to make a selection. It was concluded that the Unified Modeling Language provides the best tool for creating a communicative model.

In the final step, experimental models and results, the literature was found to be rich in techniques. A gap was found in the analysis of the outputs of stochastic simulations. Four questions resulted: 'which stochastic measures should be used in analyzing a stochastic simulation?', 'how many replications are required for an accurate estimation of the stochastic measure?', which least squares method should be used in the regression of a stochastic response?, and 'how many replications are required for an accurate regression of a stochastic

measure? Heuristics are presented for each of these questions.

A proof of concept is provided on the methodology developed within this thesis. The selected scenario is a Humanitarian Aid / Disaster Relief Mission, where the U.S. Navy has been tasked with distributing aid in an effective manner to the affected population. Upon application of the proposed methodology, it was observed that subjective decomposition and weighting of the scenario proved to be a useful tool for guiding and justifying the form of the eventual model. Shortcomings of the methodology were identified. The primary shortcomings identified were the linking of information between the steps of the model development procedure, and the difficulty in correctly identifying the structure of the system impacts decomposition.

The primary contribution of this thesis is to the field of M&S. Contributions are made to the practice of conceptual model development, a growing discussion within the literature over the past several years. The contribution to conceptual model development will aid in the development models for non-observable systems. Additional contributions are made to the analysis of stochastic simulations. The methodology presented in this thesis will provide a new and robust method to develop and validate models in a traceable and defensible manner.

## CHAPTER I

### INTRODUCTION TO MODELING AND SIMULATION

Laplace's Demon: We may regard the present state of the universe as the effect of its past and the cause of its future. An intellect which at a certain moment would know all forces that set nature in motion, and all positions of all items of which nature is composed, if this intellect were also vast enough to submit these data to analysis, it would embrace in a single formula the movements of the greatest bodies of the universe and those of the tiniest atom; for such an intellect nothing would be uncertain and the future just like the past would be present before its eyes. (Pierre Simon Laplace) [95]

Laplace's Demon is a perfect articulation of the philosophy of scientific determinism. Assuming classical mechanics, having perfect and infinite knowledge of the universe one would be able to predict the totality of past and present. Though flawed, Laplace's Demon does reflect humanity's nature to understand and predict the world. Humans often attempt the prediction of reality through the use of abstracted understandings of the real world. These abstractions are called models. To many, the term 'model' often elicits the thought of the use and application of Modeling and Simulation (M&S) which is pervasive in today's world. The use of M&S is commonly found in science and engineering, two fields of study that focus on understanding and predicting the world. Examples of models used in the sciences are climate models such as the Community Earth System Model developed by the National Center for Atmospheric Research [71], weather models such as the Weather Research & Forecasting Model developed by a variety of United States Government Institutes [88], biological models such as the Epidemic Simulation System developed by Los Alamos [31], social dynamic models such as Sakoda's famous Checker-board Model of Social Interaction [167], and physics models such as Newton's law of universal gravitation [125]. Models developed for use in engineering include sizing models such as the Flight



Optimization System (FLOPS) developed by NASA [107], structural models such as the Finite Element Method [185], fluid models such as Computational Fluid Dynamics [9], and supply chain models such as Markov Chains [162]. Models can take many different forms in addition to those listed above. Some of these forms are business models, conceptual models, and physical models.

Models and simulations come in many forms and are used for a variety of purposes. A better understanding of what defines models and simulations is needed. This is addressed in the following section. Additionally, it must be better understood as to why models and simulations are used. The use of models and simulations is addressed in Section 1.2. Given that models and simulations are tools and are used for specific reasons, it needs to be determined what defines good models and simulations. It will be revealed that this is determined through verification and validation, which is covered in Section 1.3. Finally, shortcomings within the current literature with respect to the previous questions are identified. These shortcomings lead to the research objective of this thesis, which is covered in Section 1.4.

### ***1.1 The Definition of Models and Simulations***

Given this wide variety of models and their applications, it is important to address the definition and purpose of a model. There are countless definitions of a model offered in the literature; however, there are a number definitions found that describe models in the broadest and most appropriate form. Four insightful definitions from the literature are listed as follows.

‘A model is a representation and an abstraction of anything such as a system, concept, problem, or phenomena.’ (Balci, 1994) [20]

‘... a model is conceived as any physical, mathematical, or logical representation of a system, entity, phenomenon, or process.’ (Zeigler, 2000) [217]

‘An abstract representation of reality in any form (including mathematical, physical, symbolic, graphical, or descriptive form) to present a certain aspect of that reality for answering the questions studied.’ (Ebert, 2005) [62]

‘M is a model of A with respect to question set Q if and only if M may be used to answer questions about A in Q within tolerance T.’ (Ross, 1977)[161, 3]

The first definition of a model is from Balci. In his statement, a model is intended to represent either a concept, problem, or phenomenon. Within the literature, the subject that is to be modeled is often referred to as the system or the entity. In this thesis both terms will be used. The selection of the term used is based on the usage from work that is being described. A system is defined as a set of interacting components [2]. Balci’s definition covers all of the entities that were modeled in the examples above; however, his definition does leave out the form of these representations.

The second quote, by Zeigler, then refines the definition of a model by including the ‘form’ of the model in addition to its intent to represent some subject. Zeigler defines the form that a model can take as physical, e.g. a statue of a person is a model of the person; mathematical, e.g. Newton’s Law of Gravitation is a model of gravity; and logical, e.g. a mental understanding of a business process.

Ebert then expands the definition of a model further by presenting its ‘purpose’. The purpose of a model is to answer a set of questions. Combining the definition offered by Ebert, who states the purpose of models, with Balci’s definition which identifies the entities to be modeled, one can expand the purpose of a model to either aid in answering questions or communicating ideas. Zeigler addresses in his work three problems that models can be used to aid in solving: analysis, inference, and design [217]. Analysis is the attempt to understand the entity to be modeled. Given knowledge about the internal structure and behavior of the entity, one is attempting to understand its macro behavior with respect to the environment in which it exists. Inference operates in the opposite direction. Given knowledge about the entity’s macro behavior within the environment, one is trying to understand the internal structure and behavior of the entity. Design is attempting to find or construct an entity that will exhibit a desired macro behavior within the greater system.

The final definition, offered by Ross, is the most technical, general, and far reaching. His definition does leave out that the purpose of a model could be to communicate an idea, as do many others. The one unique aspect of his definition is that it offers a statement of quality.

He states that the questions must be answered within a given tolerance; however, there are many models that have been developed and used that can be considered inadequate. For this reason, this author believes that the quality assessment of a model should not be part of the definition of a model, for there exist good and bad models. Considering the above discussion, a formal definition of a model is presented:

**Definition:** A model is an abstraction of reality or one's concept that is used as an aid in answering a set of questions or to aid in communication.

Like the term model, 'simulation' has a number of definitions offered in the literature; however, the definitions offered for simulation fail to converge on a universal concept. There are three philosophies regarding the term simulation. The first philosophy is to use the word simulation interchangeably with the word model. Additionally, the term simulation has been used as a substitute for the paradigms of computer modeling: discrete event simulation, Monte Carlo simulation, and system dynamics, where discrete event simulation dominates [120, 87, 153, 66, 100, 82, 72]. The second definition given to simulation is the entire process of modeling a system, i.e. building the model, conducting experiments, and analyzing said system [20, 179, 76]. Finally, the United States Department of Defense (DoD) uses the term simulation to refer to an execution or computation of a model [199, 56, 21]. This is the denotation that will be used here. Applying this terminology to Newton's Gravitation Model, the model would be the equation and a simulation would be the calculation of the trajectory of two or more objects interacting through the gravitational force. Applying this terminology to computational fluid dynamics, the Navier-Stokes equations used will be a model of the behavior of the fluid. Additionally, the computer program that is used to perform analysis on a fluid would also be referred to as a model. The execution of the computer program would be referred to as the simulation.

**Definition:** A simulation is the execution of a model.

## *1.2 The Successful Use and Application of Models and Simulations*

Given the definition of a model and a simulation, a question emerges. Why is M&S used? The definition of a model presented by Ebert, presented in the previous section, suggests that a model is used in the process of answering questions. Based on a review of the literature, the most general response to the question posed is that M&S is used to better understand reality or act as a medium of communication [153, 62, 161, 3, 20, 145, 14]. The use of M&S has numerous advantages including but not limited to the following:

- Experimental test bed: M&S offers an ability to experiment with systems that cannot be tested or are very difficult to test. [153, 179, 14, 43, 202]
- What-if questions: Allows an analyst to perform what-if analysis on the fly, enabling a better understanding of the system in question. [153, 179, 145]
- Test yet to be created systems: Simulation is the only method available to test and experiment with a system that currently does not exist. [153, 179, 43]
- Cost effective: Conducting a simulation from a model can be much cheaper than performing experiments with the real system. [153]
- Control of conditions: Unlike many real systems, everything within a simulation can be monitored and controlled. [153, 179, 66]
- Time Scaling: Offers the ability to simulate a process that may take years in the real world but may only take a number of minutes in simulation. [179, 43, 66]
- Repeatable: Offers the ability to repeat exact conditions to determine cause and effect. In the real world one cannot control every aspect; therefore, experiments are not perfectly repeatable. [66]

M&S has many applications and benefits, which is why it is so ubiquitous in today's world; however, not all models are created equal. Undoubtedly, there are good models and there are bad models; therefore, what makes a good model? Revisiting the definition of a model, Ross [161] provides a definition of a model which includes a statement on the

quality of its representation of the entity which it is emulating. He states that the model must resemble the represented system in question within a tolerance. Kleijnen states that a model should be 'good enough' based on the goals of the model [87], and Balci offers the following statement on the success of a model:

An M&S project is considered successful if it produces sufficiently credible M&S results that are accepted and used by the decision makers or sponsor. (Balci 2011) [22]

Each of these views addresses the need for the model to represent the system in question with a high enough fidelity to answer the questions posed to the system. Sanchez expands on this concept with a quip borrowed from Einstein, 'as simple as possible, but no simpler.' [168] Sanchez confirms that a model must adequately represent the system in question, but models with additional complexity beyond the needs of the study are not good models. An overly complicated model is often a more resource intensive model requiring longer simulation times and potentially greater computational resources. A model can represent the system to such a high degree that it is no longer useful. With this information, a good model can now be defined as one that represents the system that is to be emulated to the appropriate level of fidelity based on the questions presented for the study. Determining whether a model represents reality to the appropriate level of fidelity is accomplished through a process called Verification and Validation (V&V).

### ***1.3 Verification and Validation of Models and Simulations***

Remember that all models are wrong; the practical question is how wrong do they have to be to not be useful. (George E.P. Box) [38]

Before using a model to aid in answering a set of questions, one must be sure that the model has the appropriate level of fidelity based on the questions asked of the system. This is achieved through V&V. A common and incorrect interpretation of the task of V&V is to prove that the model is correct [47]. Instead, the goal of V&V is to attempt to prove the model is incorrect or not capable for answering the questions asked [152, 106]. This

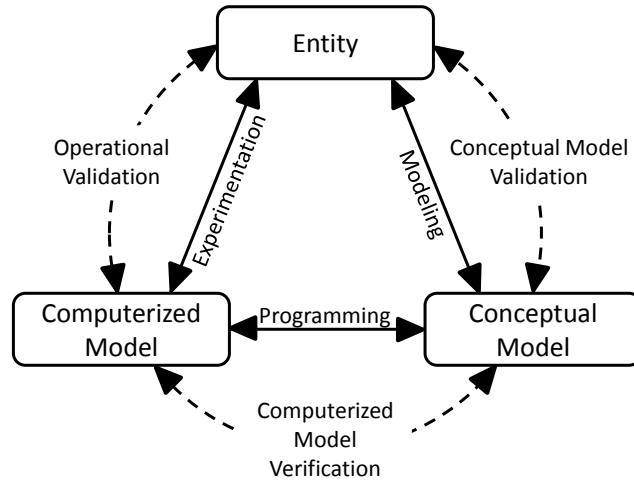
stems from the concept that there does not exist a model that perfectly represents reality; therefore, absolute model validity is impossible [171, 139, 168, 49, 170]. Essentially, model verification and validation is a process of increasing the user's confidence in the model [152]. The definitions for verification and validation are as follows.

**Definition** Verification is the process of determining if model transformations from one form to another were accomplished as intended. Verification is focused on making sure the model was built right.

**Definition** Validation is the process of determining if the model sufficiently represents the system for study based on the questions asked. Validation is focused in making sure the right model was built.

A basic overview of how V&V is used throughout a model's life is shown by the Sargent Circle shown in Figure 1 [171, 170]. The circle starts with the entity. The entity, referred to in this thesis as the system, is the object or process that is to be modeled. The conceptual model is the manner in which the entity is intended to be represented within a computerized framework. Generally, the conceptual model is the way in which the entity should be modeled in the mind of the modeler [154, 25, 158, 133]. A better definition of a conceptual model will be presented in Chapter 4. Assessing the conceptual model's ability to accurately represent the entity with respect to the research questions and to the fidelity requirements imposed by the research questions is referred to as 'conceptual model validation'. This is the first appearance of validation. The computerized model is the implementation of the conceptual model in a computer for execution. This is accomplished through the process of programming. Ensuring that the conceptual model was translated into the computerized model correctly is referred to as 'computerized model verification'. This is also the common interpretation of verification. Once the computerized model is developed, experiments are performed on the model that will address the questions of the study. Ensuring that the computerized model adequately resembles the entity under study is referred to as 'operational validation'. This is the common interpretation of validation for M&S. An important takeaway from Sargent's Circle is that V&V activities occur throughout the life of an M&S

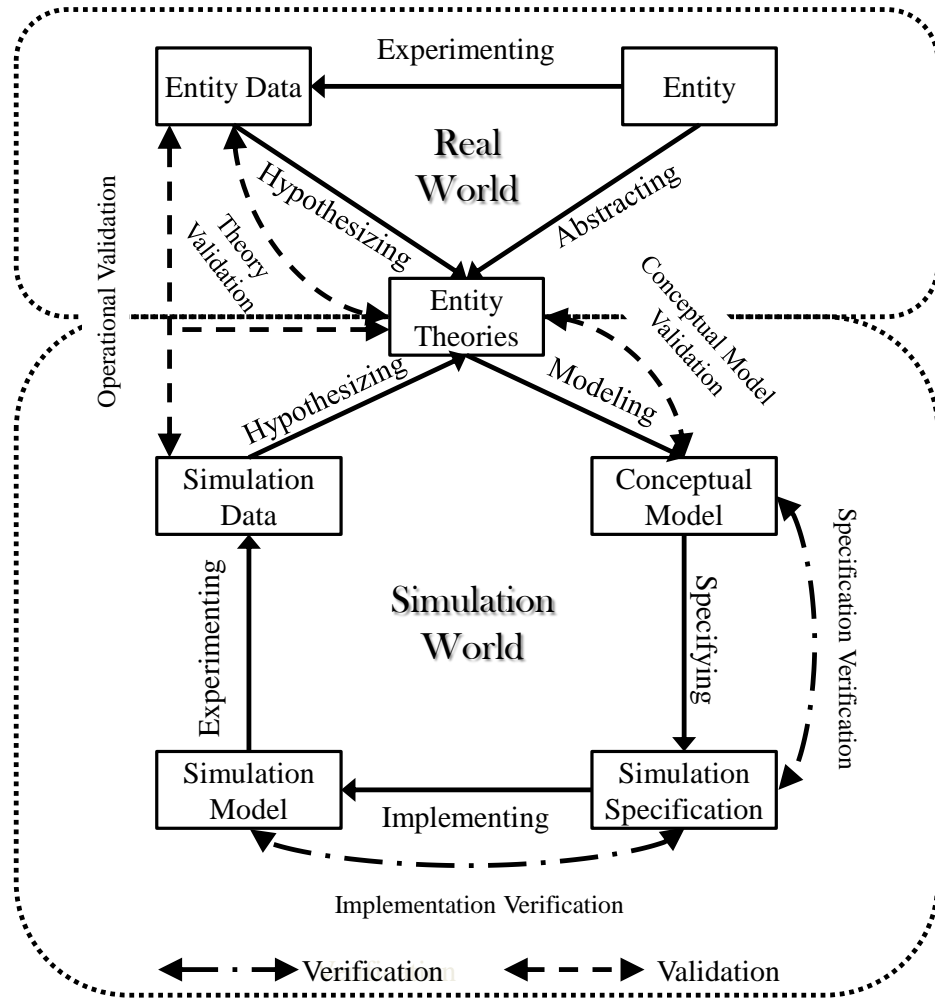
program. Another takeaway is that validation occurs between the modeling processes and the system, and verification occurs between the modeling processes.



**Figure 1:** Simple Modeling Process

Given a basic understanding on how model development and V&V activities occur throughout the life of an M&S program, a more detailed description of the interaction between the system knowledge and the model is presented. This detailed description of how V&V is used throughout a models life is shown by the Evolved Sargent Circle shown in Figure 2 [169, 172, 170]. The Evolved Sargent Circle is composed of two main sections: the real world and the simulation world. The interaction of the simulation with the real world is the primary consideration in this section. The simulation world will be reviewed in greater detail in Chapter 2.

The entity, or system, that is being modeled and simulated is shown in the upper right box of the real world. The goal of any M&S effort is to gain understanding, or knowledge, about the identified entity. This is represented within the Evolved Sargent Circle as the Entity Theories box between the two worlds. The entity theories include characteristics, causal relationships, and the behavior of the entity or system under study [169, 172, 170]. These are listed as theories because the theories could be proven to be false. There are two approaches to developing entity theories. The first is to gather data directly from the entity through experimentation. This data allows the creation of hypotheses about the



**Figure 2:** Evolved Sargent Circle

entity which can be falsified through theory validation with more entity data. Another method for developing entity theories is through abstraction. Abstraction is the process of creating entity theories through observations of the entity. This is commonly performed by Subject Matter Experts (SMEs) who have extensive experience interacting with the entity. Given the entity data and theories, the model can be validated in three different ways. The first is through conceptual model validation, first presented within the Sargent Circle. The other two are forms of operational validation. Traditionally, operational validation involves the comparison between the simulation output data and entity data; however, operational validation can also be performed between the simulation output data and the entity theories.



The latter case involves observing the simulation data and determining whether the data performs as expected.

Given an understanding on how V&V and M&S development contribute to each other, the specific process of V&V must be investigated. Unfortunately, there is no widely accepted methodology in V&V of models [65, 30, 87, 41, 16, 17, 46, 137, 49, 172, 170]; however, there are numerous techniques. Balci presents 77 V&V techniques in his 1998 paper [17]. These techniques are applied throughout the model's life-cycle. Numerous techniques for verification are available within the literature. These include but are not limited to animation, traces, structured walkthroughs, white box testing, and black box testing. A significant amount of research is devoted to the study of verification; however, for the subject of this thesis, verification is not a primary concern. Instead, focus is placed on validation. This is due to the unique challenges to validation presented by non-observable systems. Non-observable systems and the challenges they present will be expanded upon shortly.

In addition to the application of validation in three forms, shown in the Evolved Sargent Circle, validation can be looked at as occurring in three stages: data validation, model development validation, and model operational validation. 'Data validation' is the process of determining that the data used, either as numerical inputs or qualitative observations, are accurate representations of the real system to be studied. This was not shown in the Sargent Circles shown above; however, it is an important part of validation. 'Model development validation' is concerned with ensuring that the logic and assumptions made for developing the model are sufficient to addressing the questions of the system. Conceptual model validation would be included within model development validation. Finally, 'model operational validation' is concerned with comparing the results of the simulation to observations of the real system or the theories of the real system.

In each stage of validation there exist many techniques that can be utilized [16]. These techniques can be broadly categorized into 'objective validation techniques' and 'subjective validation techniques' [171]. An objective technique would use a mathematical or statistical test. A subjective technique is one in which a determination of validity is made through the opinion of a SME, the model developer, the end user, or a third party.

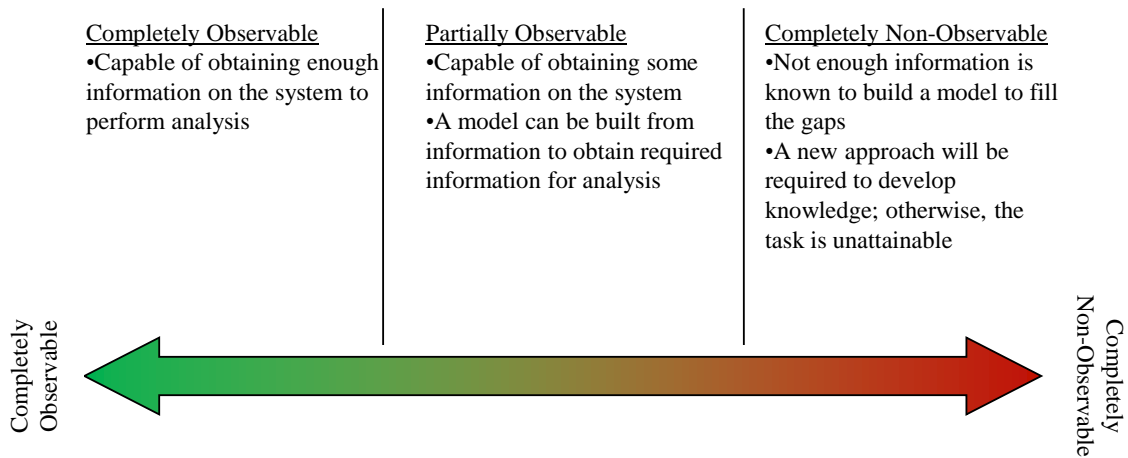
The stages of validation and the two categories of validation techniques can be applied to observable or non-observable systems. A completely observable system is one that allows the ability to gather the required information on the system in a timely, feasible, and viable way. That is a completely observable system is one that enables the gathering of all relevant information that is needed for analysis. The only tasks required to answer the questions about the entity or system are to gather the information and analyze it. Applying this concept to the Evolved Sargent Circle, a completely observable system would provide sufficient entity data to form the entity theories. A M&S effort would not be required.

On the other end of the spectrum is a completely non-observable system. A completely non-observable system is one in which the gathering of the required information is either not timely, feasible, or viable to the extent that no analysis can be performed to discover more knowledge of the entity. An example of this would be that a question on a system is posed but not enough information is known to develop a model that can illuminate the gaps in knowledge. Applying this concept to the Evolved Sargent Circle, a completely non-observable system would not provide sufficient information to form the entity theories. Without the entity theories an M&S effort cannot be attempted.

In reality most modeling efforts are based on a partially observable system that fall between the two extremes. Figure 3 shows this spectrum between observable and non-observable systems. On the far left is the completely observable system which has been addressed. In the middle is a partially observable system. A partially observable system can provide enough information to enable the development of a model which will answer the specific questions. An example of this is estimating the drag of an object in a fluid. Enough is known about the system from previous research and observations to create a CFD model. This model can then be executed to receive more information that will answer the specific question. Another example of a system of systems is a ballistic missile defense model. In this model all the physical attributes are known; however, the interaction between the parts is not known. A model is then built to fill these gaps in knowledge. On the far end is the completely non-observable system, which has been defined. An example of this would be a Human Behavior Representation (HBR) model. Suppose an anti-piracy operation must

be analyzed; however, there is too great of an uncertainty about how the pirates behave to build a model to study the system. Under this circumstance a non-traditional approach is required.

Commonly, systems that fall on the observable side of partially observable are referred to as observable, and systems that fall on the non-observable side of partially observable are referred to as non-observable systems. This is the notation that will be used for the remainder of this paper. More specifically, an observable system is one that provides entity data for which operational validation can be performed. This validation approach would be an objective approach. A non-observable system is one that lacks entity data, and therefore operational validation must be performed between the simulation data and the entity theories. This validation approach would be a subjective approach. Finally, it should be noted that every system will contain elements that exist on different parts of the observable/non-observable spectrum. For example, in the anti-piracy example the pirate behavior is completely non-observable; however, the specifications of the patrol vehicles is completely observable.



**Figure 3:** Observable / Non-Observable Spectrum

The validation approaches for observable and non-observable systems were presented by Sargent [171] as a table, reproduced in Table 1. Objective validation of observable entities is the ultimate form of validation and should always be performed if possible. Validation techniques that fall under this category include F-Tests [47], Smith-Satterthwaite Test[47],

Non-parametric Rank Sum Test[47], Mann-Whitney test [178], and Theil's inequality [178]. These tests can be classified as tests of the means, variance, and correlations between the model and the entity. Objective approaches can also be used for validating models of non-observable entities. The difference between the two is that with non-observable entities the simulation results are compared to that of an accredited simulation as opposed to the real entity. Though this does make one wonder why the effort would be made to develop a model and validate it to another model, if that other model already existed and access to its simulation results is available. As discussed earlier, models of systems are developed to address specific questions; therefore, a new model would not be expected to behave as the other accredited model unless they address the same problem, which makes the model development pointless. Objective validation of models of non-observable systems is not expected to be common.

**Table 1:** Sargent Validation Breakdown

	Observable System	Non-Observable System
Objective Approach	Comparison using statistical tests and procedures	Comparison to other models using statistical tests and procedures
Subjective Approach	Comparison using graphical displays Explore model behavior	Explore model behavior Comparison to other models

Subjective validation of observable entities is exceedingly common and often one of the first steps in validating a model. Validation techniques that fall under this category include graphical comparisons. Graphical comparison is the displaying of the information to check to see if the statistical metrics, e.g., mean, variance, are within reasonable bounds. A more rigorous example of graphical comparison is the Turing-Schruben test [176].

Subjective validation of models developed for both observable and non-observable entities can be accomplished through exploring the model behavior. This can be accomplished through animation, degenerate tests, face validity, sensitivity analysis, and regression analysis [171, 172, 87]. In each of these techniques, an expert determines if the observed behaviors

resemble that of the real world.

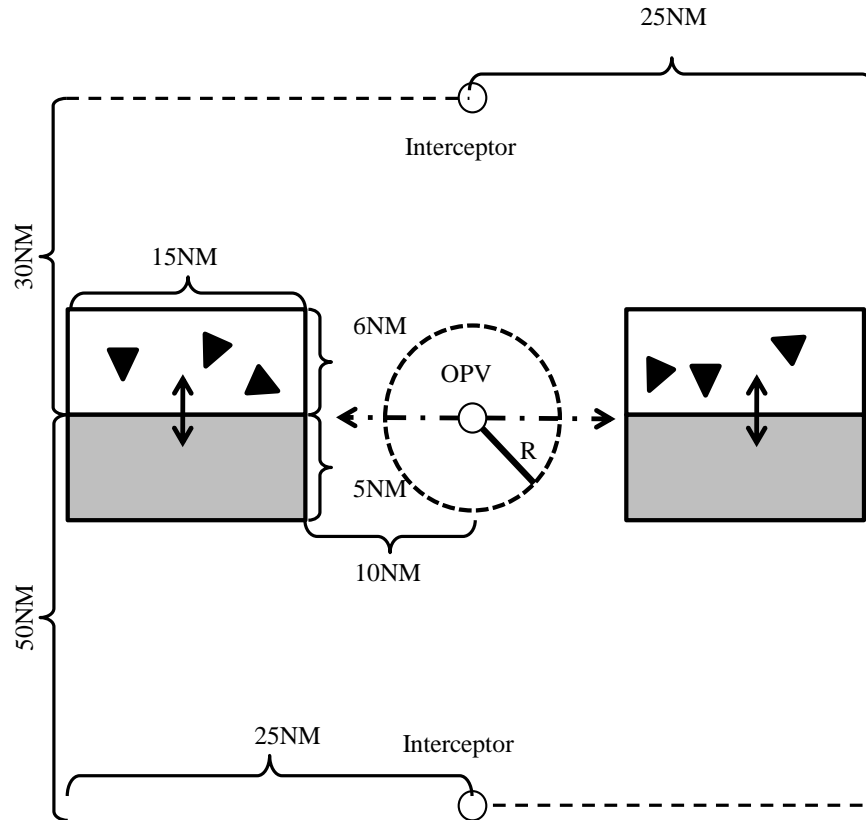
Finally, models of non-observable systems can be validated subjectively through the comparison to other accredited models. The techniques used here include comparison using graphical displays and exploring model behavior. The main difference is that the comparison is made between the model to be validated and the model that has been accredited.

For non-observable systems, only conceptual model validation and subjective operational validation between the simulation data and the entity theories is available. Unfortunately, conceptual modeling is given little attention within the M&S community [153, 154, 156, 39]. Without proper conceptual model development, the validation efforts of models for non-observable systems are solely based on subjective operational validation. An example of subjective operational validation of a non-observable system is given in the next section.

### **1.3.1 Subjective Validation Example and Observations**

This section reviews the validation efforts of three different conceptual models from a previous research project [196] conducted for the Office of Naval Research. The project required the development of an agent based model of a naval patrol scenario. The model was used to help size an Offshore Patrol Vehicle (OPV). A visual description of the conceptual model is shown in Figure 4. In this scenario there are two legal fishing zones that have the dimensions 15x6 nmi. This is shown as a white box in Figure 4. Below the two legal fishing zones are the illegal fishing zones that have the dimensions 15x5 nmi. Within the fishing area there are fishing vessels. These fishing vessels primarily remain in the legal zone but will cross with some kind of modeled behavior to be discussed shortly. An OPV, called Foxtrot, is tasked with patrolling these waters to enforce adherence to the legal fishing zones and to protect the vessels from being commandeered by interceptor vessels protecting their claim to the illegal fishing zone. The interceptor vessels are on the north and south end of the model. The north interceptor is called Yankee, and the south interceptor is called Zulu. Both interceptors are from country Charlie.

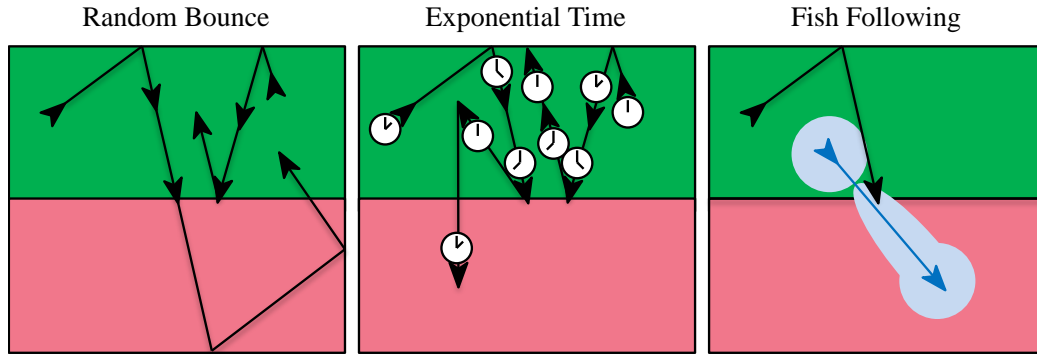
When developing the model there was some concern on the proper representation of the fishing vessel movement behavior. The behavior of the OPV and the interceptors was



**Figure 4:** Economic Exclusion Zone Layout

easier to estimate because they followed very simple procedures. The fishing vessel behavior was not well defined nor were there SMEs available who had extensive knowledge of their behavior. There existed no data on their crossing rates nor their movement behavior. Three concepts were suggested: a random bounce, exponential time, and following fish schools [10]. A visual representation of these three conceptual models is shown in Figure 5. The random bounce representation described the vessels' behavior by traveling in an initial random direction. Once the vessel came into contact with a border, the vessel would turn to some new random direction and continue. If the border was that of the illegal zone, there was a probability the vessel would cross. When the vessel traveled in the illegal zone and came into contact with the border to the legal zone it would cross every time. The exponential time representation described the vessels' behavior similar to that of the random bounce. The difference between the two representations is that the vessels would only cross once an internal variable of time had expired. Once the time had expired the vessel would turn and

cross into the other fishing zone. Finally, the following fish schools concept defined another set of agents to represent schooling fish. These schooling fish would follow a random walk. The fishing vessels would track these schools and cross into the illegal zone depending on their perceived risks and rewards.



**Figure 5:** Three Conceptual Models

Each concept was developed into a computational model and simulated. These simulations were subjected to the following validation techniques: animation, degenerate tests, face validity, and sensitivity analysis. Under this analysis no issues were found for any of the concepts. During analysis the three concepts showed very different results. Shown in Figure 6 are the sorted parameter estimates for the three conceptual models. Sorted parameter estimates are useful for screening and determining variables of importance [174]. The parameters are sorted in decreasing order of significance. The top set of parameters is that of the random bounce conceptual model. The middle set of parameters is the exponential time conceptual model. The bottom set of parameters is the fish following conceptual model. The parameters have been highlighted based on vehicle type. The yellow highlights represent variables that belong to the Yankee or Zulu interceptor. The red highlights represent variables that belong to the OPV. The green highlights represent variables that belong to the fishing vessels.

The first observation is that for all three conceptual models the uncontrollable variables that belonged to the interceptors and fishing vessels determined the majority of the behavior. It is observed that for the first concept, random bounce, the cruise speed and radar range

were the most important OPV parameter. The second concept, exponential time, showed that the radar range was the most important OPV parameter. The third concept, following fish schools, showed that none of the OPV parameters had an impact on the outcome of the simulation.

This study resulted in three potential conceptual models that could all pass subjective validation and resulted in widely different results. For non-observable systems, only subjective operational validation and conceptual model validation is available to validate the model. It was shown here that subjective validation can be misleading. These results lead to the following observation.

**Observation:** Subjective validation methods in isolation are insufficient for models of non-observable systems.

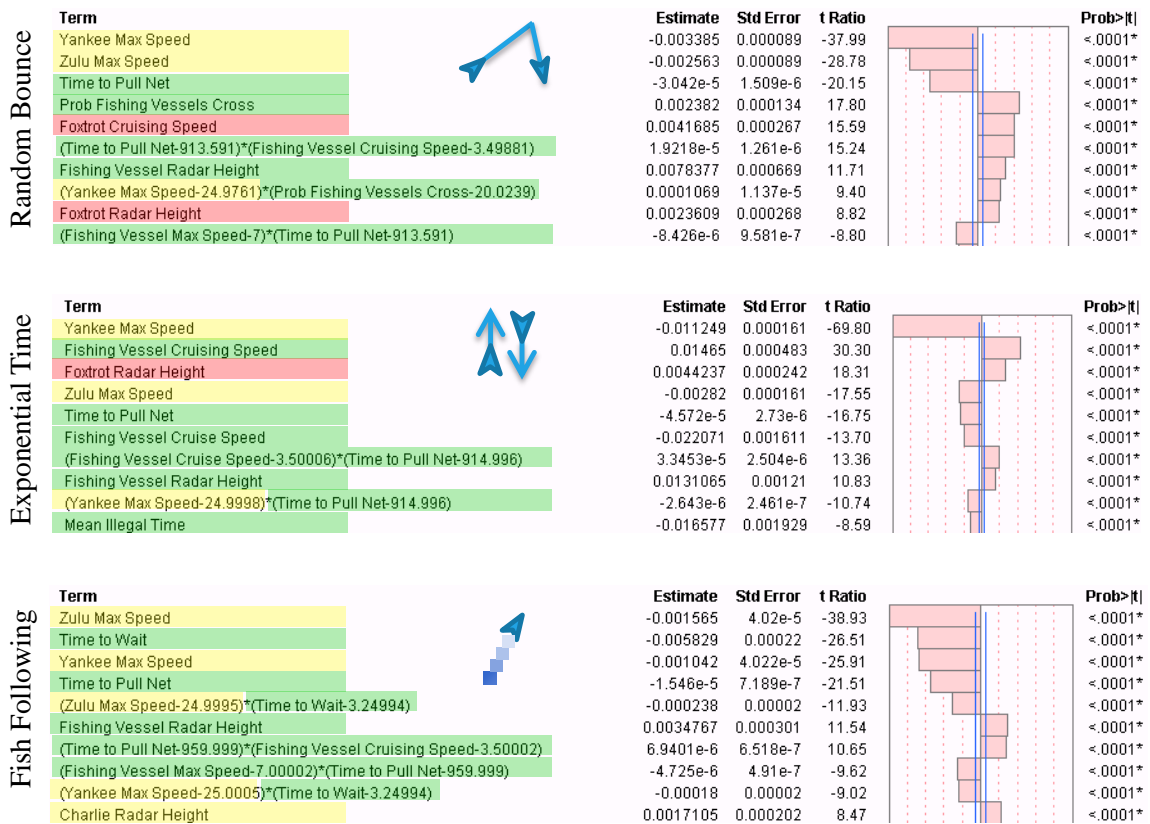


Figure 6: Sorted Parameter Estimates of the Three Conceptual Models



## 1.4 *Summary and Motivation*

Models are ubiquitous in the modern world. The application of these models range from simple models of gravity to models of the weather to models of epidemics. A quality model was defined as representing the system that is to be emulated to the appropriate level of fidelity based on the questions presented for the study. This quality is determined through the process of V&V.

Models that have been developed for observable systems prove to be an ideal case for model validation. These systems allow the model to be objectively and subjectively validated. Unfortunately, this is rarely the case. Many organizations, including the United States Department of Defense (U.S. DoD), are concerned with the modeling and simulation of non-observable systems, e.g. counter insurgency operations, anti-piracy operations, humanitarian aid / disaster relief missions. Model validation of non-observable systems is limited to subjective operational validation and conceptual model validation. The previous section showed that subjective operational validation in isolation is insufficient. The remaining validation technique is the process of validating the model during the development process. For the non-observable system case, model development acts as the primary source of model validation. The concept that model development can be the primary source of model validation is strongly supported by the literature. Pace articulates this best in the following quote.

‘Conceptual validation should be the foundation for simulation credibility... without validation of the concepts and algorithms of the simulation, one has no basis for judgement about how well the simulation can be expected to perform for any other conditions.’ (Pace, 2004) [136]

Some industries that face non-observable systems, such as industrial systems, have a long history of guidelines for proper model development and system representation. In these situations historical precedence will guide proper model development; however, not all modeling efforts have the benefit of a historical perspective. The modeling of non-observable systems with little to no historical precedence will require a new approach to

model development. This leads to the following research objective of this thesis.

**Research Objective:** Identify a methodology to develop a model in a traceable and defensible manner for a non-observable system that has a limited or non-existent history of modeling.

This research objective will be addressed through the following efforts. The first will be to define a potential set of model development procedures within the literature that can be used for model development of non-observable systems. A selection will then be made as to which procedure is best suited for the development based on a set of criteria. Once a procedure is selected, an assessment will be made as to whether the model development procedure is sufficient for this type of problem. This work is covered in Chapter 2. Once a model development procedure has been selected, the literature is revisited to determine how each step of the process can be completed. Chapters 3 through 5 address specific steps in the selected model development procedure. Chapter 3 discusses methods for problem formulation, objective definition, and system definition. Chapter 4 provides a new method for proper conceptual model development. Chapter 5 investigates the experiments and results steps in the selected model development procedure. Chapter 6 applies the methodology that was developed in chapters 2 through 5. Finally, a conclusion is presented on the proposed methodology with additional observations and suggestions for future research.

## CHAPTER II

### MODEL DEVELOPMENT PROCEDURES

Every model ever built had to go through a process of development, whether explicitly stated or not; however, the formal procedure and documentation of model development varies widely from discipline to discipline and model to model. The highest abstraction of model development can be discretized into three realms: The entity, conceptual model, and computerized model [171, 152, 181, 126, 153, 172, 123]. A common representation of this model development abstraction can be seen in Figure 1, often called the Sargent Circle. This was first presented in chapter one. The Sargent Circle was originally developed as a means to communicate concepts of V&V; however, model development and V&V are two sides to the same coin. It is difficult to address one without addressing the other; therefore, the Sargent Circle can double as a road map to model development. In the interest of thoroughness, the Sargent Circle will now be reviewed.

The entity, sometimes referred to as the system, is the object or process that is to be modeled. The entity represents reality. The conceptual model is the manner in which the entity is to be represented within a computerized framework. The conceptual model is the way in which the entity should be modeled in the mind of the modeler [154, 25, 158, 133]. This concept will be expanded on in greater detail in chapter three. Transitioning from the entity to the conceptual model is referred to as a process of modeling. This is the primary interest of this thesis based on the research objective stated in chapter one. The modeling process can be conducted in a variety of ways. This process will be addressed in the next section. Assessing the conceptual model's ability to accurately represent the entity with respect to the research questions and fidelity requirements imposed by the research questions is referred to as conceptual model validation. The computerized model is the implementation of the conceptual model in a computer for execution. This is accomplished through the process of Programming. Assuring that the conceptual model was translated

into the computerized model correctly is referred to as computerized model verification. This is also the common interpretation of verification for M&S, as detailed previously. Once the computerized model is developed, experiments are performed on the model that will address the questions of the study. This process is called experimentation and will be addressed in further detail in chapter three. Ensuring that the computerized model adequately resembles the entity under study is referred to as operational validation. This is the common interpretation of validation for M&S.

The above modeling process can be used to describe a wide range of computer modeling, ranging from modeling physics to operational models, such as fluid modeling, structure modeling, supply chain modeling, industrial operation models, military models, and biological models. The focus of this thesis is the development of M&S for non-observable systems. There is a significant volume of literature dedicated to this subject of model development over the past five decades. The most influential works and those deemed important to this study will be covered; however, significant semantic irregularities exist within the literature. For this reason, some definitions are required before various model development approaches are addressed. The definitions that follow are heavily leveraged from the literature; however, the precise definitions and their use is unique to this thesis.

The first definition is for the phases of the model life cycle. Phases resemble phases of design for many engineering applications. Commonly, phases of the model life cycle are sequential and do not repeat. The second definition is the procedure for model development and use. This is very similar to phases of the model life cycle; however their distinction is important. The procedure can be thought of as detailing the steps of model development. For example, the model development phase can be populated with the following activities: conceptual modeling, communicative model development, programming, and the V&V associated with the activities. Iteration is expected between these activities. The next definition is what is referred to as a framework or methodology. This is a more detailed account of how to accomplish the steps presented in a procedure. This may include an approach to system decomposition, model definition, and a documentation language. The methodologies found in the literature contained these three attributes to a varying degree.

System decomposition can be described as a process of defining a system as an aggregate of its smaller parts in an effort to better understand and model the system in question. Model definition is similar to system decomposition; however, in this context model definition is focused on decomposition of the modeled system. Model definition is the identification of the objects that will exist within the model, their attributes, and behaviors. The model definition will most likely resemble that of the system decomposition. The documentation language would be a formal method for communicating the functioning of a model. This can include a set of definitions, images, or documents that are standardized. The purpose of a documentation language is to create consistency among different fields of study enabling better communication. The final definition required is the modeling support system. A modeling support system is a tool that aids in the management of M&S development. This often takes the form of a computer program.

In the following sections some of the above definitions will be explored further. The current literature will be explored to support these definitions and to develop an understanding of how models and simulations are developed. First the phases of model development will be explored. This will be followed by an expansive review of the literature of current delineations of the activities that exist within the procedures for model development and use. Finally, in chapter three an investigation is made into the different methodologies and frameworks that exist to aid in the development of models and simulations.

### ***2.1 Phases of the Model Life Cycle***

The phases of the model life cycle can be broken into many different sections. Of the definitions to be explored, the phases of the model life cycle has received the smallest attention in the literature. One reason for this may be that the phases of the model life cycle is primarily a semantic discussion. The primary interest for model development lies in the activities and methodologies to develop models. The earliest found phase breakdown comes from a 1976 report from the Government Accountability Office (GAO) [202]. In the report five phases are defined: problem definition, preliminary design, detail design, evaluation, and maintenance. The purpose of the first phase, problem definition, is to define: what

problem is to be solved, who will be building the model, how the problem should be modeled, and how the model will be used. The preliminary and detail design phases involve building the model. Preliminary design is focused on detailing what the model will do and how it will be executed. Detail design is focused on the actual development of the model. The evaluation phase is a final V&V that the model undergoes. The final phase, maintenance, is to document any and all changes to the model. This effort will make the use of the model throughout its life much easier for current and future users.

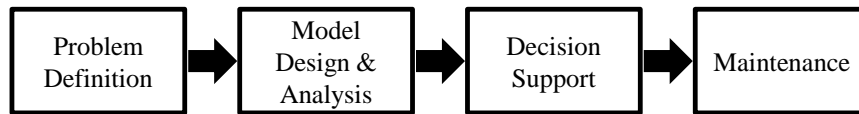
Another breakdown of the phases of model development comes from Nance and Balci's Model and Simulation Life Cycle [118, 20]. This work can find its origins in The Conical Methodology developed by Nance and Balci, which was developed in response to the 1976 GAO report [120, 118, 121, 122, 20]. In their approach three phases exist: problem definition, model development, and decision support. The problem definition is a process of understanding the customer's problem and correctly formulating the requirements of the model that is to be designed. The model development phase includes processes that translate the modeling requirements into an actual programmed model and analyzing said model. The decision support phase includes the process of communicating the simulation results and findings to the decision makers.

Another suggestion on the phases of model development made by Chance et al. [43]. They present the following three phases: model design, development, and deployment. They make a very important statement in that their phases can be iterated. They state that identified omissions, a change in project scope, and errors found in later phases can cause a return to earlier phases. This is a different interpretation than in this thesis on the definition of phases. In this delineation, model design phase contains activities that prepare for the actual model design and construction. These activities include identifying customers, goals, time allotment, and performance measures. The phase ends with a document detailing the requirements of the model. Model development entails selecting a model paradigm, e.g. discrete event simulation and agent based model, data collection, building the model, and V&V. The final phase, project deployment, includes analyzing the model results, presenting results to the concerned party, and maintaining the model.

GAO's five phase description is mostly complete; however, it is missing a phase in which the results are presented to the concerned party as the Nance-Balci and Chance approaches suggest. Additionally, the preliminary and detailed design phases seem to be redundant. In other approaches this is defined within the model development phase. GAO's preliminary and detailed design phases can be considered as two steps in the model development procedure, conceptual model development and program model development, respectively [20]. Chance's approach makes a similar breakdown to GAO with the model design and development phase. Because the phases of model development are intended to be completed in a linear fashion all activities relating directly to designing, programming, and Verification, Validation and Testing (VV&T) of the model will be defined under a model design phase. The Nance-Balci approach provides a good suggestion for the phases of model development; however, the maintenance of the model is not explicitly defined. Chance's approach also provides a good suggestion for the phases of model development; however, the important phase of problem definition is not given its own phase and is included with model design phase.

Noting the positives and negatives of the phase breakdowns listed above, a conclusion can be drawn for a breakdown of the phases of model development: problem definition, model design, decision support, and maintenance. In this delineation, the problem definition phase is the same as the GAO and Nance-Balci approach. The purpose of this phase is to translate the customer needs into model requirements in terms of questions to be answered, fidelity requirements, budget available, and time available. The model design phase includes all activities relating directly to designing, programming, and VV&T of the model. This includes the preliminary design, detail design, and evaluation phases of the GAO approach and the model design and development phases of the Chance approach. The decision support phase includes the analysis of the model in combination with communicating the analysis and notable findings to the concerned parties. Finally, the maintenance phase includes activities to insure that the model can be reused in the future. This includes final documentation and continued documentation throughout the model's life. These phases are designed to be completed in a successive manner. It is possible for a return to earlier

phases as was noted by Chance et al. [43]; however, these phases are not designed to be a spiral development. The returning to earlier phases, though common, is an indication that there was an error in the model development that must be corrected. It should be noted that new projects that reuse the model will still require a restart of the model development phases resulting in updating the model through each phase.



**Figure 7:** Phases of the Model Life Cycle

## 2.2 *Procedures for the Development of Models and Use*

**Research Question 1:** Out of the literature, which procedure is best suited for model development of the defined type of entities?

There are many suggested procedures available in the literature that detail the steps in the development of models and simulations. In this section the more popular procedures available in the literature will be reviewed. This section will focus on what the steps are rather than how the steps are completed. Once the most appropriate procedure is selected for this application, the literature will be reviewed to determine how each step is accomplished.

What makes a good procedure? There are a number of criteria that will help determine the quality of a model development and use procedure for modeling a non-observable system. These are completeness, iterative, traceable, and flexible [23]. The attribute of completeness is one that determines if the procedure listed includes all steps in the model life cycle. For example, a procedure may exclude the creation of a conceptual model or the steps of documentation and reporting results. The primary steps that are being looked for are problem formulation, setting objectives, defining assumptions, conceptual model development, programming, verification, validation, experimental design, analysis, documentation, application, and maintenance or storage. These may exist as individual steps or



within other steps. The second criterion is iterative. All procedures should be iterative, for model development is iterative in nature. A procedure should display both global and local iterative features. A global iterative feature is one that links the simulation results to the initial system decomposition or model conceptualization. A local iterative feature would be one that displays iterations between adjacent steps in the model development procedure. The movement from one step to another does not occur at once but instead is a process requiring iteration between the two steps. Third, an M&S procedure should lend itself to traceability. One should be able to trace the observations from the results to the assumptions made about the model and those assumptions to the problem statement. Given that there are an infinite number of ways to develop a model and simulation, these decisions must be traceable and defensible. This will aid in not only the credibility of a model but also its reuse for future application. An increase in the number of steps within a procedure and the number of V&V activities aid in traceability. Finally, a procedure should be broad enough such that it can be applied to a wide number of applications; however, it must be focused enough so that it is usable. This is largely subjective, therefore the procedure will be evaluated based on if it is applicable to the following model paradigms: system dynamic models, discrete event simulations, and agent based models. Initially, each procedure will be given a subjective score of poor, moderate, good, and very good for each of the criteria. A more rigorous method will later be applied to help select the best procedure for use in developing a model for a non-observable system.

### **2.2.1 The Sargent Circle**

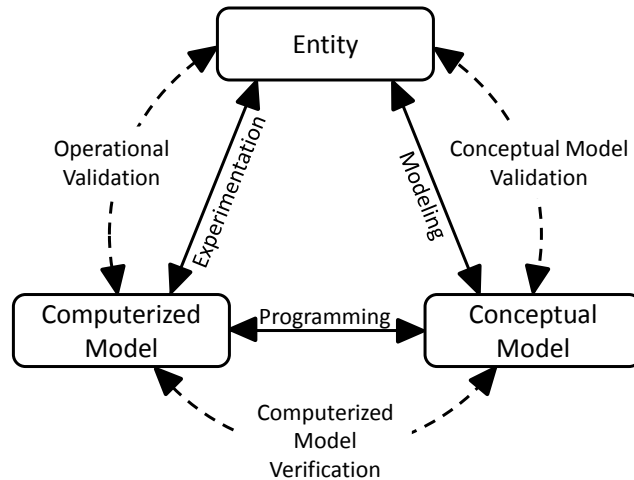
The first presented procedure for model development and use was the Sargent Circle. The procedure is reproduced in Figure 8. It was presented by the Society for Computer Simulation in 1979 [182, 171, 172]. The circle was initially posed as an aid to describe verification and validation of models through their life-cycle. It is difficult to discuss V&V without discussing model development; therefore, the Sargent Circle doubles as a model development procedure. The circle contains three steps. The first, called the system or entity, represents

that which is to be modeled within the real world. In Sargent's words "The problem entity is the system (real or proposed), idea, situation, policy, or phenomena to be modeled" [171, 169, 172, 170]. This is followed by the conceptual model. The conceptual model was earlier presented as the concept for representing the entity as a model that exists within the mind of the modeler [19, 15, 20, 25]. A more formal and modern definition of a conceptual model is that it contains a description of what is to be modeled accompanied by the objectives, inputs, outputs, content, and assumptions [171, 152, 46, 156, 94, 172, 22, 18, 23, 48]. Finally, the computerized model is the conceptual model implemented onto a computer [171, 169, 172, 170].

The Sargent Circle is a very simple depiction of model development and V&V. The figure is very useful for communicating how the two are related; therefore, it is understandable that the circle is considered poor for the criterion of completeness. The procedure contains the major steps; however, it is missing the problem formulation, application, documentation, and maintenance. The other steps that are included under completeness can be considered as part of the steps shown in the circle. The Sargent Circle is considered to be very good when considering the criterion of iterative. The procedure displays global iteration and local iteration between each step. Due to the simplicity of the procedure the traceability is considered poor. A breakup of the steps shown would aid in the traceability of the procedure. Finally, the flexibility of the procedure is considered to be very good. The simplicity of the procedure is a great benefit to the flexibility.

### 2.2.2 The Shannon Life Cycle

Robert Shannon presents another model development procedure in his 1975 book *System Simulation: The Art and Science* [178]. Shannon's Life Cycle is shown in Figure 9. Shannon begins his procedure as all procedures should be started: with the formulation of the problem. This is the process of determining exactly the problem that is to be studied. He states that "the research team must understand and articulate a set of germane objectives and goals" [178]. He further states that formulating the problem is a continuous process that is conducted throughout the study. This step is followed by system definition. He defines



**Figure 8:** Sargent Circle

the system as a combination of subsystems and a part of a larger system. He identifies two functional boundaries of the system. The first is the boundary that separates the problem from the rest of the universe. The second is the boundary that separates the system of interest from the environment. After defining the boundaries Shannon suggest the use of logical flow diagrams or static models. Once this is done it must be determined whether simulation is still required to answer the questions of the system. If so the next step is the model formulation. This is similar to conceptual modeling of the other procedures. The next step is data preparation which is a process of gathering the relevant data and determining how to use the data. Model translation is the process of developing the computerized model. This becomes the computer model that is able to execute the simulations. Shannon's procedure then hits a validation check point. Though the block is stated as a validation activity, verification is also included. If this step fails, the modeler is to return to previous steps to right the issues; however, it is unclear to which step the modeler must return. It is assumed that this is left open to the situation at hand. After passing the validation check, strategic planning is addressed. In this step the experiments are designed to answer the questions posed to the system. Tactical planning is concerned with how to execute each of the experiments identified in strategic planning. The experimentation step is the execution of the experiments. Included in this step is sensitivity analysis of the model

parameters. The final check of interpretation is to determine whether the results are useful. If not, then the process returns to an earlier state. It is unclear to which step the process should return; however, this author believes that this was left vague by design. The final two steps are implementation and documentation. Once the results are available, they must be put into action. This constitutes the implementation step. Finally, no model development is complete without documentation of the assumptions, development, and results of the study.

Shannon's procedure for model development is mostly complete containing many of the steps of interest; however, it is missing the maintenance phase. For this reason the procedure is given a value of good for the metric of completeness. There are many feedback loops for the procedure; however, they primarily exist only if there is an observed problem. Additionally, it is unclear where the feedback loops are supposed to point. For these reasons, the iterative nature of the procedure is considered to be moderate. The traceability of Shannon's procedure is considered good. There are many steps and multiple check points that aid in the traceability of the model development. Finally, the flexibility is also considered good. The procedure does a good job in defining what must be done without limiting its application.

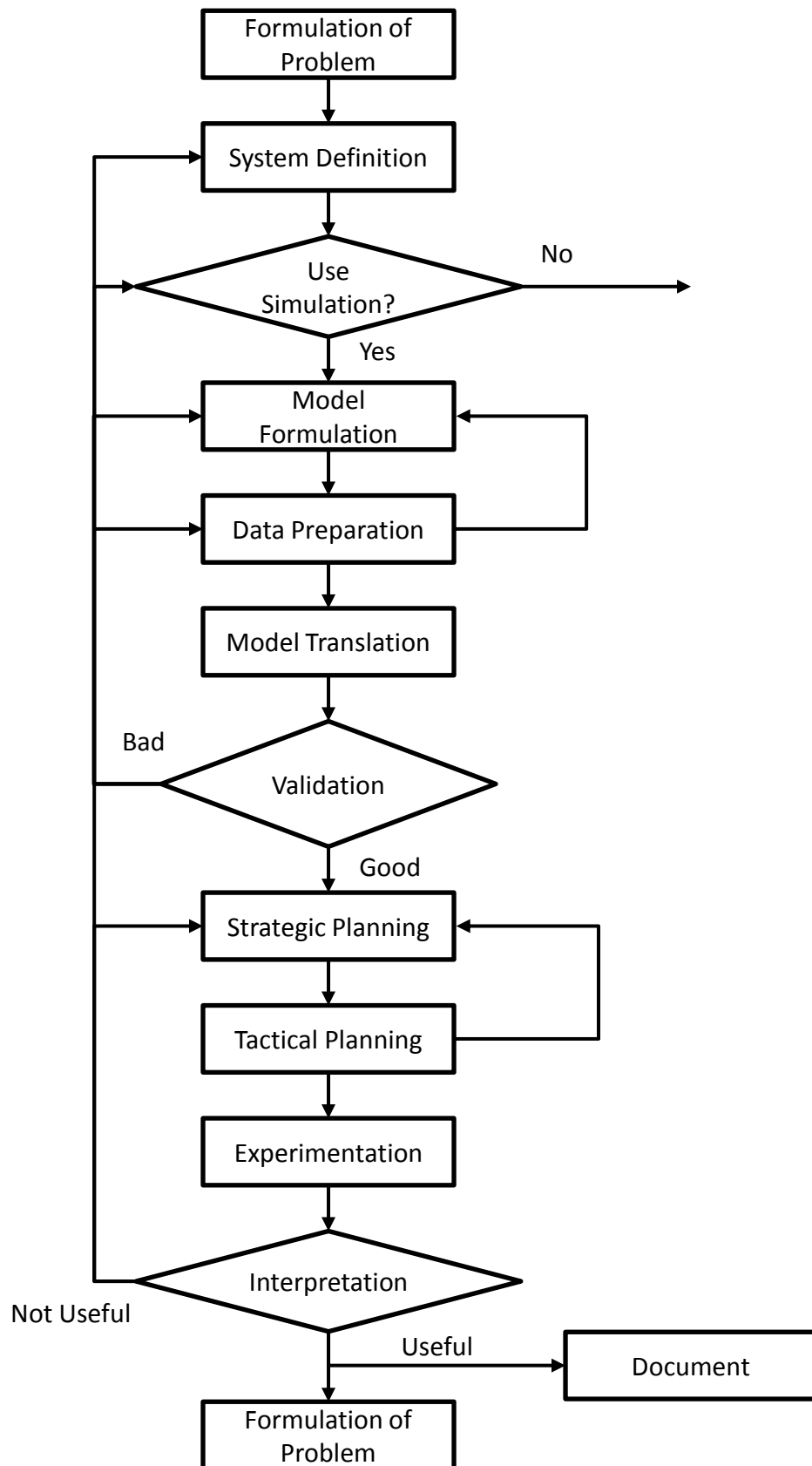


Figure 9: Shannon's Life Cycle

### 2.2.3 The Evolved Sargent Circle

Sargent produced a more complicated description of model development and V&V in 2001 to help the American Society of Mechanical Engineers develop a V&V guide for computational solid mechanics [136, 137]. This is sometimes referred to as the Evolved Sargent Circle [137] and is shown in Figure 10. This procedure shows the two worlds, which are the real world and the simulation world. The real world deals with experimentation with the actual entity. The simulation world deals with model development based on the theories of the entity that have been gathered through abstraction and testing the real system. This section is more concerned with the simulation world which begins with system theories. System theories include characteristics, causal relationships, and the behavior of the entity or system under study [169, 172, 170]. This is then followed by the conceptual model. Sargent describes the conceptual model as a "mathematical/logical/graphical of the system developed for the objectives of a particular study" [170]. The next step is simulation model specification which is a description of the software design. The simulation model is the computer programmed model that is based on the conceptual model and is capable of simulating the system in question. The final step is the simulation model data/results. This is the output from the simulation based on the experiments run.

When evaluating the Evolved Sargent Circle, many obvious improvements are made over the Sargent Circle. First, there is an improvement in the completeness of the procedure; however, it is still missing documentation, presentation, and maintenance. This is understandable, considering the circle is intended to aid in V&V. The completeness is rated as moderate. The iterative criterion remains very good for displaying both local and global iteration. It should be noted that the evolved circle does not necessarily show local iteration. The local iteration is assumed to be implied, because both circles are often discussed in the same paper [169, 172, 170]. The added steps in the evolved circle over the simple circle improve the traceability of this procedure. The addition of how the model interacts with real world testing and how that generates knowledge about a system is a benefit; therefore, the procedure is rated as a good for traceable. Finally, the circle is very generic and can be applied to a wide variety of problems; therefore, the procedure is rated as very good for

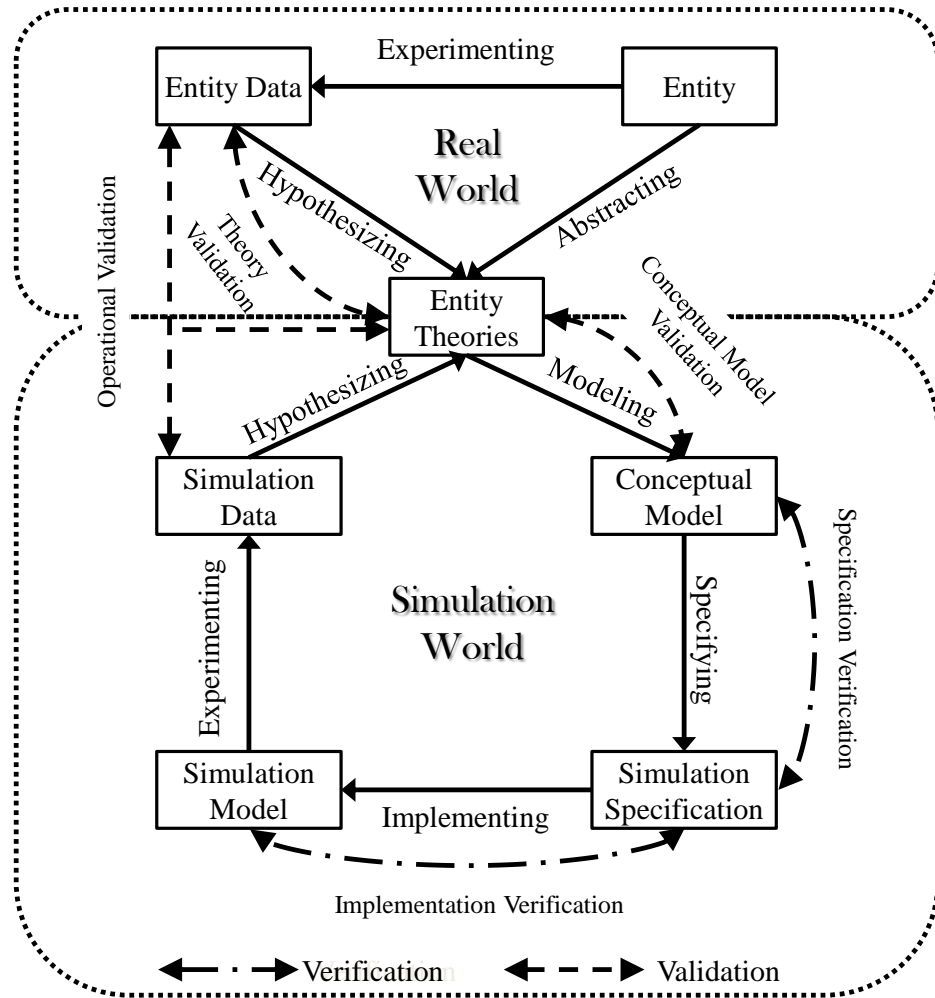


Figure 10: Evolved Sargent Circle

the criterion of flexibility.

#### 2.2.4 Banks' Steps in a Simulation Study

Banks provides a very famous and useful model development procedure [29, 27, 28], which contains 12 steps and is shown in Figure 11. Banks' approach starts with problem formulation. Banks describes problem formulation as a statement of the problem accompanied by the assumptions which are agreed upon by the client and analyst. The second step, setting of objectives and overall project plan, is a proposal of what is to be completed. The objectives state the questions that the simulation will attempt to answer. Statements on

the assumptions of the study should be included in this step. In addition, managerial information is included in this step, e.g. time scale, manpower needed, hardware requirements, software requirements. The next two steps are conducted in parallel. The model conceptualization step is simply the development of the conceptual model. The data collection step involves pulling real world data that can be used as an input for the model. Banks approaches model development from the standpoint of industrial systems, e.g. manufacturing, warehouses. In this approach statistics would be collected from current existing systems, e.g. process times, travel times. The next step of model translation is the process of programming the model. The next two steps are verification and validation. The eighth step, experimental design, is the process of defining what inputs are to be varied, the number of runs required, the duration of the simulation, etc. The runs are then analyzed to estimate the defined measures of performance. It may be found that more runs are required, at which point the previous step is repeated. Once the analysis is completed with adequate number of runs, the 11th step is conducted, which is documentation and reporting. This involves reporting the analysis to the client and creating documentation about the model to enable reuse. The final step, implementation, is the action made in the real world. This is based on the findings of the simulation study.

Banks' Steps in a Simulation Study is a phenomenal starting point for the development of a model for analysis. Banks' procedure is mostly complete, including all primary steps excluding the final phase of maintenance. Banks' procedure is given a value of good for the criterion of completeness. This author maintains two points of disagreement about the order of steps. The assumptions of the model are made before the conceptual model is developed; however, this author considers the development of a conceptual model and the stating of assumptions as linked and should be considered in the same step, i.e., assumptions should be made in step three. The second point of disagreement is that the collection of data should follow the development of a conceptual model. Until a conceptual model is developed it is uncertain what data will be needed. Different conceptual models will require different sets of data. The iterative nature of Banks' approach is judged to be poor. Banks' procedure does not display the full picture of the iterative nature of model development.



The iteration on the model development only occurs between V&V and model translation and conceptualization, respectively. It is very possible, and arguably common, for the objectives and assumptions to be revisited after the analysis has been performed as more is discovered about the system. The traceability of this procedure is moderate. The structure is there to provide traceability; however, V&V occurs only once in the procedure and the objectives and analysis is combined in one step. Thus, the traceability is only moderate. The iterability and traceability of the procedure can be improved by adding additional steps that detail the more complicated steps, adding additional feedback loops between analysis and assumptions, and adding V&V checks throughout the procedure. The procedure is very strong on its flexibility. The procedure is considered to be very good and can be applied to a wide variety of modeling paradigms.

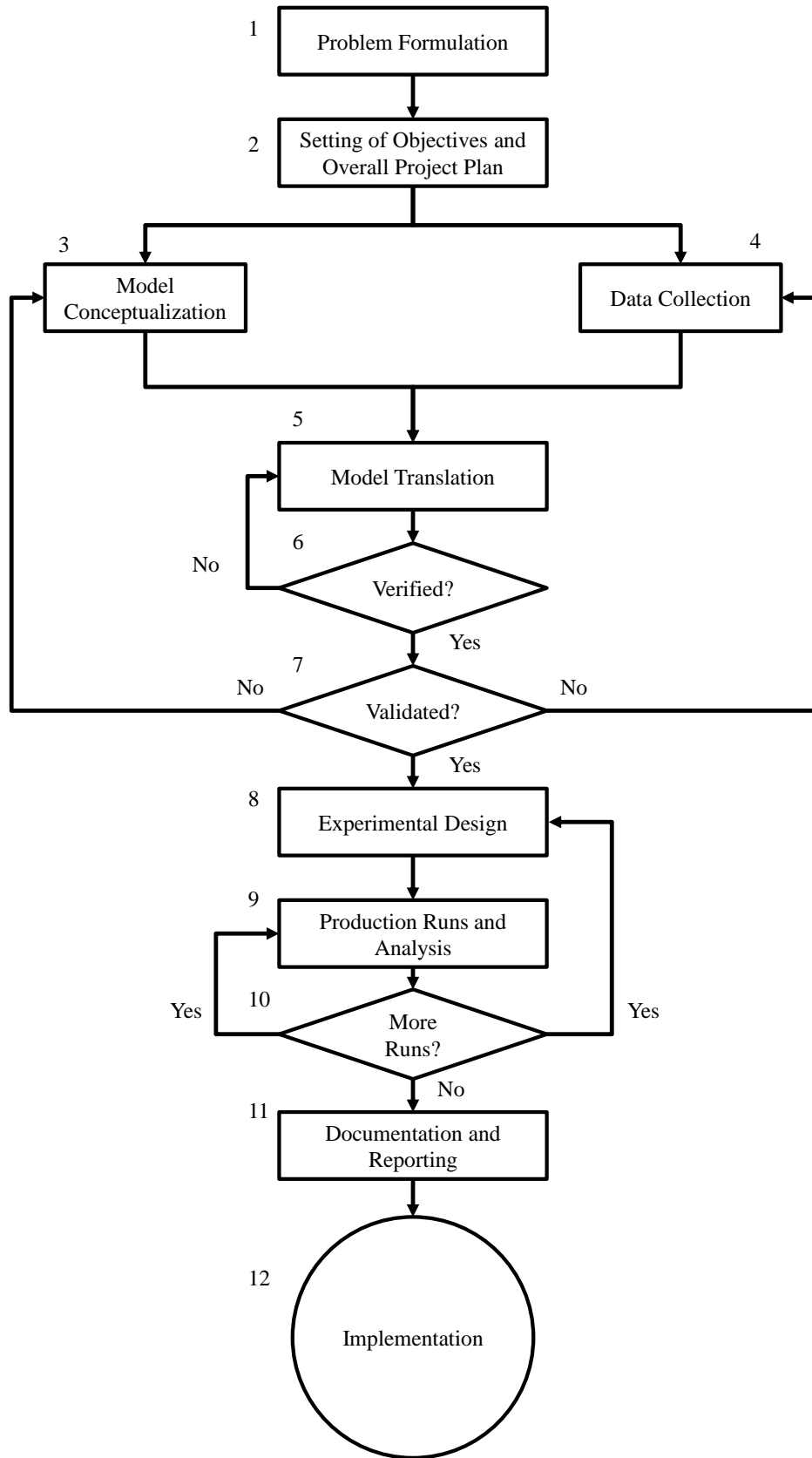


Figure 11: Steps in a Simulation Study by Banks

### 2.2.5 Law's Steps in a Simulation Study

Averill Law presents another procedure involved in model development and use in his famous book *Simulation Modeling & Analysis* [100]. Law's steps are very similar to those presented by Banks and can be seen in Figure 12 [98, 99, 100]. The first step is to formulate problem and plan the study. Included in this step are determining the objectives, questions to be answered, the scope, and time frame of the study. Additionally, Law suggests that the software to be used is selected in this step. The second step is collect data and define a model. This is similar to a combination of Banks' model conceptualization and data collection. Law instructs for data collection to collect information on the system, its operations, and its performance. In regards to model design Law suggests starting with simple models and 'embellish it as needed'[100]. The conceptual model and assumptions are documented here. Law's third step is a validation check on the assumptions made on the model. This check is preceded by programming the defined model. Next, a verification check is made to determine if bugs exist. Once the model is verified, pilot runs are made for validation purposes. After validation, the experiments are designed. This includes defining the length of the simulation runs, the number of repetitions required, and the system configuration that will be experimented. These experiments are then conducted and used in analysis in step nine - analyze output data. Law defines two major objectives of the analysis: determine absolute performance and compare alternative system configurations. Law ends with a documentation and presentation step. He states that the assumptions, the results, and the model should be documented for potential future projects. For presentation, animations are suggested to foster communication. Additionally, Law suggests that the model development process and the validation process should be presented in an effort to promote credibility.

Law's Steps in a Simulation Study comes up short compared to Banks' procedure for the type of problems addressed in this thesis. Law's procedure is mostly complete; however, many of the desired steps exist implicitly within a larger step. For example, the first step can be broken into problem formulation, objective definition, and system decomposition. Conversely, Law combines CM development and data collection, an improvement over Bank's procedure. This procedure also omits the steps in the maintenance phase of

model development. The completeness of Law's approach is judged to be moderate. Law's iterability is an improvement over Banks' in that the assumptions are validated. This also aids in the traceability of the model; however, this approach is only considered moderate for being iterative and traceable. The iteration on the model development only occurs between V&V and model translation and conceptualization, respectively with assumption validation. It is very possible, and arguably common, for the objectives and assumptions to be revisited after the analysis has been performed and more is discovered about the system. This approach is considered to have poor iteration. Though this procedure can be made for traceability, it does not reinforce traceability. The traceability of Law's procedure is moderate. The iterability and traceability of the procedure can be improved by adding additional steps that detail the more complicated steps, adding additional feedback loops between analysis and assumptions, and adding V&V checks throughout the procedure. Finally, this approach is not as flexible based on when the selection of the modeling approach is made. Selecting the type of model to use in the first step would require the application of this procedure to a very familiar and common problem. This is not the case here. Studies on new and unfamiliar systems will require the development of a conceptual model before a modeling approach can be decided. Despite this shortfall, Law's procedure is considered moderately flexible due to its broad and encompassing steps.

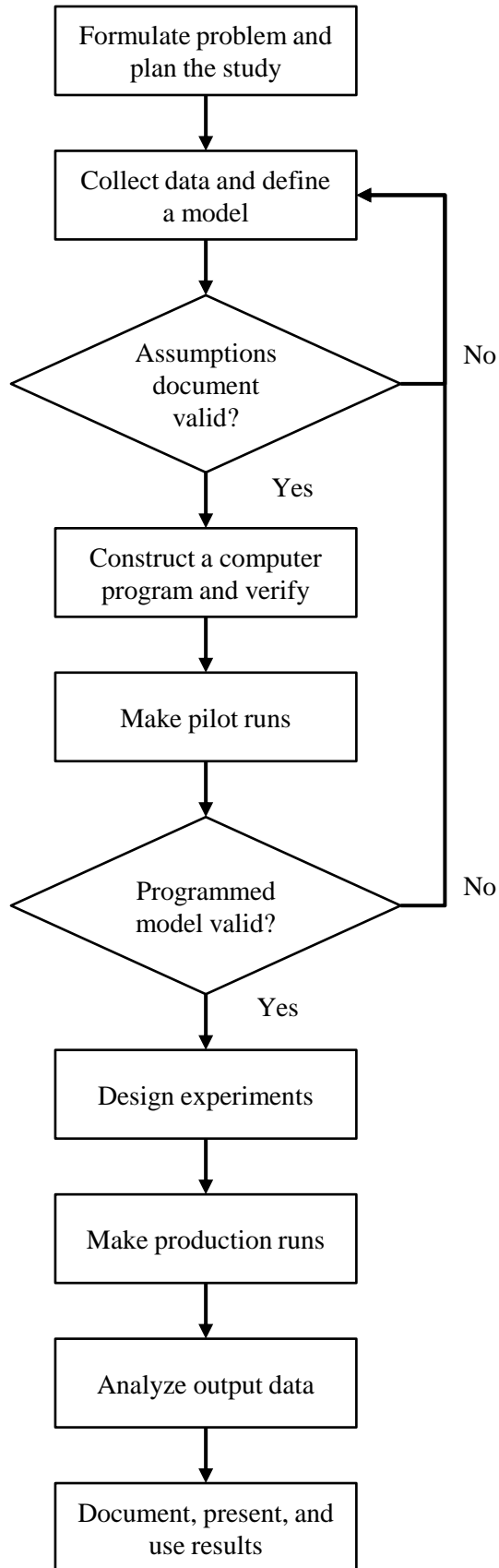


Figure 12: Steps in a Simulation Study by Law

### 2.2.6 Balci's 1986 Simulation Life-Cycle

Osman Balci presents a phenomenal procedure called the Life Cycle of a Simulation Study shown in Figure 13 [19, 15, 20, 122]. This description largely evolved through collaboration with Nance in the early 1980's [121, 19, 122]. Balci's Simulation Life Cycle contains three phases, 10 processes, 10 steps, and 13 credibility assessment stages. A unique difference about Balci's procedure compared to the previous ones is that the steps are more like sign posts along the way and the work exists along the arrows connecting the steps. These are referred to as processes. Beginning the problem definition stage is the communicated problem. The communicated problem is the problem that is expressed by the customer and is rarely clear, specific, or organized. Once the communicated problem is received the process of problem formulation is conducted. Balci describes this as the process of translating the communicated problem into a well defined problem that will clearly state the objectives and requirements of the simulation study. It is sometimes also referred to as problem structuring or problem definition. The well formed problem is referred to as the formulated problem. From the formulated problem solution techniques are investigated. It is important to note that problems can be solved by other means than simulation, despite simulation being the subject of this thesis and Balci's Simulation Life Cycle. This thesis is focused on modeling and simulation, therefore a simulation solution is assumed. A system investigation then takes place. Balci invokes Shannon's six major system characteristics: change, environment, counter-intuitive behavior, drift to low performance, interdependence, and organization [178]. This is largely hinged on industrial systems. Other applications may require different system investigation. Additionally, the objectives of the simulation study must be clearly stated. Once the system is decomposed into subsystems and its characteristics are documented, the development of the conceptual model can be started. This is called model formulation. From Nance's work, which Balci's approach is derived, the conceptual model is defined as the following:

“The conceptual model is that model which exists in the mind of the modeler.

The form of the conceptual model is influenced by the system, the perceptions of

the system held by the modeler (which are affected by the modelers background and experience and those external factors affecting the particular modelling task), and the objectives of the study” [120, 119].

The next step is to create a communicated model through the process of model representation. A communicative model is one that can be used to communicate to other humans the conceptual model so that it can be judged and compared to the actual system. The communicative model must be based on: the applicability of describing the system, the technical background of audience, its ability to lend itself to formal analysis and verification, its ability to support model documentation, its maintainability, and its translatability into the programmed model. Once the communicated model is accepted, the model is programmed creating the programmed model. This is the executable model that provides simulation. Plans are then made to experiment with the programmed model using a Design of Experiment (DOE). A DOE is a group of experiments that makes purposeful changes in order to gather the greatest amount of information with the smallest effort. The set up of the model, such that it can execute the DOE, is referred to as the experimental model. The process of experiments are the actual runs of the simulation, which leads to simulation results. The process of redefinition is one of updating or modifying the model for other uses or to better represent the true system. The final step is to interpret and present the findings to the decision makers. This is done to aid the decision maker in the decision making process.

Balci’s Life Cycle of a Simulation Study performs very well for the evaluation criteria for model development and use procedures. This procedure is mostly complete containing all initially required steps with the addition of several others; however, it is lacking the phase for maintenance. The procedure is given the rating of good for completeness. This procedure is primarily designed to be iterative. There is an extensive use of local and global iteration; therefore, the procedure is rated as very good for its iterativeness. The extensive use of V&V throughout the procedure and the many steps shown greatly improves the traceability of model development. Traceability is rated as very good. Finally, the procedure can be used for the various modeling paradigms and even includes a step that mentions that other methods can be used for analysis beyond simulation. The increased number of steps in

Balic's procedure is a great benefit; however, it does make it less generic. Therefore, the procedure is rated as good for its flexibility.



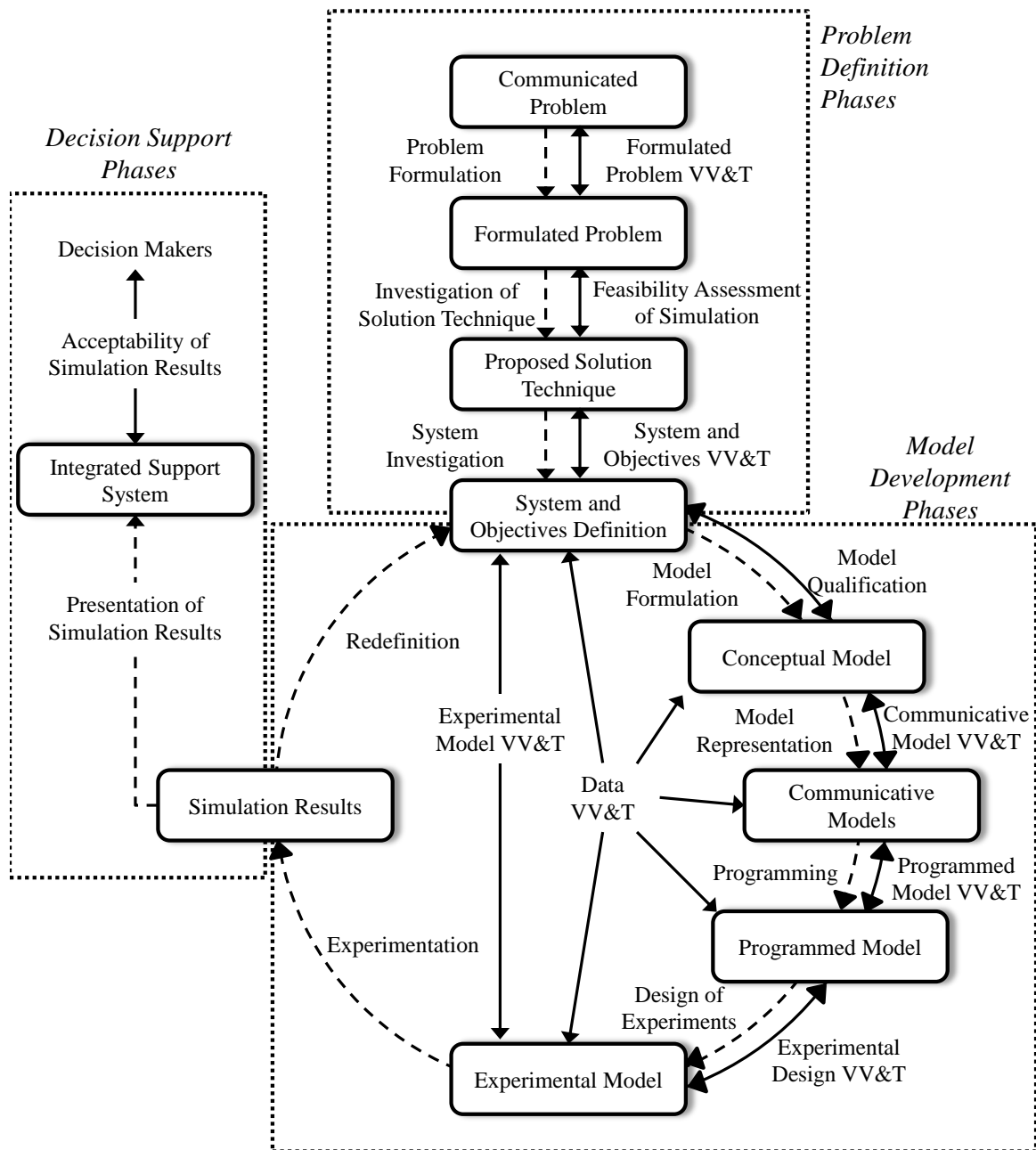


Figure 13: Balci's 1986 Simulation Life-Cycle

### 2.2.7 Balci's 2011 Simulation Life-Cycle

In 2011 Balci released an updated life cycle of a simulation study, which can be seen in Figure 14 [22, 23]. There are many similarities to Balci's older simulation life cycle. In the name of brevity only the differences will be highlighted. This approach begins with a universe of discourse, which is the problem domain for a community of interest. Balci uses the example of a model developed for Ballistic Missile Defense (BMD). The universe of discourse will refer to all entities and theories involved in BMD. The formulated problem is similar to the one stated above; however, instead of translating a poorly defined problem into a formal one, this focuses on analysing the universe of discourse in order to develop the formulated problem. The requirements specification is similar to the system and objectives definition. The former life cycle places greater emphasis on the understanding of the system where as that task is addressed by the universe of discourse and formulated problem in this procedure. The requirements specification places greater focus on how the simulation will actually function. Balci suggests specifying the M&S intended uses, use cases, non-functional requirements, functional requirements for the Use Cases, and overall requirements. The next difference is the architecture specification. Here the organization of how the simulation components interoperate is defined. This can be communication over a network or among the simulation components. Example architectures include the Department of Defense Architecture (DODAF) or the High Level Architecture (HLA) [22, 23]. The next stage is simply the instantiation from the selected architecture. This includes the decomposition of the simulation into the various levels of the submodels. The executable submodels and the simulation model are similar to the programmed model from Balci's earlier work. The difference is that here the process is broken into the programming of the submodels and the integration of the submodels to produce a full model for simulation. The simulation results is a combination of the experimental model and simulation results from the former work. The presented results is similar to the integrated decision support from the former work. The major advances that Balci's newer work provides is the inclusion of the certified simulation model and the repository of certified simulation models. It is important to note the need of certification beyond in house V&V in the life-cycle of a model.

In addition, Balci notes how the model should be used and reused by the community, a step that is overlooked by other approaches.

In general, Balci's latest procedure offers the greatest overview of the steps in the model's life cycle. The procedure is very extensive, including all desired steps and additional ones. Therefore, this procedure is the only one presented that is fully complete. The inclusion of the role of the model in the realm of the greater community is a good addition as well. The completeness of this procedure is considered to be very good. The one criterion that has regressed compared to the older procedure is iterability. This new procedure does not include the iteration between the simulation results and the requirements specification. The approach includes numerous iterations between each step; however, the macro level interaction is absent. Balci makes a statement that the user can iterate as desired; however, the statement is not a sufficient substitute for explicit feedback loops. Thus, the iteration on Balci's new procedure is good. Similar to Balci's former approach, this procedure is highly traceable. The many steps and V&V activities make the procedure's traceable measure very good. Finally, the procedure is lacking in flexibility. The new procedure is not as flexible as Balci's former approach. The inclusion of a universe of discourse, architectures, certification, and certified model repositories make the procedure less flexible. The inclusion of all of these steps will only be needed for certain domains of study, e.g. military analysis, industrial systems, and not others, where there is not a centralized organization or a homogeneous set of modeling efforts, e.g. biological sciences. The flexibility of Balci's 2011 procedure is considered poor.

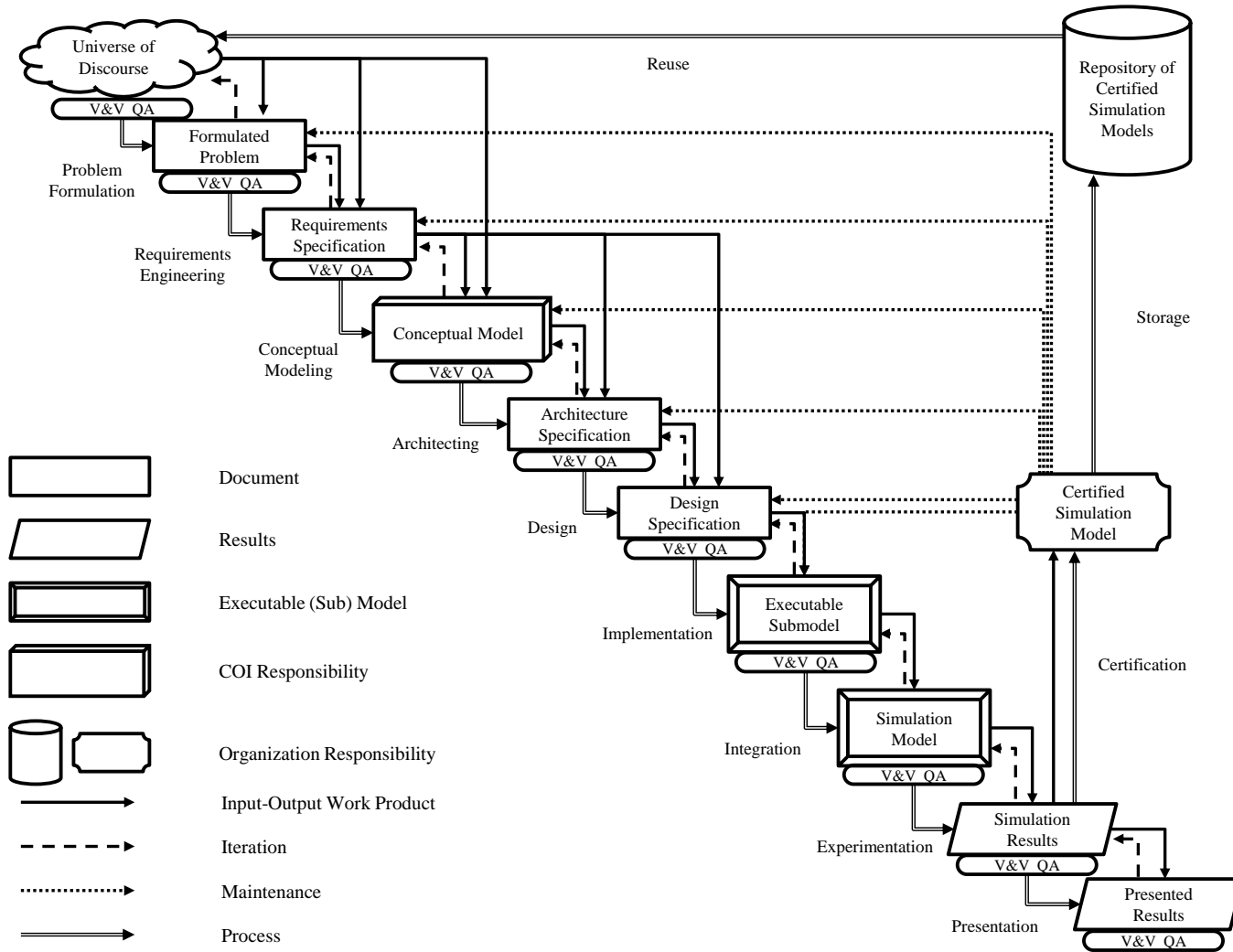


Figure 14: Balci's 2011 Simulation Life Cycle

### *2.3 Selection of a Model Development Procedure*

A selection needs to be made on which procedure should be used for the development of models for non-observable systems. This procedure will then form the backbone of the proceeding discussion on model development methodologies, i.e., how the procedure is enacted. This decision will be made utilizing literature available in the field of study referred to as Multi Attribute Decision Making (MADM). MADM methods help identify the best solution from a finite set of alternatives [105]. Many techniques are available in the literature. Of the available techniques the Analytical Hierarchy Process (AHP) was selected. AHP was developed in 1970 by Thomas Saaty with the intention to facilitate decision making for problems with a hierarchical structure of attributes. AHP is a widely used technique spanning over forty years [164, 165, 166]. The benefit of the use of AHP in this application is its reliance on ratio scaling. This information can be provided based on conclusions from the literature survey. AHP has been criticized for some of its short comings. The major criticisms of AHP are rank reversal [61, 60, 195] and accurately assigning appropriate weightings or ratios [210, 36]. Despite the criticisms AHP has proven its usefulness and remains one of the most popular decision making techniques. AHP uses qualitative pairwise comparisons for each attribute and for each alternative presented. The procedure for AHP is detailed below from Saaty's 2008 article [166].

1. Define the problem and determine the kind of knowledge sought.
2. Structure the decision hierarchy from the top with the goal of the decision, then the objectives from a broad perspective, through the intermediate levels (criteria on which subsequent elements depend) to the lowest level (which usually is a set of the alternatives).
3. Construct a set of pairwise comparison matrices. Each element in an upper level is used to compare the elements in the level immediately below with respect to it.
4. Use the priorities obtained from the comparisons to weigh the priorities in the level immediately below. Do this for every element. Then for each

element in the level below add its weighed values and obtain its overall or global priority. Continue this process of weighing and adding until the final priorities of the alternatives in the bottom most level are obtained.

Following Saaty's approach, the problem is defined as to determine the best model development procedure available in the literature for the application of model development for non-observable systems. The criteria that are being used to determine the best model development procedure are complete, iterative, traceable, and flexible. The alternatives that are being weighed are the Sargent Circle, the Evolved Sargent Circle, Shannon's Procedure, Bank's procedure, Law's procedure, Balci's 1986 procedure, and Balci's 2011 procedure. The pairwise comparisons are made using the scale shown in Table 2.

**Table 2:** Pairwise Comparison Scale [165]

<b>Verbal Judgement of Preference</b>	<b>Numerical Rating</b>
Extremely Preferred	9
Very Strong to Extremely	8
Very Strongly Preferred	7
Strongly to Very Strongly	6
Strongly Preferred	5
Moderately to Strongly	4
Moderately Preferred	3
Equally to Moderately	2
Equally Preferred	1

The first pairwise comparison made is that of the criteria shown in Table 3. The values were determined through the following process. The least important criterion was selected to be the flexibility of the procedure. The primary purpose of this thesis is the investigation of how to construct models for non-observable systems. It is beneficial to have a procedure that is flexible; however, it is impossible to develop a procedure for every possible study that requires simulation. Thus, flexibility was determined to be the least important. Next it was determined that a procedure that is iterative is of greatest importance. The development of models is naturally an iterative process [171, 20, 152, 149, 153, 172]; therefore, a procedure must reflect that. In the absence of real world validation data, the traceability of a model development procedure is critical; therefore, the traceability of a procedure is

considered to be very important but not as important as iterative. Finally, completeness was addressed. Completeness becomes very important when critical processes are missing; however, once all the critical processes are included in the procedure, either as a step or within a step, completeness loses its importance. Thus, completeness was considered of medium importance. The priority of each criterion was determined through the following process. Each column was divided through by the sum of the column. Each row was then averaged to determine the priority value. This value is seen in the far right column of the table.

**Table 3:** Pairwise Comparison of Criteria

<i>Criterion</i>	Complete	Iterative	Traceable	Flexible	<i>Priority</i>
Complete	1	1/3	1/5	4	0.13
Iterative	3	1	1/3	6	0.26
Traceable	5	3	1	9	0.57
Flexible	1/4	1/6	1/9	1	0.05

The next pairwise comparison made would be for each of the optional procedures under the criterion of completeness shown in Table 4. The Sargent Circle, labeled simple, was found to be the least complete model development procedure. Shannon's and Banks' procedure were considered to be equally good for completeness. These two procedure barely outperformed Sargent's Evolved Circle and Law's model development procedure. Banks' procedure was considered equally to moderately preferred to Law's procedure. This is mainly due to the fact that Banks breaks up the steps more than Law. Due to the extensive amount of detail provided by Balci's two procedures, they were considered to be extremely preferred over the other procedures in terms of completeness. Of Balci's two procedures, his 2011 procedure was considered to be moderately more complete. This is due to the fact that it includes the final phase of the model life cycle, maintenance, and incorporates the concept of the universe of discourse.

The next pairwise comparison made would be for each of the optional procedures under the criterion of iterative shown in Table 5. Banks' procedure was considered the least iterative. Law's procedure was considered equally or moderately more iterative than Banks'

**Table 4:** Pairwise Comparison of Complete

<i>Complete</i>	Simple	Shannon	Evolved	Banks	Law	Balci 1986	Balci 2011	<i>Priority</i>
Simple	1	1/5	1/3	1/5	1/2	1/9	1/9	0.02
Shannon	5	1	3	1	2	1/8	1/9	0.08
Evolved	3	1/3	1	1/3	1	1/7	1/9	0.04
Banks	5	1	3	1	2	1/8	1/9	0.08
Law	2	1/2	1	1/2	1	1/7	1/9	0.04
Balci 1986	9	8	7	8	7	1	1/3	0.29
Balci 2011	9	9	9	9	9	3	1	0.44

due to the fact that Law has a few more V&V checks. This is followed by Shannon's procedure which had even more V&V checks; however, these three represent the bottom of the procedures. The best was found to be the Sargent Circle closely followed by Balci's 1986 approach. Balci's 2011 approach fell due to a lack of global iteration.

**Table 5:** Pairwise Comparison of Iterative

<i>Iterative</i>	Simple	Shannon	Evolved	Banks	Law	Balci 1986	Balci 2011	<i>Priority</i>
Simple	1	5	2	9	9	1	4	0.29
Shannon	1/5	1	1/3	3	4	1/7	1/7	0.06
Evolved	1/2	3	1	7	7	1	2	0.19
Banks	1/9	1/3	1/7	1	1/2	1/9	1/7	0.02
Law	1/9	1/4	1/7	2	1	1/8	1/7	0.03
Balci 1986	1	7	1	9	9	1	4	0.27
Balci 2011	1/4	7	1/2	7	7	1/4	1	0.14

The next pairwise comparison made would be for each of the optional procedures under the criterion of traceable shown in Table 6. Balci's procedures are considered to be extremely traceable due to their extensive number of steps displayed and the number validation and verification checks. Balci's 2011 procedure is moderately more traceable than his 1986 procedure due to the increased number of steps. Obviously the simple Sargent Circle



performs the worst under this criterion. The other procedures all perform equally poor for this criterion in comparison to Balci's procedures. Law's procedure and Sargent's Evolved Circle are considered to be moderately preferred over Banks' and Shannon's procedure because Law and Sargent provide an additional validation check for the assumptions of the conceptual model.

**Table 6:** Pairwise Comparison of Traceable

<i>Traceable</i>	Simple	Shannon	Evolved	Banks	Law	Balci 1986	Balci 2011	<i>Priority</i>
Simple	1	1/5	1/4	1/5	1/2	1/9	1/9	0.03
Shannon	5	1	2	1/3	1/3	1/9	1/9	0.04
Evolved	4	1/2	1	2	2	1/9	1/9	0.05
Banks	5	1	3	1	2	1/8	1/9	0.04
Law	2	3	1/2	1/2	1	1/7	1/9	0.06
Balci 1986	9	9	9	8	7	1	1/3	0.39
Balci 2011	9	9	9	9	9	3	1	0.39

Finally, pairwise comparisons are made for each of the optional procedures under the criterion of flexible shown in Table 7. Though Balci's 2011 procedure has performed well in other categories, it is not considered very flexible. This is mainly due to the immense detail given in the steps of the procedure. Law's procedure also performs poorly due to the fact that the modeling paradigm is selected too early in the development process. The best performing ones are obviously the most generic: Sargent's Simple and Evolved Circle. Shannon's and Bank's procedure performed equally well, and they are considered barely better than Balci's 1986 procedure due to the fewer steps involved.

The final calculation of AHP to determine the best procedure requires the multiplication of the priorities of each option for each criterion to the corresponding criterion priority. These values are then summed. The final score results can be seen in Table 8. From the analysis it was concluded that Balci's 1986 procedure is best suited for the development of models of non-observable systems. Now, it must be determined whether this procedure is sufficient for continued use in addressing the research objective.

**Table 7:** Pairwise Comparison of Flexible

<i>Flexible</i>	Simple	Shannon	Evolved	Banks	Law	Balci 1986	Balci 2011	<i>Priority</i>
Simple	1	5	1	5	9	7	9	0.31
Shannon	1/5	1	1/5	1	8	2	4	0.12
Evolved	1	5	1	5	9	7	9	0.31
Banks	1/5	1	1/5	1	8	2	4	0.12
Law	1/9	1/8	1/9	1/8	1	1/7	3	0.03
Balci 1986	1/7	1/2	1/7	1/2	6	1	3	0.08
Balci 2011	1/9	1/4	1/9	1/4	1/3	1/3	1	0.02

**Table 8:** AHP Score

	Simple	Shannon	Evolved	Banks	Law	Balci 1986	Balci 2011
Score	0.1095	0.0512	0.0953	0.0419	0.0507	0.3347	0.3167

#### 2.4 Assessment of Balci's 1986 Procedure

Balci's 1986 Simulation Life Cycle was selected as the best suited procedure for model development and use of non-observable systems. Though it is the best, it still must be determined if the procedure is sufficient for these types of models. The primary concerns were for a procedure that showed high levels of traceability and iterability. This is a direct result of the research objective to develop a methodology that can be traceable and defensible and aids in the validation of the model. In the literature review of Balci's 1986 work, no shortcomings were found in respect to these criteria. The only major shortcoming of Balci's work is the absence of the maintenance phase of the model life cycle. This is not considered to be a major problem given that the focus is on model development. In conclusion, it is determined that Balci's 1986 Simulation Life Cycle is sufficient for the use as the procedure for model development and use of non-observable systems.

**Research Question 1 Hypothesis:** Balci's 1986 model development procedure is the best suited procedure for the development of models for the identified type of problems.

## CHAPTER III

### PROBLEM FORMULATION THROUGH SYSTEM DEFINITION

If I had an hour to solve a problem and my life depended on the solution, I would spend the first 55 minutes determining the proper question to ask, for once I know the proper question, I could solve the problem in less than five minutes.  
(Albert Einstein)

#### ***3.1 Problem Formulation***

The problem definition phase is the most important phase in any model development effort. This phase involves taking the communicated problem from the customer or community of interest and translating it into a clear and specific problem that is to be addressed. In the end of the problem definition phase, the correct problem must be identified for which the simulation study will be tasked with answering. The failure to identify the true problem greatly affects the acceptability and credibility of the model. In the event that the wrong problem is identified, then the wrong model is built. Solving the wrong problem is referred to as a Type III error [22, 23]. It is critical that this phase is conducted correctly and the modeler and client must agree on the stated problem that the model will address [28]. Balci offers a series of steps that can be taken to help formulate the problem. The six step process is reproduced below from Balci's 2012 paper [23].

1. Establish the problem domain boundary
2. Gather data and information about the problem domain within the established boundary
3. Identify the stakeholders and decision makers who would be interested in the solution of the communicated problem
4. Specify the needs and objectives of the stakeholders and decision makers identified

5. Identify and specify the constraints
6. Specify all assumptions made clearly and explicitly

The importance of problem definition cannot be overstated. Any reputable procedure will begin with and emphasize this phase, as evidenced by the model development and use procedures presented in the previous chapter. The M&S community recognizes the importance of the problem definition phase and no shortcomings were identified in the literature. Therefore no research questions are posed for the problem definition phase.

### ***3.2 System and Objectives Definition***

Within his found published work, Balci does not go into specific detail about the intent of the objective definition. The following is inferred from Balci's numerous papers and statements made by other prominent figures in the community. Concepts from his newest life cycle are also utilized. It can be argued that the requirements engineering step in Balci's 2011 life cycle for modeling and simulation is an improvement to the objectives definition proposed in his 1986 life cycle.

The objectives definition is a clear statement of the purpose of the simulation study and flows from the problem definition phase. The objective definition can be seen as a programmatic statement guiding the model development efforts. Included in the objectives definition are statements on: the overall study objective, specific questions to be answered, the time frame allowed for the study, and available resources to complete the study [122, 98, 99]. This end product will then act as an agreement between the model developer and the model customer. Balci encourages the inclusion of an M&S acceptability criterion to avoid conflict in the final stages of the study [22, 23].

The guidelines for the development of an objective definition are vague; however, this is by necessity. The set of possible studies is so vast that no general outline can be given in a methodology that will perfectly address every situation. Instead, this step must be accomplished in its unique way for each study that arises. Instead, a methodology should offer a minimum set of necessary statements to be included in an objective definition. The minimum set of necessary statements are study objective, questions to be answered, time

frame, and available resources.

System definition is concerned with identifying the system of interest and decomposing the system into its subsystems such that its complexity can be managed and translated into a model formulation. Balci addresses system definition through the characteristics of the system to be modeled. He cites Shannon's six major system characteristics: change, environment, counterintuitive behavior, drift to low performance, interdependence, and organization [178, 19, 20]. Exploring Shannon's ideas further, he proposes two important boundaries for the system of interest. The first is the boundary that separates the problem from the rest of the universe. The second is the boundary that separates the system of interest with the environment. There are numerous methods by which this step can be accomplished. This leads to the second research question.

**Research Question 2:** How should a system and objective definition be developed that defines the necessary aspects of the entity and enables translation into conceptual modeling?

After a review of the literature, a number of approaches are available that can contribute to system and objective definition. These include Soft Systems Methodology (SSM), Unified Modeling Language (UML), Systems Modeling Language (SysML), and Department of Defense Architecture Framework (DoDAF). The last three fall into the field of Requirements Engineering (RE) which will also be discussed. It should be noted that though one of these approaches will be selected, these approaches are not mutually exclusive. For example, a SysML model can be developed that is DoDAF compliant. The techniques used in the selected approach will be used to define the system.

In order to make a selection a set of criterion must be identified. The criteria for a good system and objective definition are simplicity, flexibility, communicability, and transferability. The purpose of having a simple method is that the process of developing the system and objective definition should be worth the time and effort; otherwise, practitioners will not perform system and objective definition. If a method is proposed that requires more resources than are available, then practitioners will not utilize the method presented. The

approach should also be flexible, capable to being applied to a wide variety of problems. The approach will be judged on its flexibility if it can be applied to systems and system of systems as well as physical and behavioral models. The communicability criterion is important due to the fact that model development is an important step in validation; therefore, communication between the model developers and the system experts must be emphasized. Additionally, the life of the model must be considered as well. Persons beyond the developers and the system experts may have a need to use the model or reference the model in the future. This leads to the requirement that a method should be easily understood by the developers, the subject matter experts, and third party users. Finally, system and objective definition is only a step in the larger procedure. The methods used in this step must lend themselves to easy conceptual model development; therefore, there is a metric for transferability. The potential methods need to be investigated against the criterion to determine which methods or combination of methods should be used.

### **3.2.1 Soft Systems Methodology**

Soft Systems Methodology (SSM) is an approach to decompose and understand complex systems and was developed as a means to apply systems engineering principles to business and other socially intense problems. This method is considered a ‘soft’ technique as opposed to a ‘hard’ technique because the problems are not well defined, there is not a single objective, and the makeup of a model goes beyond physical entities and can include mental constructs [151]. The SSM approach was developed and popularized by Checkland [45, 44]. Checkland describes SSM as follows:

SSM is an approach which, in use, enables those taking part to learn their way to agree action which they perceive will ‘improve’ the problem situation; it is a consciously organized process of inquiring and learning,... [44]

The early method contains seven stages: entering the problem situation, expressing the problem situation, formulating root definitions of relevant systems, building conceptual models of human activity systems, comparing the models with the real world, defining changes that are desirable and feasible, and taking action to improve the real world situation

[45]. In the late 1980's Checkland evolved the methodology into two streams: logic-based and cultural based. This was not necessarily a new methodology as it was a refocusing of the current methodology. The logic-based stream of analysis focuses on the activity models that were present in the seven stage method. The cultural stream was new and arose from the realization that the use of his methodology did not result in action even though it was useful for understanding and knowing what must be done. The importance of decision making in human systems is best articulated by Checkland in his 2000 retrospective paper on SSM:

This version of SSM as a whole recognizes the crucially important role of history in human affairs. It is their history which determines, for a given group of people, both what will be noticed as significant and how what is noticed will be judged. It reminds us that in working in real situations we are dealing with something which is both perceived differently by different people and is continually changing [45].

In 1990, shortly after the inclusion of the two streams, Checkland redefined SSM as four major activities. These four activities subsume the two streams that were defined only a few years earlier. This new definition was developed to better represent the more flexible use of SSM which evolved over the years of implementation by industry. The four activities are as follows:

1. Finding out about a problem situation, including culturally/politically
2. Formulating some relevant purposeful activity models
3. Debating the situation, using the models, seeking from that debate both
  - (a) Changes which would improve the situation and are regarded as both desirable and culturally feasible
  - (b) The accommodations between conflicting interests which will enable action-to-improve to be taken
4. Taking action in the situation to bring about improvement

### 3.2.1.1 The Four Activities of SSM:

The first activity, finding out about the problem, involves gaining insight into the situation. Checkland identifies four approaches to gaining insight: drawing rich pictures of the problem situation, analysis one, analysis two, and analysis three.

The drawing of rich pictures of the problem situation is a major characteristic of SSM throughout its history [45]. The purpose of the rich picture is to encourage holistic thinking about the problem, provide communication between those involved, and provide clarity of the situation. There is no formal method in how these pictures must be constructed. Through reviewing Checkland's work he routinely emphasizes flexibility over standardization; therefore, the method of construction of the rich pictures is up to the users of the methodology. Examples of rich pictures are shown in Figures 15 and 16. These pictures were developed by Checkland as a means to communicate the contents of a white paper released by the NHS in 1997 entitled 'The New NHS'. These pictures were found to be very useful in conceptualizing the white paper [45].

#### *Model of the White Paper - The new NHS - Modern : Dependable*

##### Core Concept:

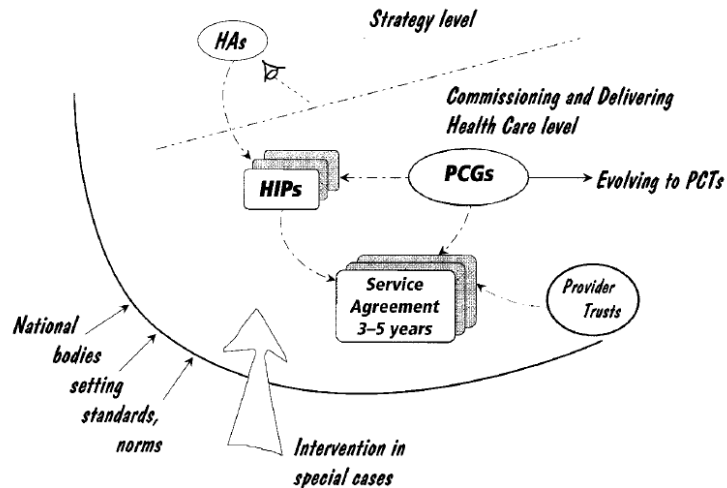
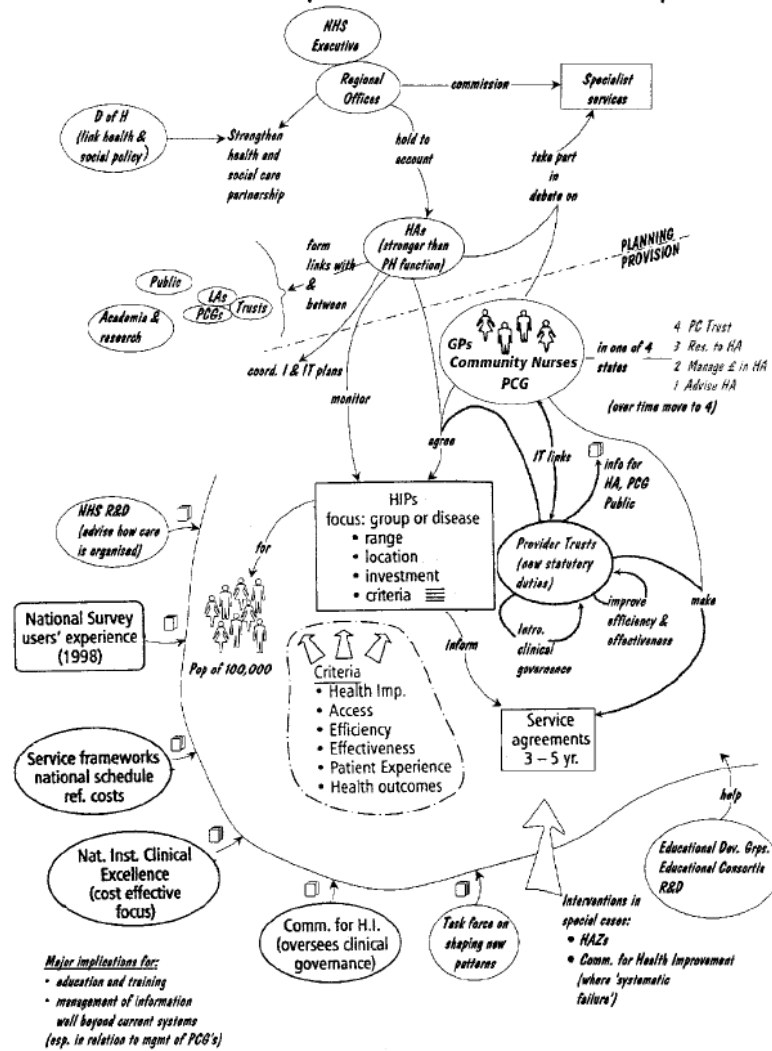


Figure 15: NHS Core Concept Rich Picture [45]



**Model of the White Paper - The new NHS - Modern : Dependable**



**Figure 16: NHS Detailed Right Picture [45]**

Analysis one is the analysis of the intervention itself. Checkland uses the term intervention as a description of the whole process of SSM as opposed to the actions to be taken at the end of SSM. For a more detailed discussion see Checkland's 2006 paper [44] which focuses its efforts on analysis one. This analysis is conducted through three roles seen within the system: the client, the problem solver, and the problem owner. The client is the person or group that initializes the intervention. The problem solver are those that carry our organized work to solve the problem. The problem owners can be described as those that have an interest in the solving of the problem. An important difference between SSM

and traditional systems engineering is the separation between the client and the problem owners. Checkland declares that this separation spurs ideas that can be used to help solve the situation.

Analysis two involves the study of the social characteristics of the problem situation. The goal is to understand the social system and the interactions between the different players. It should be noted that there is no social entity to be discovered but instead the social realm is constantly evolving and being changed by the individuals or the groups themselves.

Analysis three involves the study of the disposition of power in the problem situation. Checkland starts with the concept that everyone involved has power and the analysis here is focused on acquiring a sense of what must be done to cause some things to happen and prevent others to happen. The goal is to understand how the culture works and what changes are feasible. This analysis reverberates with the expansion of SSM in the late 1980s to help bring analytical solutions to real world action. This concludes the first activity.

The second activity is to formulate purposeful activity models (PAMs). Checkland's use of the word model does not imply a mathematical model but instead a model that is an abstraction of reality that is used to aid in communication and discussion. The concept of a model is very similar to the one presented earlier in this dissertation. PAMs are devices used to stimulate the debate [91]. The tools used in this stage are CATWOE, root definitions, and measures of performance.

CATWOE and root definitions are used similarly. The mnemonic CATWOE stands for Customer, Actors, Transformation process, Weltanschauung (worldview of an individual or group), Owner, and Environment. The primary focus of the second activity is T, the transformation process. Everything under this section can be seen as a means to fully understand how the system performs the defined transformation. The PAMs within the model need a clear definition which is provided by root definitions. The root definition is the definition of the system and can be compared to a company's mission statement [91]. The CATWOE will help to develop the root definition.

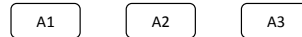
Root definitions take the form 'do P by Q to achieve R'. P represents the question, 'what to do'. Q represents the question, 'how to do it'. Finally, R represents the question, 'why

do it'. These questions structure the problem into a system level hierarchy. P identifies the system level which is also identified by the T in CATWOE, Q identifies the subsystems, and R helps to identify the wider system which takes place on the level of the problem owners. Checkland emphasizes the fact that the system level depend on the view point of the observer. One person's system is another's subsystem.

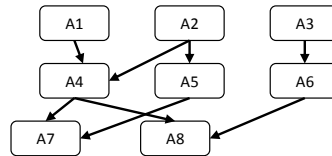
Finally, the measures of performance are used as a means to monitor the operational system to enable adaptation to changing environments. The measures of performance can be summarized as the three or five Es: efficacy, efficiency, effectiveness, ethicality, and elegance. For thoroughness these Es are checking that the desired output is produced, checking that the minimum resources are used, checking that the transformation is worth doing, checking that the transformation is ethical, and checking that the transformation is a pleasing one. It should be noted that typically only the first three Es are used.

With everything now defined the model must be built. Checkland provides a procedure for building activity models in his 2000 paper [45], which was reproduced here in Figure 17 using Checkland's words but not his emphasis. This concludes the second activity.

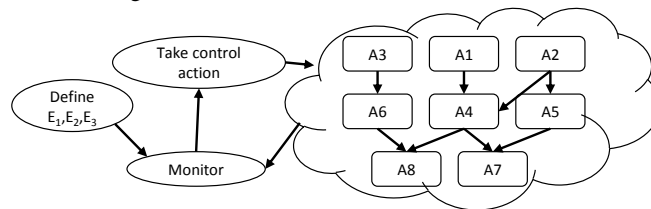
1. Using verbs in the imperative ('obtain raw material X') write down activities necessary to carry out T (obtain I, transform it, dispose of output). Aim for 5-9 activities.
2. Select activities which could be down at once (ie not dependent on others):



3. Write these out on a line, then those dependent on these first activities on a line below; continue in this fashion until all activities are accounted for. Indicate the dependencies:



4. Redraw to avoid overlapping arrows where possible and add monitoring and control



**Figure 17:** Checkland's Procedure for Building Activity Models [45]

The third activity is to explore and debate the situation. As mentioned earlier, two goals of this debate is to identify feasible changes to the system that would improve its operation and to find accommodations between conflicting interest to enable action. This activity is fairly vague on specifics to achieve the two goals [35, 91]. Those involved in the debate often start with each activity in the PAM and then the relationships considering the methods that these activities can be improved. The action may involve structural, process, or attitude change. Once a set of actions are selected the fourth activity is implemented which is to take action.

### *3.2.1.2 The Use of SSM for Simulation Development in the Literature:*

Though SSM's history is to aid in business problem solving, the method has been applied numerous times to aid in model development. Primarily SSM has been used to help define the system qualitatively among the involved parties and to develop a plan of action to improve the system; relatively little have applied SSM to model development for simulation [92, 90]. The exceptions are found within the field of healthcare simulations from Britian.

SSM has been used to aid in the simulation of healthcare systems by Lehaney in the mid 1990s [103, 102]. Lehaney's work appears to be the earliest and most influential on applying SSM to simulation. Lehaney uses the earlier 7-stage SSM. The use of SSM is primarily used to help formulate the problem to improve end user acceptance [103]. To demonstrate his work, Lehaney uses an out-patient service as an example [102].

Kotiadis, who was found to be the most prolific user of SSM for the development of simulations within the literature, often uses the SSM to aid in creating solutions to healthcare problems [92, 90, 191, 93, 193, 91]. In Kotiadis's 2006 paper [92], she presents a discussion on merging different paradigm methodologies, which she refers to as a multiparadigm multimethodology and makes an argument for the beneficial relationship between SSM and discrete event simulation. Additionally, she presents a good overview of previous attempts on merging different paradigms in relation to healthcare modeling. In her 2007 paper [90], Kotiadis makes the argument that SSM could be used more to help determine simulation study objectives of discrete event simulations and further makes the argument that SSM

has potential benefits for simulation conceptual model development. She then provides an example on developing simulation objectives for a complex health care system in Kent, UK. In 2008 Kotiadis teamed up with Stewart Robinson, who was heavily cited earlier in this dissertation for his contributions to the philosophy of conceptual models, to discuss the use of SSM in the development of a conceptual model [94]. Kotiadis and Robinson discuss the use of SSM in the knowledge acquisition and model abstraction of a conceptual model where knowledge acquisition is the process of obtaining information about the system from subject matter experts and whose model abstraction is the process of simplification of the system such that only the important aspects are modeled. They then provided an example using the same healthcare scenario from Kotiadis's 2007 paper. In 2010 with Antuela Tako and Chistos Vasilakis, Kotiadis presents a framework for the development of conceptual models for simulation which appears to be a aggregation of her previous four years of work and publications [191]. A similar paper is then given in 2012 [193]. In this paper the authors present a participative framework for developing conceptual models called PartSim. Two important steps within the framework, structure the situation of interest and specify study objectives, rely on the use of SSM [191, 193]. From these works Tako and Kotiadis create the term facilitated conceptual modeling. In 2012 they released a reflection paper on the topic [192]. In this paper they present experiences on facilitation issues such as group size and workshop organization.

In addition to the joint paper with Kotiadis in 2008, Robinson has suggested the use of soft methods to aid in modeling and simulation [151, 153, 157]. In his 2001 paper [151], Robinson discusses the potential soft operations research (OR) techniques of discrete event simulation (DES), challenging the traditional paradigm that DES is solely a hard OR technique. Though this is not an explicit use of SSM with simulation, it does help to introduce the overlaps of the two paradigms with respect to DES. The paper helps to build the case that soft techniques such as SSM can be used with hard techniques such as simulation. In his 2004 book on simulation [153], Robinson suggests that SSM can be used to help develop a conceptual model when the clients have a poor grasp of the problem. Robinson then repeats these statements in a 2008 paper. It was not found in the literature

that Robinson had used SSM in simulation; however, he does give support that it is a reasonable approach to conceptual model development. Others, Pidd and Hengst, have also made the argument for the use of SSM or soft OR techniques in simulation [143, 144, 76].

### *3.2.1.3 Rating SSM on the System and Objectives Definition Criterion:*

The criterion defined for system and objective definition are simplicity, flexibility, communicability, transferability. The SSM is a fairly simple approach to defining and understanding the system. The primary difficulty with SSM is determining the appropriate definitions for all the parts of SSM. Therefore, for the criterion of simplicity SSM is rated as good. Flexibility is a philosophy that underpins SSM. Checkland routinely emphasizes that SSM is only a guiding methodology and can be used in many ways. For this reason SSM is rated as very good for the criterion of flexibility.

The criterion communicability and transferability are a bit harder to assess. Communication must be made between the developers, the subject matter experts, and third party users. SSM lends itself to impeccable communication between the model developers and the subject matter experts, assuming they work collaboratively as intended. This is largely due to the flexibility of the methodology. Flexibility is a primary focus of the SSM. Consequently, the same flexibility leads to difficult communication with a third party. The lack of formal structure to the rich picture developments and the PAMs would make it difficult to fully understand the system unless one were part of the original development. Since SSM communicates very well for one group and poorly for another, the criterion of communicability is give a rating of moderate. Finally, SSM is rated as moderate for transferability. This is primarily due to the extreme flexibility of the method. This leads to potentially vague models which would be difficult to transfer into a more formal conceptual model.

## **3.2.2 Requirements Engineering**

Requirements Engineering (RE) is most often used in software development. It is a traceable process used in identifying, analyzing, modeling, verifying, and managing requirements [213, 146]. The definition of the requirements of a system involves all aspects of the development up to actual system design [160, 159]. In Douglas Ross' 1977 paper, which has been credited

for opening the field of requirements engineering [203], describes RE as follows.

Requirements definition is a careful assessment of the needs that a system is to fulfill. It must say why a system is needed, based on current or foreseen conditions, which may be internal operations or an external market. It must say what system features will serve and satisfy this context. And it must say how the system is to be constructed. [160].

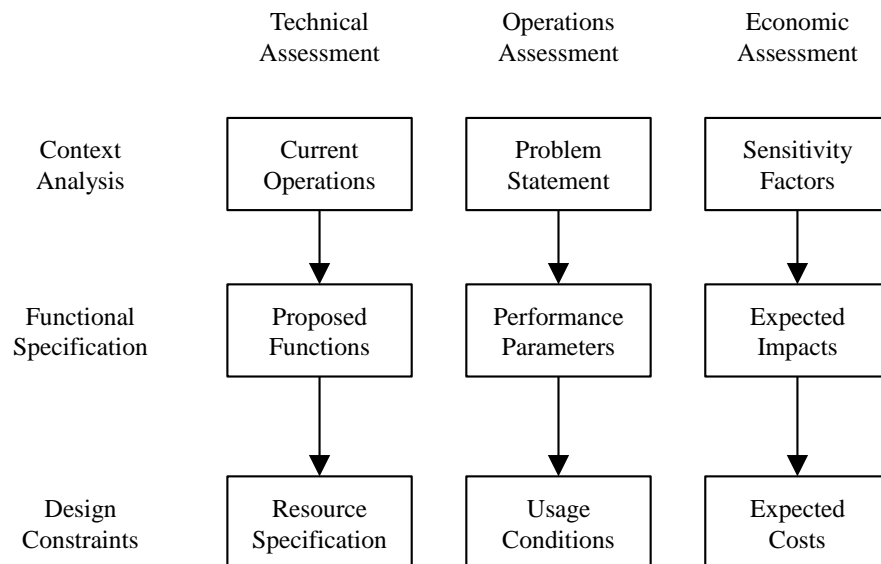
These definitions of RE provide a good overview of what RE is; however, the specifics of RE are far more muddled, because RE is a subject area that is contributed to by many people as opposed to a specific methodology that was developed by a person or group. Additionally, based on the definitions above it can be argued that every topic within the System and Objectives Definition section fits within the field of RE. For example the developments within the field of RE directly lead to the development of the Unified Modeling Language (UML) [203]. This in turn influenced the creation of the System Modeling Language which is an extension of UML 2.0 [69]. Finally, it appears that DoDAF was influenced by the developments in RE, since DoDAF traces its origins to the C4ISR Architecture Framework which was initially developed in 1996 after 20 years of RE developments [55]. The concepts in RE and DoDAF share some commonalities. Finally, SSM and RE share a lot of commonalities in their intended purpose.

Because RE is a field of study as opposed to a methodology, RE will not be considered as a potential solution to S&O Definition. Instead a general overview will be given on the history, concepts, and miscellaneous uses of RE to develop relevant computer models. This will create a better context for the discussion of UML, SysML, and DoDAF.

#### *3.2.2.1 The Beginning of Requirements Engineering:*

Requirements Engineering (RE) as a field began in the 1960s and 1970s as a response to the challenges faced by software developers. Sometimes RE is referred to as requirements definition or requirements analysis. One of the most influential papers was written by Douglas Ross as a two part publication in the IEEE Transactions on Software Engineering in 1977 [160, 159, 203]. In these papers he presents the concepts behind developing a requirements

definition [160] and then presents a methodology called Structured Analysis (SA) [159]. Ross states that requirements definitions deal with three subjects: context analysis, functional specification, and design constraints. Context analysis defines why the system is being created and why the selected criteria that bounds the system exist in their specific manner. Functional specification defines what the system must do functionally. Design constraints defines how the system is to be constructed and implemented through specifying conditions [160]. These three subjects can be described as why, what, and how the system will be developed. In Ross' discussion he wisely brings in the human element to RE stating that "any proposed methodology must be people-oriented" [160]. Every person involved in the system has different role that they fill and therefore will have different viewpoints about the system. Thus, the system inherently has multiple viewpoints. Ross defines nine viewpoints as shown in Figure 18. Ross states that the context analysis, functional specification, and design constraints must be addressed from the following points of view: technical assessment, operational assessment, and economic assessment. The technical assessment concerns the feasibility of the architecture of the system. The operational assessment concerns the performance of the system within its environment. Economic assessment concerns the costs and impacts of the system's implementation and use [160].



**Figure 18:** Ross' Viewpoints of Requirements Definition

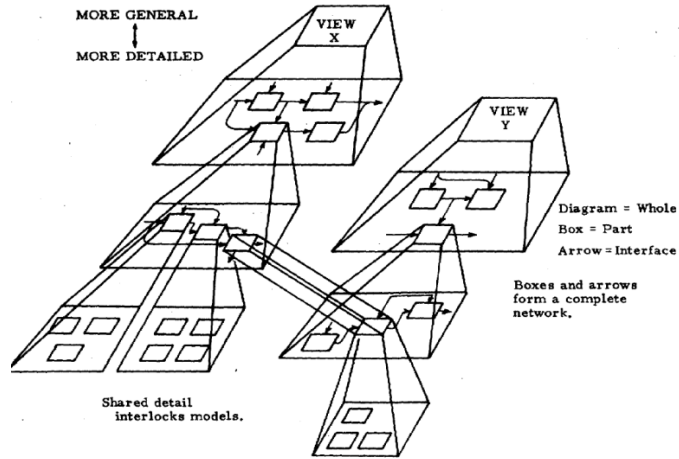


Ross proposes a method for requirements definition called Structured Analysis (SA) [159]. Ross relates SA to a blueprint for design and construction. A key component of SA is that it is hierarchical and is a top down decomposition. The output product of the SA method is a hierarchical structure that decomposes complex systems into understandable chunks. Ross defines SA as follows.

The only function of SA is to bind up, structure, and communicate units of thought expressed in any other chosen language... SA is structured decomposition, to enable structured synthesis to achieve a given end [159].

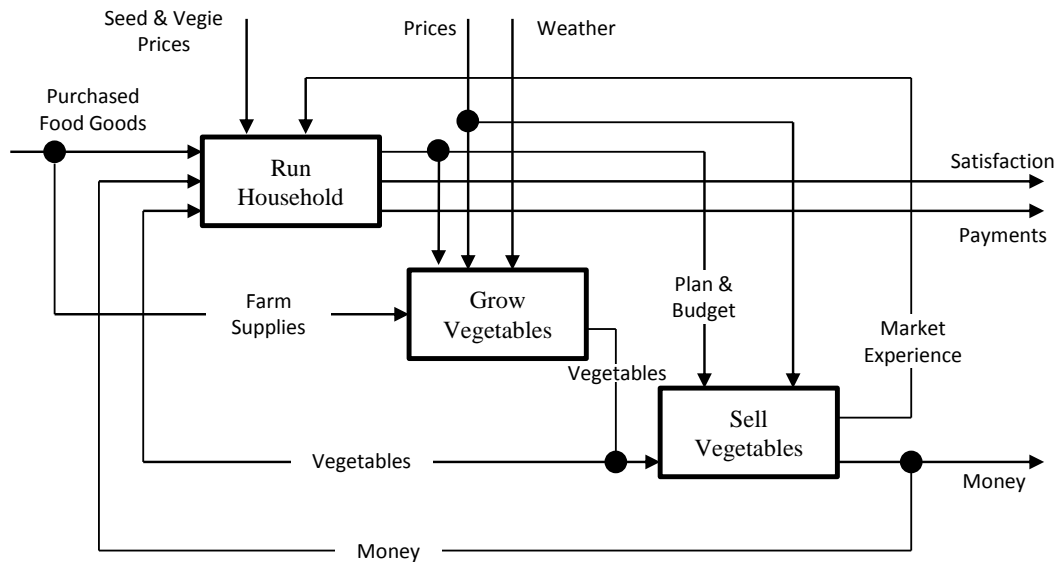
SA has two categories of decomposition: data decomposition, and activity decomposition [160, 115]. Ross refers to these as the thing aspect and the happening aspect, respectively. These decompositions are detailed in diagrams in a top-down fashion. A diagram that is higher in the hierarchy is referred to as the parent of the diagram. A diagram that is a detailed diagram of one of the activities is referred to as the child. Each diagram contains boxes and arrows. Boxes represent some part of the system. The arrows represent the relationship between the parts. They do not represent a flow or a sequence of events that must occur. Ross suggest that each diagram should have at most six boxes in it. An example of this decomposition is shown in Figure 19 and is from his 1977 paper [160]. An activity decomposition is primarily simply the boxes and the arrows, showing the happenings and their relationships. The boxes have a specific notation. All inputs into the activities enter the box on the left side. All outputs of the activity exit the box on the right side. Control variables enter the box on the top. A control variable is an influence on the activity but is not directly consumed.

Mylopoulos provides an example of these decompositions in his class notes using the scenario of running a farm [115]. Two activity diagrams are shown in Figure 20 and Figure 21. The first diagram, which could be a box activity in a higher up diagram, takes in one input, three controls, and has three outputs. This example reiterates the point that this does not represent a flow diagram. Note that the output of the grow vegetables activity feeds both downstream and upstream. The hierarchical nature of SA is shown with Figure



**Figure 19:** Ross' System Decomposition

21, which expands on the grow vegetables activity. The grow vegetables diagram is a child of the run farm diagram. The run farm diagram is the parent of the grow vegetables diagram. Note that these diagrams remain faithful to Ross' six or less rule. The data decomposition can be seen in Figure 22. The data flow is on the same level as the grow vegetable activity diagram. Review Figure 21 and Figure 22 one can see the relationship and different views.



**Figure 20:** Mylopoulos' Example: Run Farm

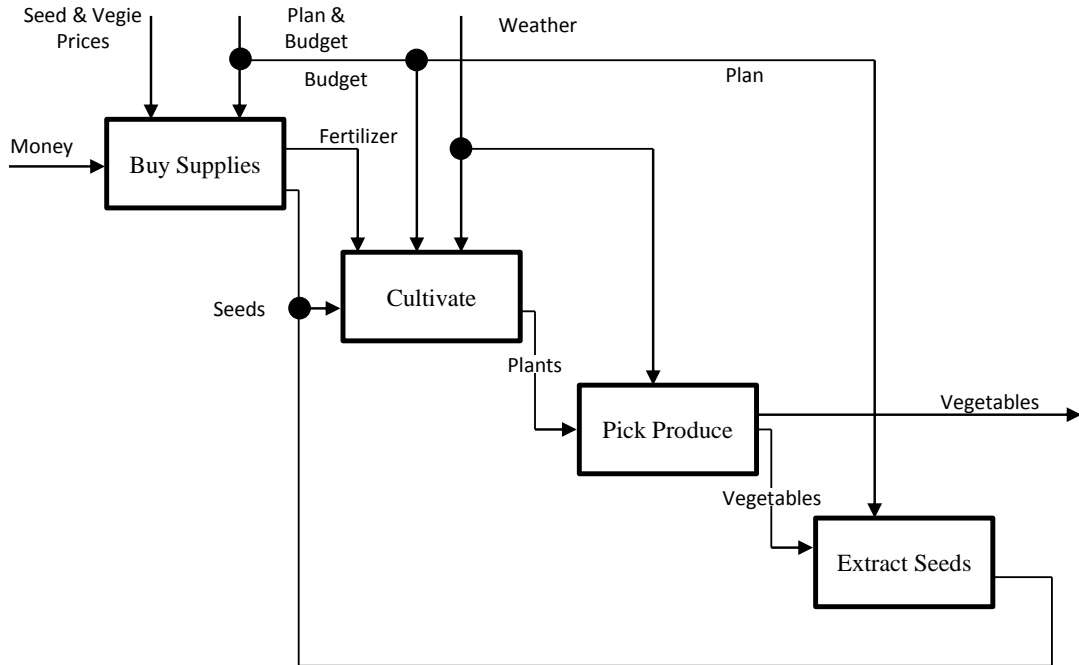


Figure 21: Mylopoulos' Example: Grow Vegetables

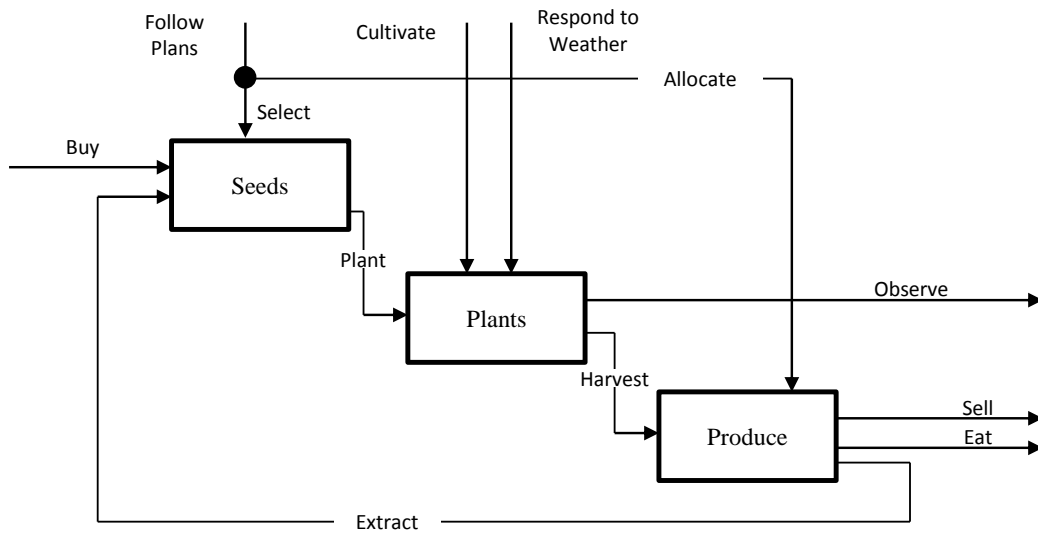


Figure 22: Mylopoulos' Example: Data Flow

### *3.2.2.2 The Development of Requirements Engineering Philosophies:*

Requirements engineering evolved over the years. A brief review of the concepts that emerged over the decades will be presented. The developments discussed include Object Oriented Method (OO-Method), Agent-Oriented Method (AO-Method), and Goal Based Reasoning.

The OO-Method, sometimes referred to as object oriented analysis, was developed in the late 1980s to aid in software development which was influenced by object-oriented programming, database design, structured analysis, and knowledge representation [116]. There are many variations on the OO-Method. These include but are not limited to Object Oriented Software Engineering (OOSE) developed by Jacobson, Hierarchical Object-Oriented Design (HOOD), Object-Oriented Design (OOD) developed by Boosh, Object-Oriented Structured Design (OOSD) developed by Wasserman, and Responsibility Driven Design developed by Wirfs-Brock [81, 37]. Additionally, UML was created based on the OO-Methods, and many of its diagrams are based on the OO-Method. Generally the OO-Method is based on objects/classes, structures, subjects, attributes, and services. The OO Methodology is executed in this manner. First objects are identified, not that classes are generic objects. The object is an entity that encapsulates attributes and behaviors. Next a structure is created which provides a taxonomy of the classes. Subjects are then defined, which groups the classes and objects into subject layers and represent a specific perspective. The attributes of the objects are then defined. Finally the services of each class is defined. These concepts are expanded further under the UML section where select diagrams are presented.

Agent-Oriented Methods are very similar to OO-Methods and are largely based on them. The driver for the development of the AO-Method was due to the fact that agents and objects are not the same thing, though they are similar. One of the major difference is that agents are decision makers and can decide how to execute actions based on the messages it receives. Another difference is that agents maintain different states that affect their decision making and interaction with the environment [81, 104]. Iglesias identifies three methods for AO-Methods: Agent-Oriented Analysis and Design, Agent Modelling Techniques for Systems of BDI agents, and Agent Oriented Methodology for Enterprise Modelling [81].

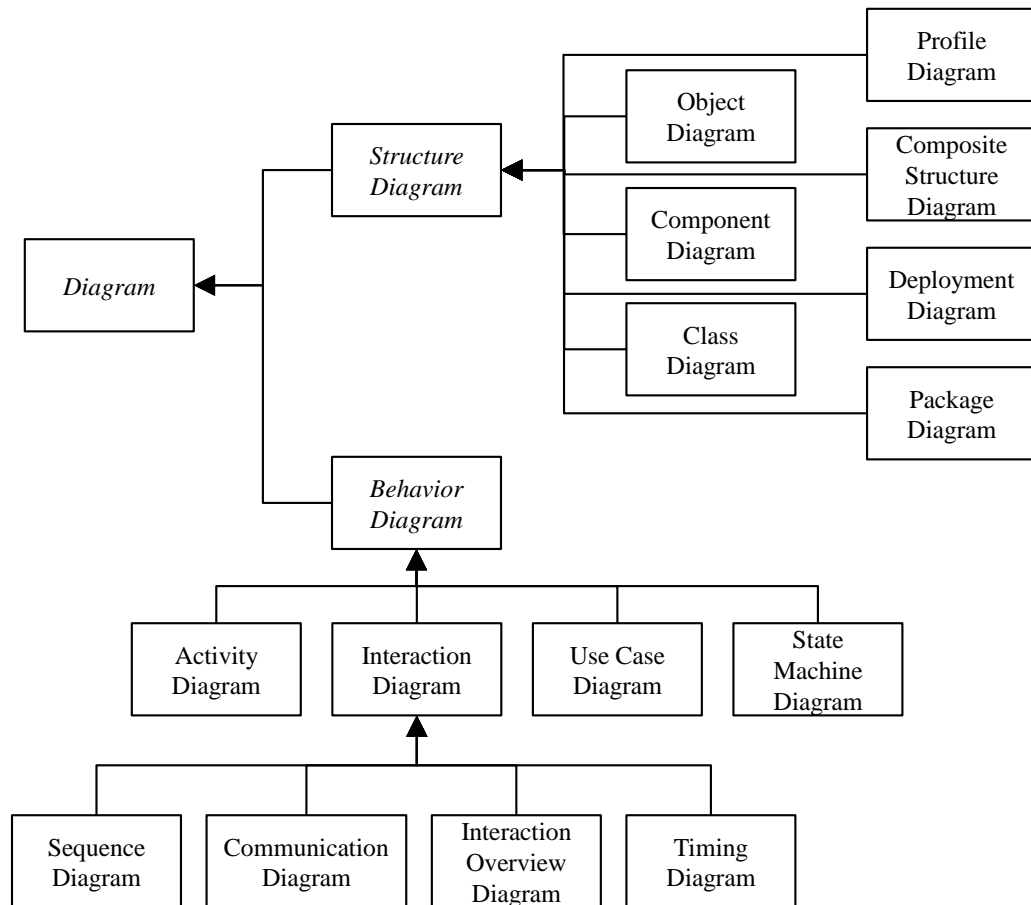
The primary differences between AO-Methods and OO-Methods is the focus on the agent, its decision making, and its goals.

Goal Based Reasoning, sometimes referred to as Goal Oriented Analysis, focuses on alternatives and their relationship to organizational objectives [114]. Van Lamsweerde claims that up until goal based reasoning was developed, the other RE efforts were focused on the what and how of RE. Goal based reasoning brought in the why question [203]. Former RE research focused on what the software system was meant to do and how it was going to do it. Little effort was given into why the system was needed or if the requirements identified truly met the needs of the problem owners [96]. The most commonly found methodology found within goal based reasoning is Knowledge Acquisition in Automated Specification (KAOS) [52]. KAOS has three primary models or diagrams: goal model, object model, and operation model [96] The major advancement of the goal based approach is the realization of the need for goals and the development of a mean to integrate that into previous RE philosophies.

### **3.2.3 Unified Modeling Language**

Used primarily in the field of software engineering, the Unified Modeling Language (UML) is a series of diagrams designed to help catalogue and visualize software architecture. The current version, UML 2.0, contains 13 different diagrams that can be categorized into three groups: structure diagrams, behavior diagrams, and interaction diagrams [132]. The motivation for the development of UML came in the early 1990s where there existed a plethora of RE methods [67, 114]; some were reviewed in the previous requirements engineering section. The issue that arose was that there existed many standard notations resulting in deterring new users of standardization and the fragmenting of the modeling tools market [194, 209]. UML was conceived by the Rational Software Corporation with three of its most prominent employees: Booch, Rumbaugh, and Jacobson, known as the Three Amigos [215]. The Object Management Group acted as a forum for the development of the standard [209] and adopted the standard in 1997. In 2005 OMG adopted UML 2.0 [1]. By 2008 70% of software development organizations were using UML [209].

UML contains 13 diagrams. Only a subset will be covered here in detail. The category of structure diagrams contain the profile diagram, the object diagram, composite structure diagram, component diagram, deployment diagram, class diagram, and package diagram. The category of behavior diagrams contains the activity diagram, interaction diagrams, use case diagram, and state machine diagram. The interaction diagrams include the sequence diagram, communication diagram, interaction overview diagram, and timing diagram. This list and its structure can be seen in Figure 23.



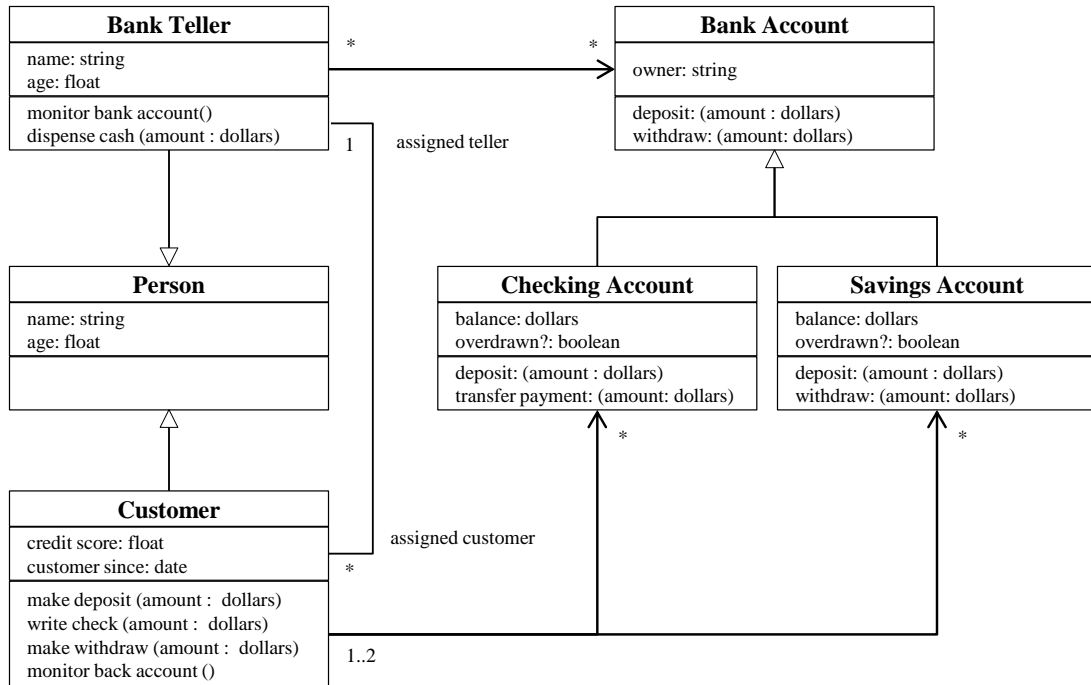
**Figure 23:** UML Diagrams

The first diagrams to be discussed are the structure diagrams. Only a subset of these will be described: the class diagram, the package diagram, and the component diagram. The class diagram is said to be the backbone of most OO-Methods [67]. The class diagram

describes the types of classes, the relationships between the classes, and the class's operations. These classes will have a name, associated attributes, and operations. The class is represented by a box with three compartments. The top one contains the name. The middle one contains the attribute list. Each attribute is listed on a new line along with its attribute type, e.g. integer, string, time. The bottom compartment lists the class's operations, also called methods [33]. The operations are the processes that the class can carry out [67]. The interactions between the classes are defined with relationships. There are two types of relationships between the classes: associations and subtypes [67, 117]. An association is a relationship between classes and commonly take one of two forms: bi-directional and uni-directional, though other forms exist. The bi-directional association is represented with a line. For this association both classes are aware of the relationship. The uni-directional association is represented with an open arrow. In this case only the class that the arrow is pointing from is aware of the other class [33]. Subtypes include inheritances between the superclasses and the subclasses through a generalization. The superclass is a generalization of many subclasses. A basic example of this is shown in Figure 23 where the class diagram is the subclass of the structure diagram which is a subclass of the UML diagrams. The structure diagram class can be considered a generalization of all the subclasses which share many characteristics. This relationship is often represented with a line and a hollow arrow head. Next to the base of the associations there may be a number. This number will represent the number of associations that the class may have. Additionally, the base of the associations may be named.

An example of the class diagram is shown in Figure 24. As can be seen the Person class is a superclass to Bank Teller and Customer. Bank Teller has uni-directional association with the Bank Account where only the Bank Teller is aware of the Bank Account. The asterisk indicates that unlimited associations can be made between the two classes. The Bank Teller has a bi-directional association with the Customer. The numbers at the base indicate that a Bank Teller can be associated with an unlimited number of customers and each Customer has exactly one Bank Teller. The Customer has a uni-directional association with both Checking and Savings Account. The number indicate that the accounts can have

one to two people associated to them and the Customer can own as many accounts as they want.

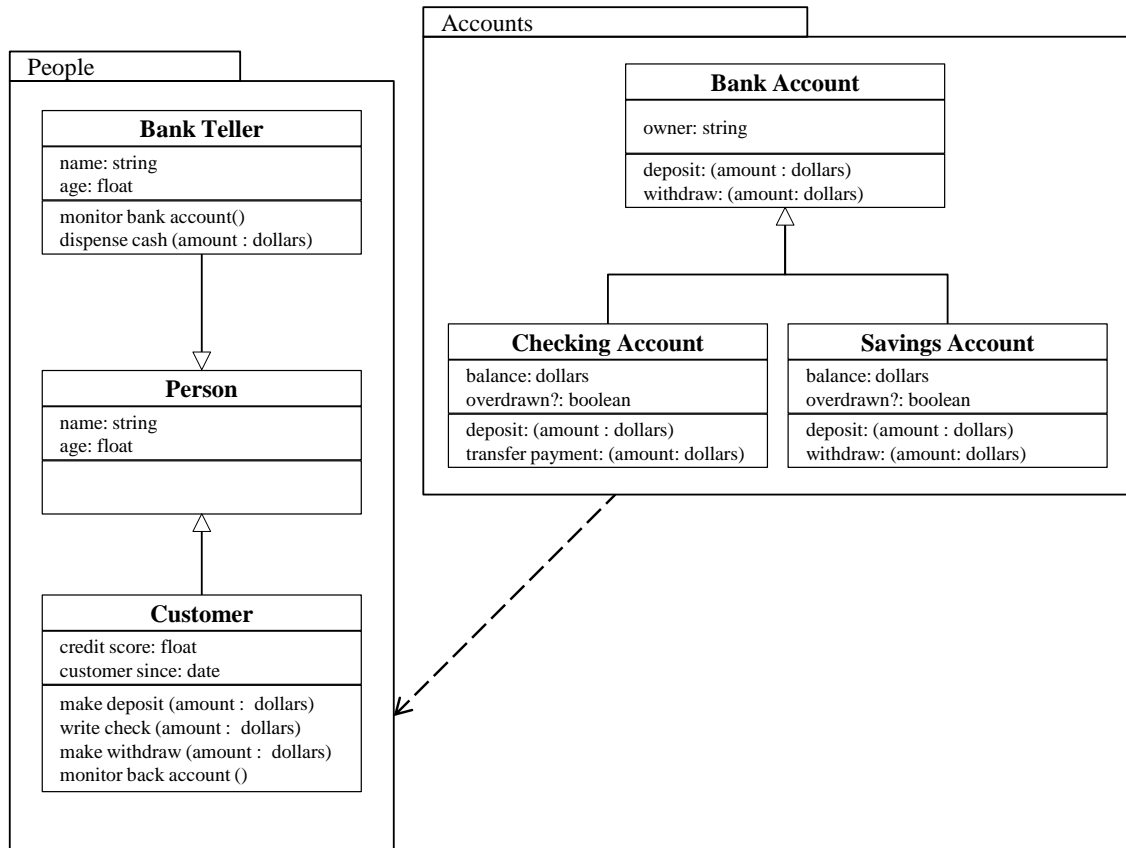


**Figure 24:** Class Diagram Example

The next structure diagram is the package diagram. The package diagram is primarily an organizational tool provided by UML for large difficult to manage systems [33]. Fowler describes a package diagram as a class diagram that only shows the packages and dependences between packages [67]. Techniques will vary. It is common to show the included elements in the package diagram. The dependences between packages are similar to those in class diagrams. A packages is used to organize model types and other packages [67, 129]. This allows the modelers to divide the system into namespaces making it easier to understand [33]. This is similar to a file directory. The specific organization of packages is left to the user; however, some guidelines are available. Classes should belong in the same package if they are in the same inheritance, are highly related to each other, or collaborate a lot [8]. Example of a package diagram using the previous class diagram is shown in Figure 25. A dependency is shown from People to Accounts because the classes under People send messages to classes under Accounts. It should be noted that there are a variety of ways to



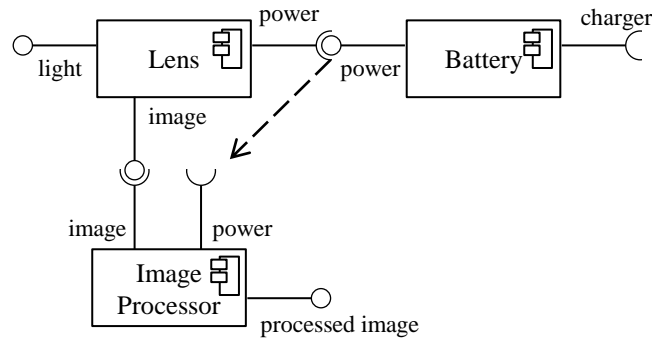
display the package diagram.



**Figure 25:** Package Diagram Example

The next structure diagram is the component diagram. Components are units of the system. They are autonomous, encapsulated units within a system that has one or more interfaces [34]. A component may contain one or more classes. The component diagram is similar to the class diagram; therefore, the classifier rules are the same [34]. A component diagram shows the high level structural relationships between components that make up the system. Component diagrams are useful for providing a architectural view of the system which is useful for communication between the various groups. A simple example of a component diagram is shown in Figure 26. Here, a basic camera is shown. The battery component supplies power to the lens and the image processor. Note that the lollipop extension from the component denotes that something is being supplied and the socket

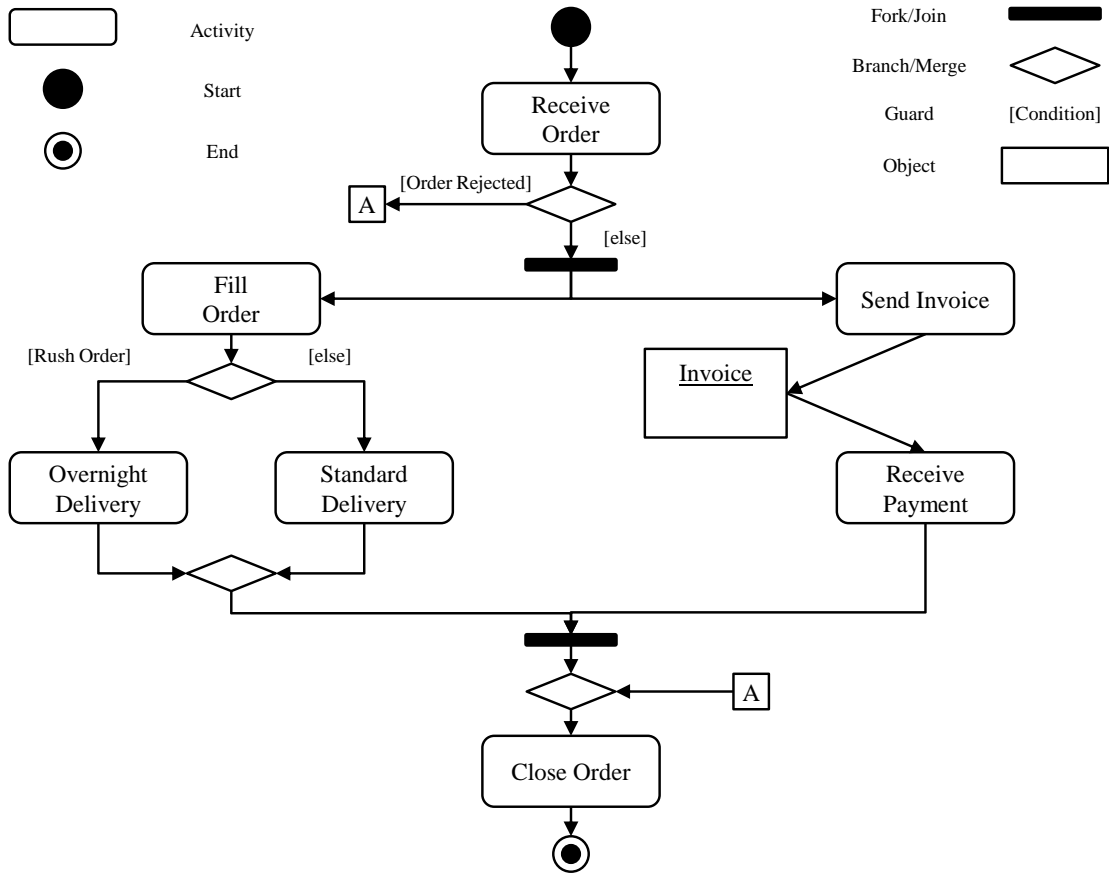
denotes that something is required. These are referred to as assembly connectors. Both the lollipop and the socket are labelled. The component requirements can be shown to be fulfilled by either directly connecting the lollipop and socket or with a dashed arrow as shown below. Each component can also be broken down into subcomponents. This would then create a new component diagram.



**Figure 26:** Component Diagram Example

The first of the behavior diagrams is the activity diagram. The activity diagram is a visual representation of a workflow. It will show the sequence of activities from start of finish. The elements within the diagram include activities, control flow, start/end node, branch/merge nodes, objects, and fork/join nodes. An example of an activity diagram can be seen in Figure 27. This example and description is based on the two following sources [67]. An activity is the act of doing something. The control flow shows the path from one activity to the next with an arrow. The start and end nodes signify the beginning and end of the full activity. The branch/merge uses the same image, but branch takes one incoming signal and outputs two mutually exclusive signals that are defined by the guard. The merge takes in multiple signals and outputs one. This signals the end of the former branch. Objects or object flows can be created in an activity diagram. The fork takes in one signal and then executes all the out going signals which operate in parallel. The join ends the fork and takes multiple signals and returns one. Though this example does not show it, it is not uncommon to include swimlanes in an activity diagram [67, 117]. As an example of using swimlanes in activity diagrams, Figure 27 can be broken into order

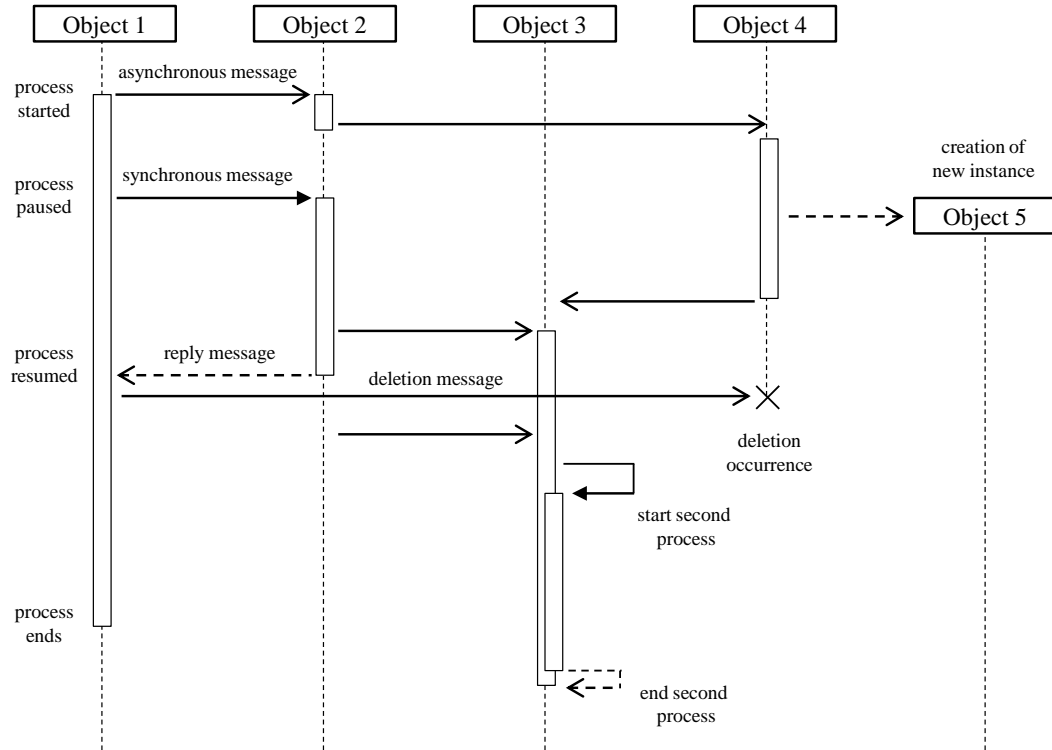
fulfillment, customer service, and finance swimlanes.



**Figure 27:** Activity Diagram Example

Of the interaction diagrams only the sequence diagram will be discussed. The sequence diagram shows multiple objects with lifelines running down the page. The objects are listed along the top with lifelines running down below them. The object's interactions are shown over time with arrows between the lifelines. Time is considered as progressing as one goes down the page. Rectangles on the lifelines indicate that a process is occurring. A solid arrow from one lifeline to another indicates that a message has been passed. These messages can be synchronous or asynchronous denoted by a solid arrowhead or a line arrowhead, respectively. A return message is an asynchronous message which is denoted by a dashed line and line arrowhead [67]. The sender of a synchronous message will wait until it receives a reply from the receiver before continuing its processes, whereas with an asynchronous

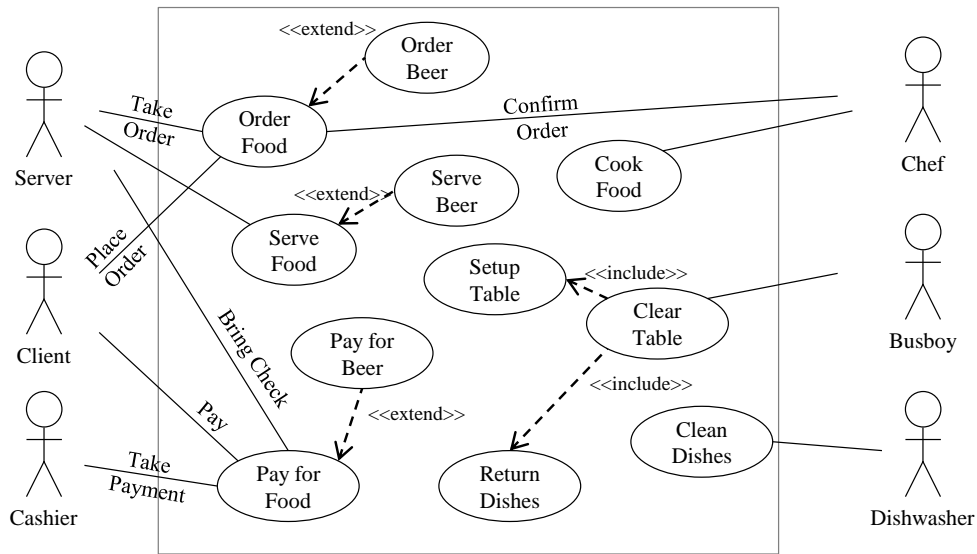
message the sender will continue to execute its processes [69]. Additionally, an object can send a message to create or delete another object. An example of a sequence diagram can be seen in Figure 28.



**Figure 28:** Sequence Diagram Example

The use case diagram communicates what the system intends to do and shows how the users and the stakeholders interact for a given scenario. Use cases were originally developed by Jacobson [67, 117]. This diagram includes actors and use cases. The actors are the users of the system or otherwise involved with the system. They do not have to represent a person or a group of people but often do. The actors carry out the use cases [67]. The use case represent the goals of a system from the actor's perspective. These use cases are described functionally in a manner of what is to be done. Actors are connected to the use cases though communication paths represented as lines. The use cases can have inclusions

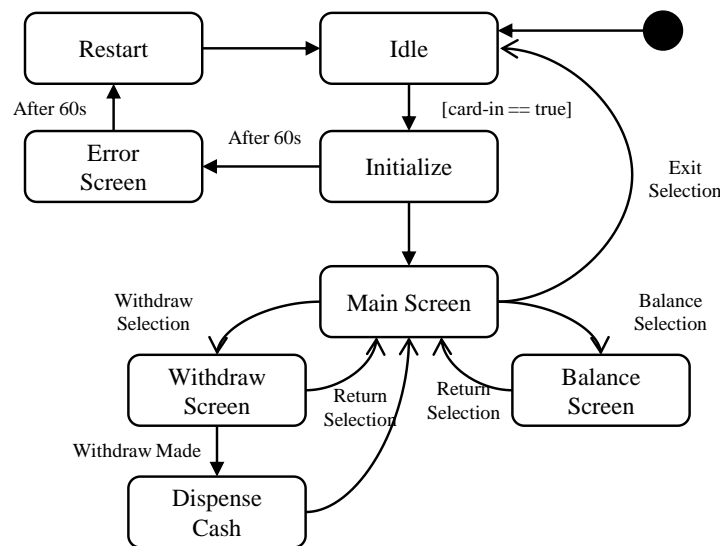
and extensions. These are represented with dashed lines with line arrows at the end. The arrow points at the thing to be included and from the thing that extends. The inclusion will include the functionality of another use case. Where as the extension refers to a fragment of functionality that may not be considered as part of the use case [69]. An example of a diner use case is shown in Figure 29. As can be seen there are six actors. The server is involved in three use cases: order food, serve food, and pay for food. The client is also involved in the order of food and the pay for food use cases. Multiple actors can use the same use case. An example of an extension is the ordering of beer. If this happens then the serve food and pay for food are also extended. An example of an include is the clear table use case which includes return dishes and setup table.



**Figure 29:** Use Case Diagram Example

The state machine diagram describes all the possible states of an object and how the object can reach that state. This diagram is useful for describing the behavior of an object through multiple use cases [67]. The use case diagram contains transitions and states. This terminology may differ. Fowler uses the terms actions and activities [67]. The former terminology will be used here. The transitions are associated with an event, guard, or action and occur quickly. These transitions are not interruptible. There are four types of events: change event, signal event, call event, and time event [117]. The change event will cause a

transition when a condition becomes true. The signal event will cause a transition when it receives the appropriate signal from another object. A call event will cause a transition when the object receives a call for an operation. Finally, a time event will cause a transition after an elapsed time [117]. The states represent long running activities that can be interrupted. The object while in a state will either satisfy some condition, perform an action, or wait for an event. An example of a state machine diagram is shown in Figure 30. This example shows a simple ATM example. The system holds at idle until a card is inserted. Then they system loads up the relevant card information. If 60 seconds pass without a transition to the main screen an error screen is shown then the system is rebooted. Once at the main screen the user may select to go to the withdraw or balance screen. Each of those screens can be returned to the main screen. The withdraw screen can make a cash withdraw. At this point the system returns to the main screen once it is finished dispensing cash.



**Figure 30:** State Machine Diagram Example

### 3.2.3.1 The Use of Unified Modeling Language in the Literature:

Similar to executable architectures, there have been attempts in the literature at developing executable UML. A paper by Risco-Martin and Mittal presents eUDEVS, an executable UML that incorporates the DEVS formalism [150]. UML has also been used by Insfran to aid in conceptual model development [83, 84]. Insfran uses RE concepts to aid in conceptual

model extractions that he sees as a short coming of current efforts. Primarily Insfran uses object oriented methods and a requirements engineering framework called TRADE to develop a conceptual model and to automatically generate software. This work uses use case diagrams, sequence diagrams, and class diagrams from UML. Many of the activities that Insfran noted as conceptual modeling would be considered a communicative model or programming in Balci's 1986 procedure. Finally, UML has been suggested as an aid to a framework for specifying a system of systems [109]; however, UML was not been suggested as a standalone tool for system of systems definition. A specific application of UML only diagrams to decompose an existing non-software system has not been found.

### *3.2.3.2 Ranking of UML on the System and Objectives Definition Criterion:*

The criterion defined for system and objective definition are simplicity, flexibility, communicability, transferability. For the criterion of simplicity UML is rated as moderate. The language contains a standardized set of 13 diagrams. The standardization helps improve the simplicity. The 13 diagrams are not a small number of diagrams to review for application, though not all are required. Additionally, the numerous details of the diagrams makes this language more complicated.

For the criterion of flexibility UML is rated as moderate. UML has been applied to numerous systems in its two decades of existence. Unfortunately, these examples have not been applied to the specific form of system decomposition that is required for this application. That is the system decomposition of a physical system not a software system. The absence of this application does not imply that it cannot be done; therefore, the criterion of flexibility remains at moderate.

For the criterion of communicability UML is rated as good. The history of UML's use and the standardization of UML aid in UML's communicability rating. The need for knowledge of UML of those involved limit the rating to good as opposed to very good.

Finally, the criterion of transferability is rated as moderate. One aspect of UML that pushes its transferability towards very good is that UML is a modeling language that can be and has been used to develop a conceptual model. Once the UML diagrams are made then

the conceptual model has mostly been built. Unfortunately, this same step would require more abstraction of the physical system than should be accomplished at this step. For this reason this criterion remains moderate.

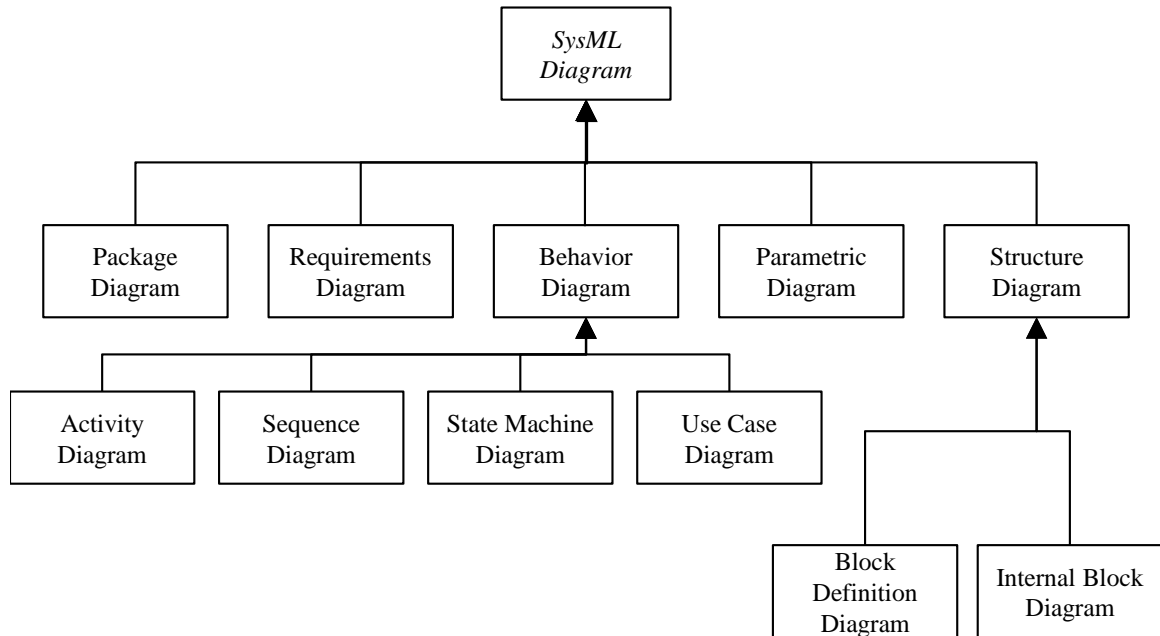
### 3.2.4 Systems Modeling Language

The Systems Modeling Language (SysML) is an extension to UML. It is referred to as a graphical modeling language that is used to support specification, analysis, design, and V&V [68]. The models that SysML produces are communicative in nature; however, there exist a number of software tools that aim to enable model based systems engineering. This is not the focus of this thesis, thus focus here will be given to only the diagrams. SysML can be applied to software, hardware, data, facilities, procedures, or personnel [69, 131].

SysML was developed by SysML Partners which was convened to respond to the UML for Systems Engineering RFP put forth by OMG in 2003 [127, 68]. SysML Partners were an informal group of software tool vendors and industry leaders that was founded and chaired by Cris Kobryn [188]. In 2007 the OMG released SysML 1.0 [128]. The latest version, 1.3, was released by OMG in 2012 [130].

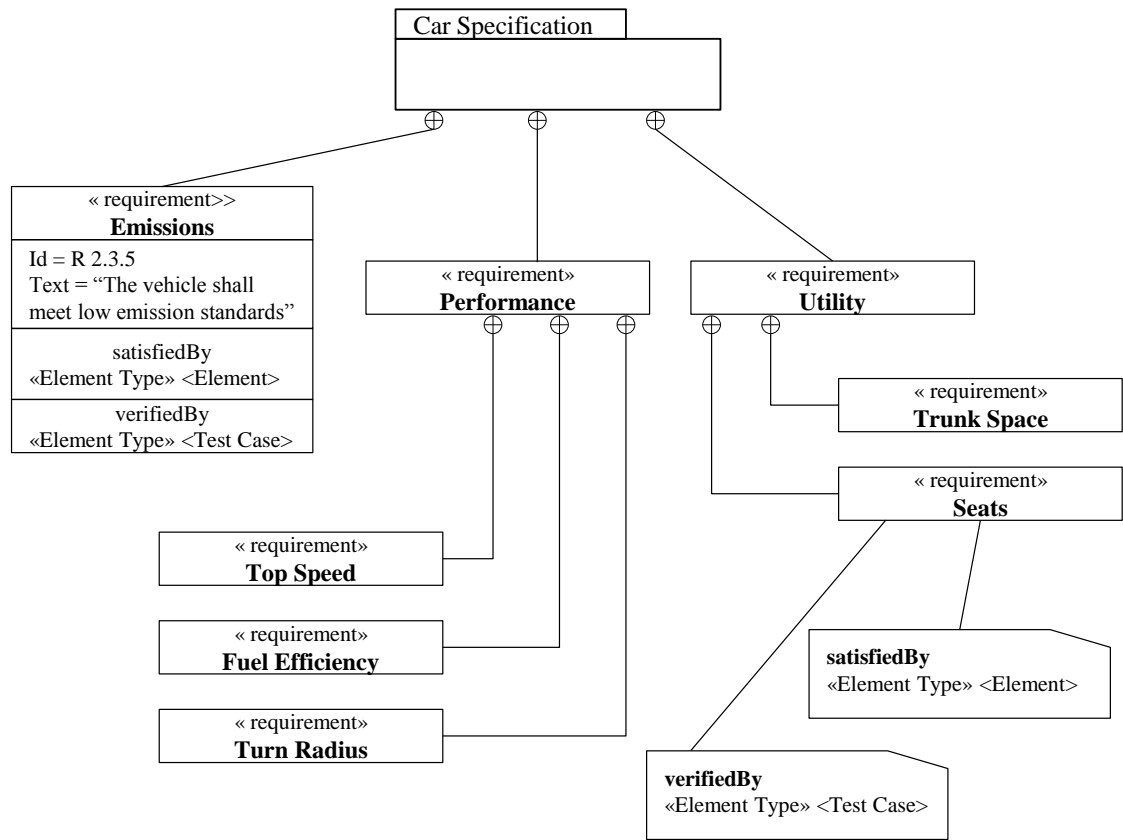
The ‘pillars’ of SysML are the structure diagrams, behavior diagrams, the requirements diagram, and the parametric diagram. In total SysML contains nine different diagrams that can be used to define a system. The breakdown of the SysML diagrams can be seen in Figure 31. The exact diagram breakdown varies depending on the source. Since SysML is based on UML many of the diagrams are the same. The package diagram and the behavior diagrams are the same or very similar as the ones used in UML. Requirement diagrams and parametric diagrams are new to SysML. Though UML has a branch of structure diagrams, the SysML structure diagrams are different and only contain the block definition diagram and the internal block diagram. These diagrams are similar to class diagrams but are made specifically for systems. A brief description is provided for these new diagrams below.





**Figure 31:** SysML Diagrams

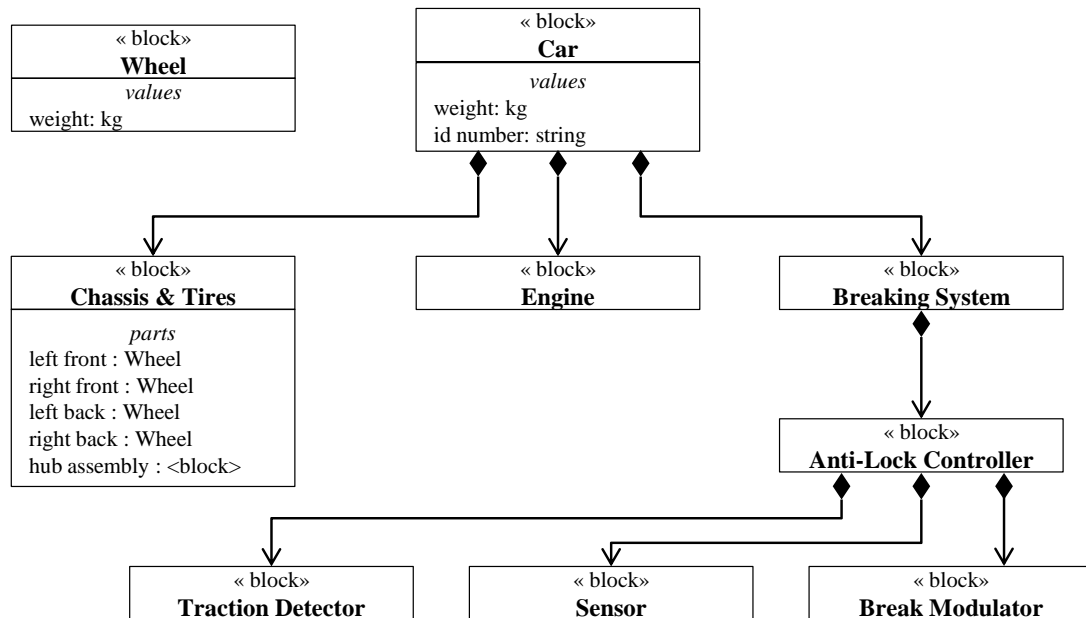
The requirements diagram is a representation of the text-based requirements that includes relationships with the other requirements and design elements. This diagram primarily supports traceability and verification that the requirements have been fulfilled [70, 69]. The requirements diagram helps to ensure that requirements are consistent, feasible, and valid to the stakeholders needs [69]. The requirements diagram includes blocks that represent the requirements and call outs that define satisfied requirements. An example requirements diagram is shown in Figure 32. This example is based on Friedenthal’s 2009 and 2010 examples [68, 70]. One can quickly see the requirements hierarchy from the requirements diagram. The car specification package includes all of the listed requirements. Below that are the emissions, performance, and utility requirements. The emissions requirements shows a little more than the others with the text of the requirement listed and the elements that satisfies and verifies it. Note that the satisfies and verifies can be listed within the box or as call outs as is shown with the seats requirements under the utility requirement.



**Figure 32:** Requirements Diagram Example

Within the category of structure diagrams, SysML includes block definition diagrams and internal block diagrams. The block definition diagram is a formalized block diagram from systems engineering [69]. The block definition diagram defines the blocks in terms of their features and their relationship to other blocks [69]. These blocks represent an element of the system, which include: hardware, software, data, facility, people, a component, or conceptual entities [68, 69]. The features that define the block can be categorized into structural features, behavioral features, or constraints. The block is represented by a rectangle with a label and a series of compartments. The label is the only required part of the block. The components that can be added include: parts, references, and values. The parts will describe what the elements that make up the block. The references will refer to parts of other blocks. The values will define some characteristic of the block such as the weight. Blocks are related to each other using composite associations. These associations relate the

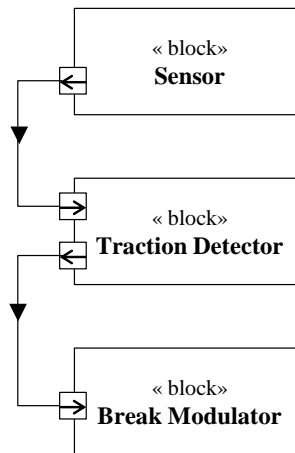
whole to the part. These are represented as solid black lines. The block that represents the whole starts with a solid diamond. The other end will represent an open arrow if it is a part property and a solid line if it is a reference property. An example block definition diagram based on Friedenthal's examples is shown in Figure 33 [68, 70]. The highest level is the car which has three parts identified. The car block lists a values component but not a parts component. This is optional. The chassis and tires block shows the parts component where four parts reference the wheel block. The associations are not shown for this block. The breaking system block is further decomposed into its parts.



**Figure 33:** Block Definition Diagram Example

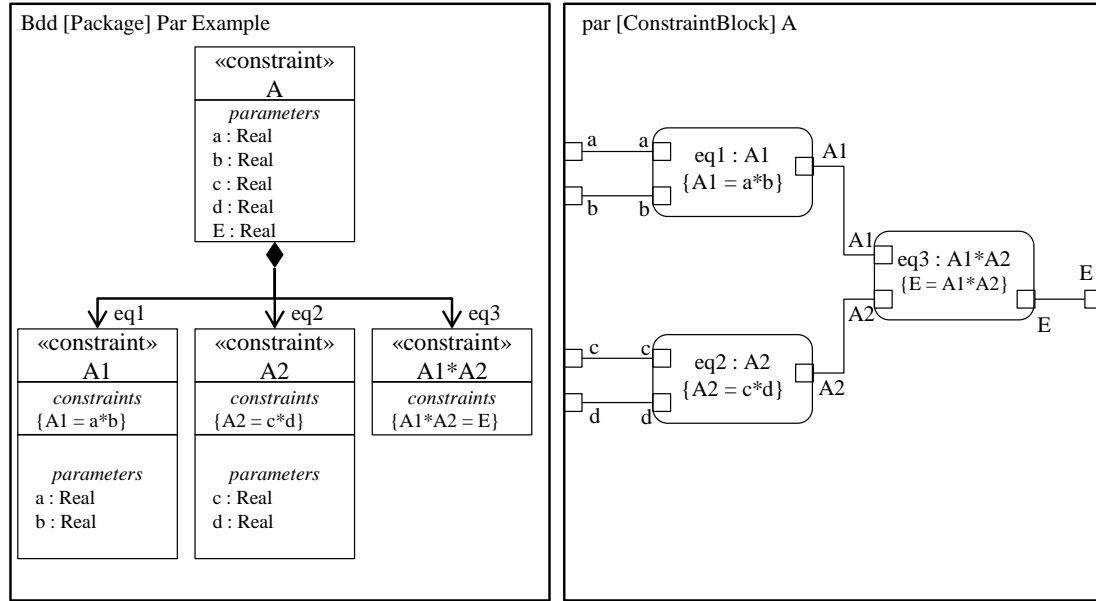
Internal block diagrams present another block composition. These resemble the component diagram from UML. This diagram allows the connection of blocks using connectors and ports. The whole block from the block definition diagram, i.e. the block that contains the parts, can be modeled as a diagram. The parts that make up the whole are represented as blocks and can have standard ports or flow ports. Standard ports are similar to UML ports. They specify the required operations or signals. The flow port specifies what is being transferred. The flow port indicates something that is being passed. These types of ports are shown in Figure 34. This figure was borrowed from Friedenthal's 2009 tutorial [68].

This internal block diagram is based on the block anti-lock controller from Figure 33. The standard ports use the same notation from UML's component diagram.



**Figure 34:** Internal Block Diagram Example

Parametric diagrams are used to describe the constraints on property values. These constraints refer to the physical or performance equations of the property values. These are represented with constraint blocks. Constraint blocks are specialized blocks. They have two main features: parameters and expressions that constrain the parameters [69]. The constraint block is defined on the block definition diagram and used on the parametric diagram. Constraints are then connected to the other constraints in a similar manner to internal block diagrams; however, instead of connecting ports parameters are connected. An example based on Paredis' tutorial is shown in Figure 35 [140]. This figure shows a block definition diagram on the left and the parametric diagram on the right. The right diagram is an expansion of the constraint block A. In the left diagram one can see on a constraint can be defined with the parameters. Constraint A does not have any constraints listed. Instead it breaks out three more constraint blocks. On the right diagram one can see how the parameters interact within the constraint block A.



**Figure 35:** Parametric Diagram Example

#### 3.2.4.1 The Use of Systems Modeling Language in the Literature:

Despite UML and SysML being listed as formal tools that can aid in conceptual model development [112], there is not an abundant number of examples within the literature of SysML being used for these purposes. Note that SysML is used extensively for model development within the field of model based systems engineering; however, this is not the same form of modeling that has been discussed in this thesis. Of the found examples within the literature Rao uses SysML to define the Global Earth Observation System of Systems. Rao then uses colored petri-nets to turn the SysML views into executable models, which are then used to help validate the SysML model [147]. Similarly, Wang uses SysML as an executable architecture in conjunction with colored petri-nets to aid in discrete event simulation modeling [207]. Both Rao and Wang hail from the Systems Engineering Department at the University of Missouri. Finally, McGinnis suggests the use of SysML to provide a means for automatic generation of object-oriented models [108]. The majority of SysML usage within the literature is to aid in model development more directly, as opposed to using SysML as a means to decompose the system in order to develop a conceptual model. An exception to this is a report by Keuning in which SysML is utilized in their methodology,

MD3S. Their methodology attempts to merge distributed simulations for the analysis of a given system where multiple simulation models are available with varying degrees of fidelity [86]. Another exception is a paper by Huang, written with McGinnis, where SysML is used to decompose a system and then provide automatic simulation generation [78].

#### *3.2.4.2 Ranking of SysML on the System and Objectives Definition Criterion:*

The criterion defined for system and objective definition are simplicity, flexibility, communicability, transferability. Since, SysML was derived from UML the two languages are very similar, therefore the ratings of the two will be very similar. For the criterion of simplicity SysML is rated as moderate. The language contains a standardized set of nine diagrams. The standardization helps improve the simplicity. The nine diagrams are not a small number of diagrams to review for application, though not all are required. Additionally, the numerous details of the diagrams makes this language more complicated.

For the criterion of flexibility SysML is rated as good. Unlike UML, SysML is used to define a system beyond software systems. The structure of SysML is very well suited to the requirements of decomposition for this application.

For the criterion of communicability SysML is rated as good. The history of SysML's use and the standardization of SysML aid in its communicability rating. The need for knowledge of SysML of those involved limit the rating to good as opposed to very good.

Finally, the criterion of transferability is rated as very good. The benefit of SysML over UML on this criterion is that SysML offers the ability to decompose the system into its parts. This then enables the modelers to identify the parts that need to be modeled more easily. This leads very well into determining what should be modeled and how it should be modeled. This would be accomplished in the next step of conceptual modeling.

### **3.2.5 Department of Defense Architecture Framework**

The Department of Defense Architecture Framework (DoDAF) is an architecture the provides organized visualizations. The purpose of DoDAF is to provide “the means of abstracting essential information from the underlying complexity and presenting it in a way

that maintains coherence and consistency” [58]. DoDAF can trace its history to the Clinger-Cohen Act of 1996 [4]. The goal of this act was to improve the way in which the Federal Agencies select and manage IT resource. As a response to the act the Command, Control, Communications, Computers, and Intelligence, Surveillance, and Reconnaissance(C4ISR) Architecture Framework was developed by the DoD and published in the December of 1997. The C4ISR Architecture Framework was intended to ensure that architectures developed by the parts of the DoD were interrelatable and comparable. Additionally, the framework would provide a clear audit trail from mission operations to current or proposed systems [42]. This work defines three viewpoints: Operational Architecture Viewpoint, Systems Architecture Viewpoint, and Technical Architecture Viewpoint. It should be noted that the C4ISR uses the term view, however, the later DODAF uses the term viewpoint. In August 2003 the DoD released DoDAF version 1.0. The new framework restructured the C4ISR Framework to broaden the applicability of the architectures to all mission areas beyond just the C4ISR community [55]. DoDAF maintained the three main viewpoints from the C4ISR Framework. In April of 2007 DoDAF version 1.5 was released. This version was aimed at improving the former version in its application to net centric warfare. Additionally, it offered guidance on its development and use. The most recent version, DoDAF version 2.0, was approved in May 2008. One of the major changes was the expansion of viewpoints to a total of eight. Additionally, DoDAF was changed from a product-centric process to a data-centric process with much of the terminology was changed for better structuring [58].

Eight categories of viewpoints are provided by DoDAF and 52 views in total [58]. These eight viewpoints are the All Viewpoint, Capability Viewpoint, Data and Information Viewpoint, Operational Viewpoint, Project Viewpoint, Services Viewpoint, Standards Viewpoint, and Systems Viewpoint. These viewpoints are described in the DoDAF Architecture Framework Version 2.0 and are reproduced in Table 9 [58]. The operation viewpoints and system viewpoints are of particular interest to system decomposition for model development; therefore, these viewpoints will be expanded upon further.

The Operational Viewpoints contain nine different views. These views with their descriptions from the DoDAF Architecture Framework Version 2.0 is shown in Tabel 10. Of

**Table 9: DoDAF Viewpoints**

<b>Viewpoint</b>	<b>Description</b>
All Viewpoint	The All Viewpoint describes the overarching aspects of architecture context that relate to all viewpoints.
Capability Viewpoint	The Capability Viewpoint articulates the capability requirements, the delivery timing, and the deployed capability.
Data and Information Viewpoint	The Data and Information Viewpoint articulates the data relationships and alignment structures in the architecture content for the capability and operational requirements, system engineering processes, and systems and services.
Operational Viewpoint	The Operational Viewpoint includes the operational scenarios, activities, and requirements that support capabilities.
Project Viewpoint	The Project Viewpoint describes the relationships between operational and capability requirements and the various projects being implemented. The Project Viewpoint also details dependencies among capability and operational requirements, system engineering processes, systems design, and services design within the Defense Acquisition System process.
Services Viewpoint	The Services Viewpoint is the design for solutions articulating the Performers, Activities, Services, and their Exchanges, providing for or supporting operational and capability functions.
Standards Viewpoint	The Standards Viewpoint articulates the applicable operational, business, technical, and industry policies, standards, guidance, constraints, and forecasts that apply to capability and operational requirements, system engineering processes, and systems and services
Systems Viewpoint	The Systems Viewpoint, for Legacy support, is the design for solutions articulating the systems, their composition, interconnectivity, and context providing for or supporting operational and capability functions.

these views the OV-1, OV-2, OV-3, OV-5a, and OV-5b will be discussed in greater detail. Additionally, these views can be represented in numerous way. For a more in depth description of the Operational Viewpoints see the Architecture Framework Version 1.5 Volume II

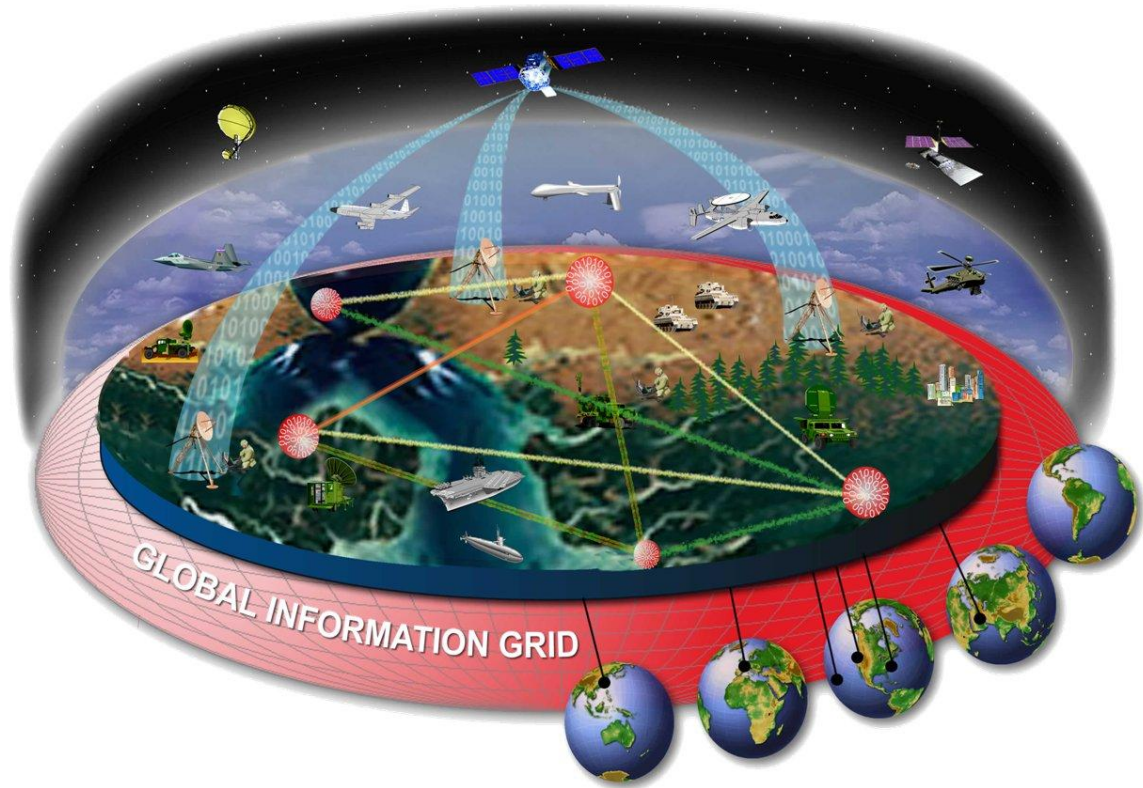


[55].

**Table 10:** DoDAF Operational Viewpoints

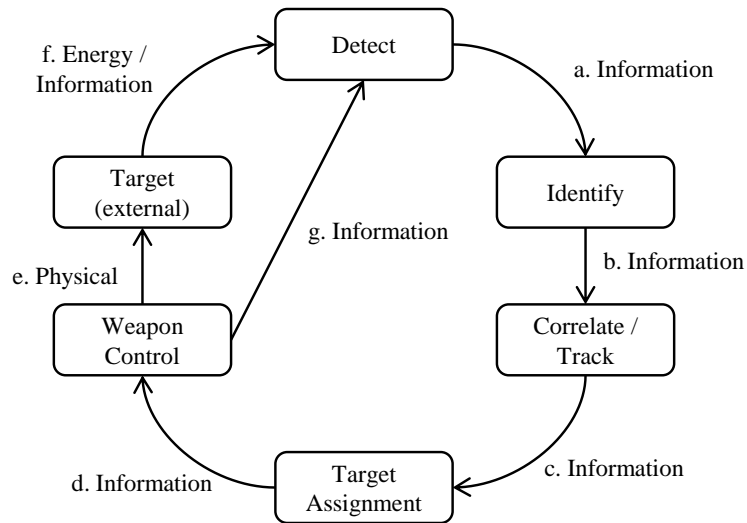
Viewpoint	Description
OV-1: High-Level Operational Concept Graphic	The high-level graphical/textual description of the operational concept.
OV-2: Operational Resource Flow Description	A description of the Resource Flows exchanged between operational activities.
OV-3: Operational Resource Flow Matrix	A description of the resources exchanged and the relevant attributes of the exchanges.
OV-4: Organizational Relationships Chart	The organizational context, role or other relationships among organizations.
OV-5a: Operational Activity Decomposition Tree	The capabilities and activities (operational activities) organized in a hierarchal structure.
OV-5b: Operational Activity Model	The context of capabilities and activities (operational activities) and their relationships among activities, inputs, and outputs; Additional data can show cost, performers or other pertinent information.
OV-6a: Operational Rules Model	One of three models used to describe activity (operational activity). It identifies business rules that constrain operations.
OV-6b: State Transition Description	One of three models used to describe operational activity (activity). It identifies business process (activity) responses to events (usually, very short activities).
OV-6c: Event-Trace Description	One of three models used to describe activity (operational activity). It traces actions in a scenario or sequence of events.

Probably the most common and known of these is the OV-1. The OV-1 is a quick, high-level image of the overall concept. Often times these images are made to be flashy even though it is stated that graphics alone are not sufficient for an OV-1. In essence the OV-1 is the highest level view of the architecture being presented. An example of an OV-1 can be seen in Figure 36. This OV-1 was developed by the DoD for the Global Information Grid project. An OV-1 has three uses: placing operational situation into context, providing a tool for discussion, and provide an aggregate illustration of the details [58].



**Figure 36: DoDAF OV-1 Example**

The OV-2 is used to define the capability requirements or capability boundary within context. The OV-2 shows flows, e.g. funding, personnel, material, information. The OV-2 describes the need for the exchange of resources with who or what defined but not the how defined. The intended usages include but are not limited to definition of operational concepts, elaboration of capability requirements, definition of collaboration needs, operational planning, supply chain analysis, and allocation of activities to resources [58]. An example of an OV-2 can be seen in Figure 37 and is based on Griendling's doctoral work [74]. Figure 37 shows an example of a suppression of enemy air defenses. Once a detection is made of a target, information is passed to the identify node which determines if the target is an enemy. Information is passed through the kill chain until it gets to the weapon control node. Here the flow from the weapon control node to the target is physical, such as a guided bomb. Information is then obtained by the detection node.



**Figure 37:** DoDAF OV-2 Example

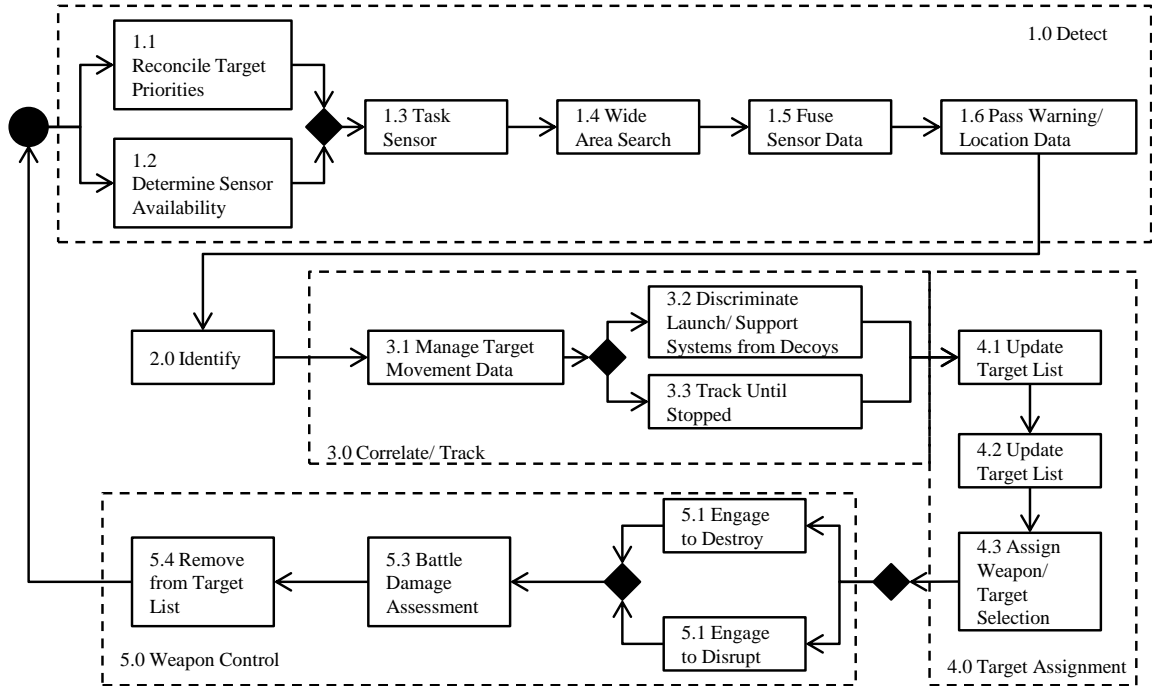
The OV-3 is used to define the interoperability requirements. The OV-3 is based on the OV-2, but it provides more detailed information. Based on the DoDAF Architecture Framework 2.0 “The Operational Resource Flow Matrix details Resource Flow exchanges by identifying which Operational Activity and locations exchange what resources, with whom, why the resource is necessary, and the key attributes of the associated resources. [58]” The purpose of the OV-3 is to capture the most important aspects of the resource flow not an exhaustive list of every possible resource. An example of an OV-3 is shown in Table 11 and is based on Griendling’s doctoral work [74]. Table 11 shows an example of a suppression of enemy air defenses; the same scenario shown with the OV-2 in Figure 37. As can be seen, each of the resource flows is displayed with greater information than is shown in the OV-2.

The OV-5a and OV-5b define the tasks that are required to achieve a mission or goal. These views describe the input and output flows between activities. These are used to define lines of responsibility in relation to the OV-2, make decisions about streamlining, and provide a foundation for the OV-6 views. The OV-5 and OV-2 are complements of each other in a sense. The OV-2 describes the operational resource flow and the OV-5 describes the operational activities. There are two ways to depict an activity model. The first is to decompose an activity in a tree structure; this would be an OV-5a. The second is to connect the activities by the resource flows; this would be an OV-5b [58]. An example

**Table 11: DoDAF OV-2 Example**

ID	Source Node	Node Activity	Type	Resource	Sink Node
a	Detect	Coordinate Sensors	Information	Warning/ Location Data	Identify
b	Identify	Identify Friend from Foe	Information	Positive Enemy Target ID	Correlate/ Track
c	Correlate/ Track	Maintain Positive Target Track	Information	Updated Target Track	Target Assignment
d	Target Assignment	Target Assessment and Assignment	Information	Target Assignment Plan and Permission to Fire	Weapon Control
e	Weapon Control	Engage Target	Physical	Destructive or Disruptive Weapon	Target
f	Target	Sense Target	Energy/ Information	Sensor Energy/Data	Detect
g	Weapon Control	Target Engagement	Information	Battle Damage Assessment	Detect

of the OV-5 is shown in Figure 38 and is based on Domercant's doctoral work [59]. This example of an OV-5 relates to the previous OV-2 and OV-3. These three views can be compared to see how they support each other.



**Figure 38:** DoDAF OV-5 Example

The System Viewpoints contain 13 different views. These viewpoints describe the systems and their interconnections. They also associate the resources to the capability requirements. These views with their descriptions from the DoDAF Architecture Framework Version 2.0 is shown in Table 12. Of these views the SV-1, SV-2, and SV-4 will be discussed in greater detail. Additionally, these views can be represented in numerous way. For a more in depth description of the Operational Viewpoints see the Architecture Framework Version 1.5 Volume II [55].

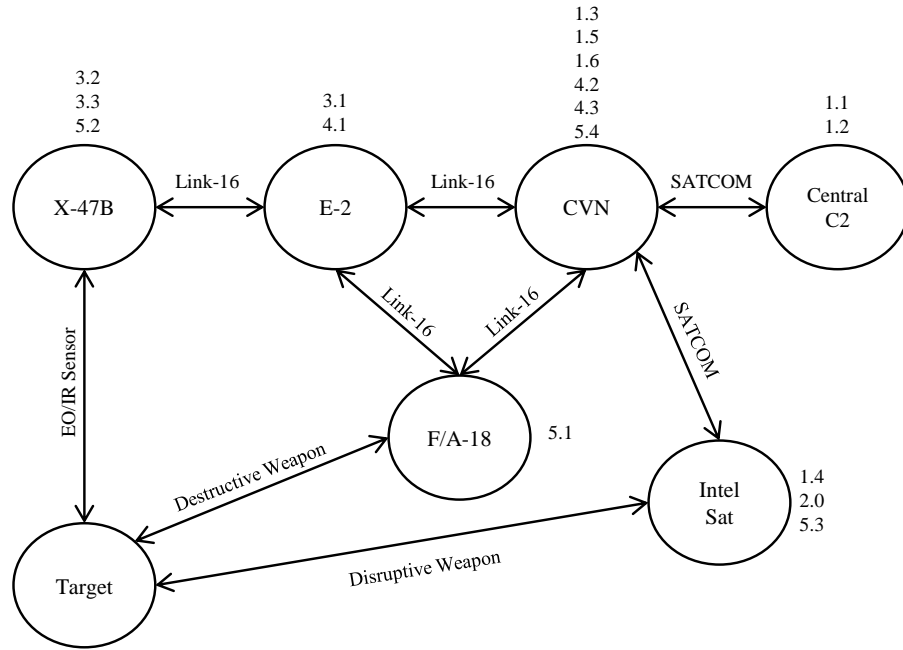
The SV-1 can be the realization of the requirements specified by the OV-2. The SV-1 links together the operational and system architecture by depicting how the resources interact. OV-2 links together activities through resource flow, where the SV-1 will link together the systems through the same resource flows. The SV-1 both describes the resource flow between systems or components in the architecture and describes a solution in terms of systems or components that satisfy the operational needs. The intended usage of an SV-1 includes but is not limited to definition of system concepts or options, system resource flow requirements capture, and operational planning [58].

**Table 12: DoDAF System Viewpoints**

<b>Viewpoint</b>	<b>Description</b>
SV-1: Systems Interface Description	The identification of systems, system items, and their interconnections.
SV-2: Systems Resource Flow Description	A description of Resource Flows exchanged between systems.
SV-3: Systems-Systems Matrix	The relationships among systems in a given Architectural Description. It can be designed to show relationships of interest, (e.g., system-type interfaces, planned vs. existing interfaces).
SV-4: Systems Functionality Description	The functions (activities) performed by systems and the system data flows among system functions (activities).
SV-5a: Operational Activity to Systems Function Traceability Matrix	A mapping of system functions (activities) back to operational activities (activities).
SV-5b: Operational Activity to Systems Traceability Matrix	A mapping of systems back to capabilities or operational activities (activities).
SV-6: Systems Resource Flow Matrix	Provides details of system resource flow elements being exchanged between systems and the attributes of that exchange.
SV-7: Systems Measures Matrix	The measures (metrics) of Systems Model elements for the appropriate timeframe(s).
SV-8: Systems Evolution Description	The planned incremental steps toward migrating a suite of systems to a more efficient suite, or toward evolving a current system to a future implementation.
SV-9: Systems Technology and Skills Forecast	The emerging technologies, software/hardware products, and skills that are expected to be available in a given set of time frames and that will affect future system development.
SV-10a: Systems Rules Model	One of three models used to describe system functionality. It identifies constraints that are imposed on systems functionality due to some aspect of system design or implementation.
SV-10b: Systems State Transition Description	One of three models used to describe system functionality. It identifies responses of systems to events.
SV-10c: Systems Event-Trace Description	One of three models used to describe system functionality. It identifies system-specific refinements of critical sequences of events described in the Operational Viewpoint.

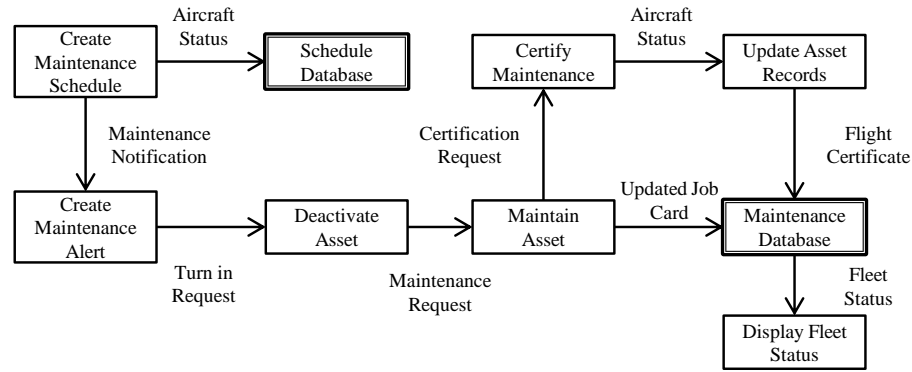
The SV-2 specifies the resource flows between the systems and gives precise specification of the connection between the systems. The SV-2 can show the connection between ports,

and the systems the ports belong to well as show the resource flows in terms of the physical or logical connectivity and the protocols [58]. The SV-2 further specifies the SV-1. These two views are combined into one example shown in Figure 39. This example is based on Domercant's doctoral work [59].



**Figure 39:** DoDAF SV-1/SV-2 Example

The SV-4 is the equivalent to the OV-5b for the Systems Viewpoint. The SV-4 is intended to describe the task workflow, identify functional system requirements, provide functional decomposition of the systems, and relate human and system functions. The two basic ways of depicting a SV-4 are to show a decomposition of functions depicted in a tree structure or with a data flow diagram showing functions connected by data flow arrows and data stores [58]. An example of the later, based on an IBM example, of a SV-4 is shown in Figure 40 [80].



**Figure 40:** DoDAF SV-4 Example

### 3.2.5.1 The Use of DoDAF in the Literature:

The most common application of DoDAF with M&S is the creation of executable architectures [214, 32, 75]. Often, an executable architecture attempts to combine the DoDAF and modeling. A user interface may be developed that allowed the created DoDAF views to be then run as a discrete event simulation or system dynamic model. In this application developing the DoDAF views would also act as model programming. Executable architectures use M&S to expand on the usefulness of DoDAF. Conversely, DoDAF has also been used to expand the usefulness and credibility of M&S. Zeigler and Mittal have used DoDAF to aid in the initial stages of model development [216, 110, 109]. Particularly, they have used DoDAF as a precursor and an enhancement to the DEVS formalism developed by Zeigler in the 1970s [217].

### 3.2.5.2 Ranking of DoDAF on the System and Objectives Definition Criterion:

The criterion defined for system and objective definition are simplicity, flexibility, communicability, transferability. For the criterion of simplicity DoDAF is rated as poor. The language contains a standardized set of 52 diagrams. The standardization helps improve the simplicity; however, the 52 diagrams is a large number of diagrams to review for application, though not all are required.

For the criterion of flexibility DoDAF is rated as good. One major benefit to flexibility is that there is no strict standardization in the DoDAF views. Each view is intended to



accomplish a goal. For the criterion of communicability DoDAF is rated as moderate.

Two issues limit the DoDAF for its communicability. The first is that DoDAF is primarily used by the United States Defense Industry, which represents a small part of all modeling requirements. For this reason many would be required to learn DoDAF in order to use it. The second is that the DoDAF does not have a strict standardization such as UML or SysML.

Finally, the criterion of transferability is rated as very good. DoDAF offers the ability to decompose the system into its parts. This then enables the modelers to identify the parts that need to be modeled more easily. This leads very well into determining what should be modeled and how it should be modeled. This would be accomplished in the next step of conceptual modeling.

### ***3.3 Selection of a Method for System and Objectives Definition***

A selection will be made in the same manner in which Research Question One was answered, using AHP. The comparisons will be based on the Comparison Scale shown in Table 2. Before creating comparisons of the criteria for the system definition, the criteria are placed in order from most desirable to least. This ranking is as follows: transferability, flexibility, simplicity, and communicability.

Transferability was easily selected as the most important because the purpose of the system definition is to better enable conceptual model development. Flexibility is a close second, because a methodology must be able to work for a wide array of problems. The last two, simplicity and communicability, are more difficult to determine preference. It was determined that simplicity was preferable to communicability because each of the methods discussed above will provide communication between the modeler and the SME. Primarily the differences between the methods will occur in communication with other audiences for model reuse. Model reuse is not the primary goal of this research objective; however, it is a concern. For this reason the criterion of simplicity is preferred over communicability.

The comparisons are then made between the criteria. This is detailed in Table 13. The results of this comparison show that transferability comprises of 49% of the priority and

flexibility comprises of 30% of the priority. These two constitute the bulk of the impact of the criteria. The remaining two, simplicity and communicability represent 13% and 8% of the priority.

**Table 13:** Pairwise Comparison of Criterion

<i>Criterion</i>	Simplicity	Flexibility	Communicability	Transferability	<i>Priority</i>
Simplicity	1	1/3	2	1/4	0.13
Flexibility	3	1	4	1/2	0.31
Communicability	1/2	1/4	1	1/5	0.08
Transferability	4	2	5	1	0.49

The next step is to compare the four system definition methods in the criterion of simplicity. The four methods Soft Systems Methodology, Unified Modeling Language, Systems Modeling Language, and Department of Defense Architecture Framework were rated for simplicity as good, moderate, moderate, and poor, respectively. The comparisons are then made between the methods. This is detailed in Table 14. As can be seen, SSM is rated at 52%. UML and SysML were considered equivalent each rated at 20%. Finally, DoDAF was considered the most complicated resulting in a rating of 8%.

**Table 14:** Pairwise Comparison of Simplicity

<i>Simplicity</i>	SSM	UML	SysML	DoDAF	<i>Priority</i>
SSM	1	3	3	5	0.52
UML	1/3	1	1	3	0.20
SysML	1/3	1	1	3	0.20
DoDAF	1/5	1/3	1/3	1	0.08

The next step is to compare the four system definition methods in the criterion of flexibility. The four methods SSM, UML, SysML, and DoDAF were rated for flexibility as very good, moderate, good, and good, respectively. The comparisons are then made between the methods. This is detailed in Table 15. As can be seen SSM is rated at 52%.

The next highest rated, DoDAF is rated at 24%. SysML is rated at 15%. Finally, UML was considered the least flexible resulting in a rating of 9%.

**Table 15:** Pairwise Comparison of Flexibility

<i>Flexibility</i>	SSM	UML	SysML	DoDAF	<i>Priority</i>
SSM	1	5	3	3	0.52
UML	1/5	1	1/2	1/3	0.09
SysML	1/3	2	1	1/2	0.15
DoDAF	1/3	3	2	1	0.24

The next step is to compare the four system definition methods in the criterion of flexibility. The four methods SSM, UML, SysML, and DoDAF were rated for communicability as moderate, good, good, and moderate, respectively. The comparisons are then made between the methods. This is detailed in Table 16. As can be seen UML and SysML are considered equivalent and are rated at 39%. The next highest rated, DoDAF, is rated at 15%. Finally, SSM was considered the least communicability resulting in a rating of 7%.

**Table 16:** Pairwise Comparison of Communicability

<i>Communicability</i>	SSM	UML	SysML	DoDAF	<i>Priority</i>
SSM	1	1/5	1/5	1/3	0.07
UML	5	1	1	3	0.39
SysML	5	1	1	3	0.39
DoDAF	3	1/3	1/3	1	0.15

The next step is to compare the four system definition methods in the criterion of flexibility. The four methods SSM, UML, SysML, and DoDAF were rated for transferability as moderate, moderate, very good, and very good, respectively. The comparisons are then made between the methods. This is detailed in Table 17. As can be seen SysML and DoDAF are considered equivalent and are rated at 40%. The next highest rated, UML, is rated at 15%. Finally, SSM was considered the least transferability resulting in a rating of 6%.

The results of the AHP method is shown in Table 18. As can be seen SysML, DoDAF, and SSM are very closely related. SysML and DoDAF are almost identical. From this

**Table 17:** Pairwise Comparison of Transferability

<i>Transferability</i>	SSM	UML	SysML	DoDAF	<i>Priority</i>
SSM	1	1/3	1/6	1/6	0.06
UML	3	1	1/3	1/3	0.15
SysML	6	3	1	1	0.40
DoDAF	6	3	1	1	0.40

analysis, any of the three methods would be sufficient to fulfill the system definition step in the Balci model development procedure. For this methodology SysML will be used. An additional supporting argument for the use of SysML is that later in the model development a communicative model and model specification must be developed. For this step it will be later shown that UML is the best suited. Since UML and SysML are so similar, the methodology will perform better if both similar methods were used.

**Table 18:** AHP Score RQ2

	SSM	UML	SysML	DoDAF
Score	0.259	0.155	0.297	0.290

**Research Question 2 Hypothesis:** The Systems Modeling Language is the best suited method for the system and objective phase of model development

## CHAPTER IV

### CONCEPTUAL MODELS AND COMMUNICATIVE MODELS

The definition of the conceptual model was addressed briefly earlier in this thesis. A more thorough understanding of a conceptual model will now be addressed. There exist a number of different definitions within the literature. Of these definitions, the scope of the conceptual model varies widely. Many consider the steps of system and objectives definition, conceptual model, and the communicative model as a part of the conceptual model. Various viewpoints will be presented and decomposed to better understand what a conceptual model is in respect to Balci's procedure.

The first conceptual model description is provided by Robinson. Robinson emphasizes in his work that conceptual modeling is the most important aspect of the modeling process; however, it is also the least understood and most difficult [153, 155, 156]. Robinson defines a conceptual model as the following.

‘The conceptual model is a non-software specific description of the simulation model that is to be developed, describing the objectives, inputs, outputs, content, assumptions and simplifications of the model.

*Objectives:* the purpose of the model and modelling project.

*Inputs:* those elements of the model that can be altered to effect an improvement in, or better understanding of, the real world; otherwise known as the experimental factors.

*Outputs:* report the results from simulation runs.

*Content:* the components that are represented in the model and their interconnections.

*Assumptions:* made either when there are uncertainties or beliefs about the real world being modelled.

*Simplifications*: incorporated in the model to enable more rapid model development and use.’ [153]

The objectives are included in Robinson’s conceptual model definition. This thesis places the objectives before conceptual models in the system and objectives stage. Other than the objectives placement, Robinson’s viewpoint is in line with this thesis. Additionally, Robinson offers four criteria to define a good conceptual model: validity, credibility, utility, and feasibility [153, 156].

Dale Pace emphasizes conceptual modeling by stating that conceptual model validation should be the foundation of model credibility [134]. At the same time he criticizes the term for being overused and having too many meanings [134]. Pace defaults to the definition given by the *DoD Recommended Practices Guide for Verification, Validation, and Accreditation (VV&A)* [135, 138]. This definition is provided below. In his work he provides four criteria for a conceptual model: completeness, consistency, coherence, and correctness. The criterion of completeness is concerned with identifying all of the entities, control characteristics, and operating characteristics in the problem domain. The criterion of consistency is concerned with addressing coordinate systems, units, levels of aggregation, precision, and accuracy from the same perspective. The criterion of coherence is concerned with making sure all identified aspects in completeness have a function in the conceptual model and are able to be translated into the programmed model. Finally, the criterion of correctness is conceptual model validity in the terminology used in this thesis.

A simulation conceptual model is the simulation developer’s way of translating model requirements (i.e., what is to be represented by the simulation) into a detailed design framework (i.e., how it is to be done), from which the software, hardware, networks (in the case of distributed simulation), and systems/equipment that will make up the simulation can be built. (DoD Recommended Practices Guide for VV&A) [53]

Balci has also contributed to the discussion on conceptual models. In his early work he defines a conceptual model as the model that is formulated in the mind of the modeler

[19, 15, 20, 25]. This definition is quite general and not of much use. In his later work, starting in 2008, he redefines a conceptual model as follows:

A simulation conceptual model is a repository of high-level conceptual constructs and knowledge specified in a variety of communicative forms (e.g., animation, audio, chart, diagram, drawing, equation, graph, image, text, and video) intended to assist in the design of any type of large-scale complex M&S application. [24, 18, 22, 23]

In Balci's work he has stressed the use of conceptual models to aid in model reuse [25, 24, 18]. Some of the additional uses of conceptual models that Balci has identified are as follows: to assist in the design of models, enable communication between all those involved, overcome complexity of the problem, and assist in verification and validation.

From the discussion above, a coherent understanding of a conceptual model can be made. A new definition is not needed, for the ones identified from the literature all sufficiently represent the conceptual model. Of the definitions listed the one that is chosen to be used in this thesis is the one presented by the Department of Defense in the 1996 report of Recommended Practices Guide for (VV&A). What is missing from many papers on the discussion of conceptual models is a quantification of a good conceptual model. Robinson and Pace offer a set of criteria. Combining the criteria presented it can be concluded that a good conceptual model is valid, credible, complete, consistent, coherent, and feasible.

Despite conceptual modeling being the most important part of model development, little interest is given to the subject [153, 154, 156, 39]. One of the arguments for this is that M&S is often referred to as an art more than a science. Around 2005 the community began investigating the subject of conceptual modeling in depth [25, 155, 94, 48]. One of the primary issues identified with conceptual modeling is that there was no discussion on how to conceptual model [135, 157]. Addressing this issue Robinson [157], Balci [25], and Chwif [48] produced papers discussing how to develop a conceptual model. Of the few methods available in the literature, the primary focus is on the application to industrial systems modeling. Another shortcoming found in the literature is that determining how much

fidelity to model specific components of the system is often brushed over. Primarily subject matter experts are used to define this fidelity requirement based on their experience. This approach works well for industrial systems, where research into modeling and simulating these systems has spanned six decades; however, in the absence of a rich history there exists a gap in identifying the important parts of the system that need greater fidelity. For these reasons a new conceptual modeling approach is needed. This leads to research question 3.

**Research Question 3:** How should a conceptual model be developed in a traceable and defensible manner that addresses the fidelity requirements for the components of the model?

#### ***4.1 Proposed Conceptual Model Approach***

Based on the review of the literature on conceptual models, there is a gap in current approaches in addressing the levels of fidelity for each component of the model. Few approaches are presented in the literature. The ones that are present in the literature do not address how to model a component and to what fidelity the component should be modeled. The current approaches treat M&S more like an art than a science. A need exists for an approach to decompose the system and identify the relationships that most impact the system in a traceable and defensible manner. This will then lead into the modeler making educated decisions on how to represent the components of the system based on the sensitivity of the overall system behavior. An added benefit of identifying important relationships of the system is that once the model is developed the results can be compared to the decomposed system as an additional validation process. The identified need of a conceptual modeling approach leads directly to a generic hypothesis to research question three. The details to hypothesis three will be thoroughly explored in the following subsections through a series of sub-research questions and hypotheses.

#### **Research Question 3 Methodological Hypothesis:**

1. Utilize subject matter experts to decompose the system and assign impact relationships between measures



2. Determine importance of relationships through analysis
3. Based on relationship importance, decide how the entity is represented and to what fidelity based on a selection scheme
4. Compare results to impact relationships

#### 4.1.1 Visualization of System Decomposition

The first sub-research question is based on the first step in the methodological hypothesis. A visual framework is needed to aid the subject matter experts in decomposing the system and assigning relationships that can also be used for the subsequent steps. This need leads to the research question 3.A shown below.

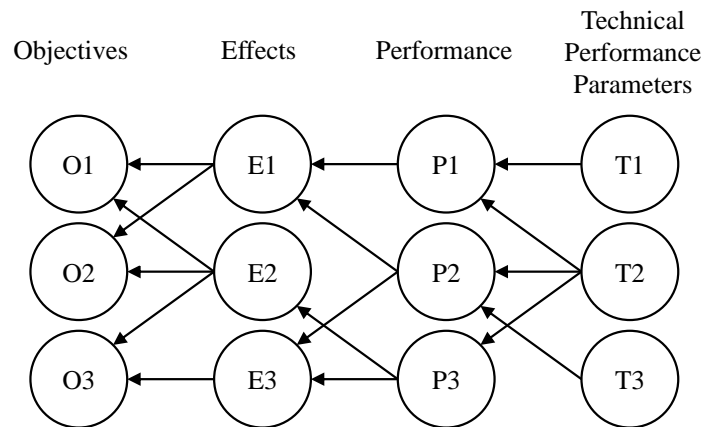
**Research Question 3.A:** What visual framework should be used to aid in the system decomposition?

Addressing this research question, the proposed conceptual modeling approach will take inspiration from both the Qualitative Function Deployment (QFD) [57] and the Analytic Hierarchy Process (AHP) [165], because both methods provide an approach for decomposing a problem. QFD aids its users in identifying metrics that have the greatest impact on the customer needs. AHP aids its users in decision making through pairwise comparison based on the subjective judgements of experts. Though different, in their most fundamental form they are mapping matrices that contain multiple levels of abstraction and relationships between these levels. These mapping matrices can be represented as a simple matrix or as a mathematical graph.

An example of a mathematical graph of a hypothetical system is shown in Figure 41. Here the system has been divided into four levels. These levels are broken up based on the work and definitions created by the Military Operation Research Society (MORS) on Measures of Effectiveness for Command and Control [73]. It must be noted that this example is not using the MORS approach but is only based on it. Additionally, this specific breakdown is being used here only as a means of a mapping matrix decomposition and not a suggestion of the specific manner that a system must be decomposed for conceptual modeling.

The decomposition is started by defining the objectives. An objective is a statement on the desired behavior of the system, e.g. increase military aircraft capabilities. Next, effects are defined that impact the objectives. An effect is a measure of how a system performs within a larger system, e.g. the number of enemies neutralized. Next, performance variables are defined that impact the effects. A performance measure is a measure of the system, e.g. range, speed. Finally, technical performance parameters are defined. A technical performance parameter is a characteristic of a system, e.g. wing span, gross weight.

In this breakdown the direction of impact travels from the technical performance parameters to the objectives. In the figure below each node represents some metric that is measurable. The arrows linking those metrics, referred to as edges, represent a relationship between the two levels. For example, technical performance parameter one, T1, has some impact on performance parameter one, P1, e.g. wing span has an impact on range, and P1 has some impact on effect parameter one (E1), e.g. range will impact the number of enemies the aircraft can neutralize. Another way in which it could be read is that P1 is a function of T1 and T2.



**Figure 41:** Example System Decomposition Graph

Figure 41 shows only edges between two metrics with a direct impact. Two other relationships could be represented with this type of visualization framework: indirect impacts and same level correlations. An indirect impact is one that spans multiple levels. For example, T1 has some impact on P1, and P1 has some impact on E1. Therefore, T1 has

an impact on E1. A same level correlation is a relationship between two metrics on the same level that do not directly impact each other, but instead are both impacted by the same lower level metric. For example, T2 impacts P1 and P2; therefore, it is expected that P1 and P2 would have a correlation based on their mutual reliance on T2. The question presents itself, should these relationships be represented on the mathematical graph?

Two arguments are made against including indirect impacts and same level correlations on the visual graph. The first argument is that this information is already presented in the graph, though they are not specifically defined. The second argument is that the introduction of new edges would only add clutter and confusion. Not only would there be more edges complicating the graph, but also there would be three types of edges: direct impact, indirect impact, and same level correlations. Using the three types of edges would only complicate the graph and add no benefit to the SMEs. Therefore, the conclusion reached is the indirect impacts and same level correlations should not be represented on the mathematical graph. The increased complication of the mathematical graph outweighs the increased information displayed.

A method to represent the same information as was presented in Figure 41 is to present it as a matrix. This matrix would be a 12 by 12 matrix that mapped every metric against each other. Along the diagonal would be a series of 3 by 3 matrices that represent the correlations between the metrics on the same level. Everything in the upper right side would represent the impact of one lower level metric on another higher level metric. These would include both direct impacts and indirect impacts. Everything in the lower left side would contain the value zero, for the metrics are unidirectional. An example of this is shown in Table 19.

Both visual frameworks provide a benefit to the SMEs in system decomposition. The mathematical graph gives a visual understanding of the relationships present in the system. The matrix provides a greater, in-depth presentation of the information that the graph does not show, e.g. indirect impacts and same level correlations. For these reasons, it is concluded that the visual framework will use both: the mathematical graph and the matrix. These visual frameworks will aid the SMEs in assigning the relationships between metrics.

**Table 19:** Conceptual Impact Model Matrix Concept

	Objectives	Effects	Performance	TPP
Objectives	Correlation Matrix L1	Impact Matrix L1.2	Impact Matrix L1.3	Impact Matrix L1.4
Effects	NA	Correlation Matrix L2	Impact Matrix L2.3	Impact Matrix L2.4
Performance	NA	NA	Correlation Matrix L3	Impact Matrix L3.4
TPP	NA	NA	NA	Correlation Matrix L4

**Research Question 3.A Hypothesis:** Visual framework will use both the mathematical graph and the matrix to aid the SMEs in system decomposition.

#### 4.1.2 Defining the Impact Relationship between Metrics

The previous section addressed how a system decomposition should be represented visually. It was concluded that a mathematical graph and an impact matrix would represent the impact relationship between the identified metrics. The next task is to determine what is meant by an impact relationship between two metrics. Eventually, analysis must be performed; therefore, the impact relationship is required to be mathematical in nature. The statement for this research question is presented below.

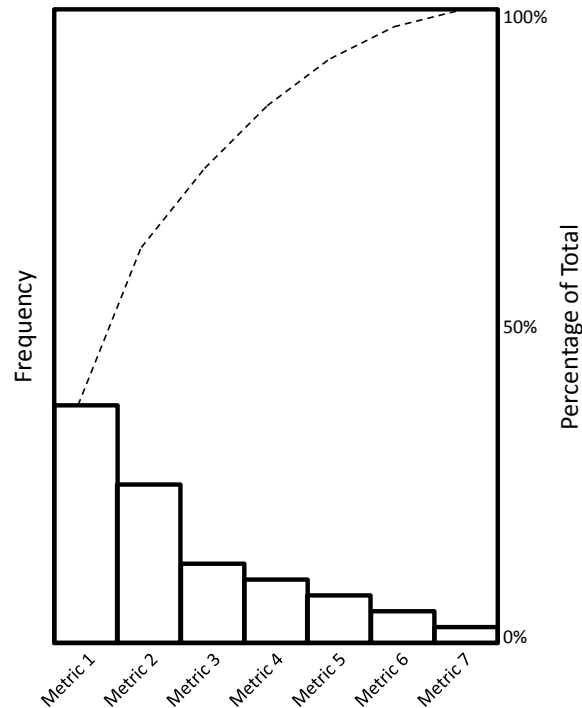
**Research Question 3.B:** What does the SME-defined impact relationship between a metric and a lower level metric best relate to mathematically?

Other methods, e.g. QFD, AHP, are vague about the precise meaning of an impact relationship between metrics. Even more so, the linkage between the impact relationships and the mathematical meaning is missing. To address this research question a number of mathematical candidates will be analyzed. The requirements for the impact relationship are as follows:

1. Answers the questions, what is meant by behavior; metric *A* contributes to what percentage of the behavior of metric *B*?
2. The sum of all impact relationship contributions to a metric must sum to 100%.

3. The mathematical measure must provide a clear understanding of the relationship between the two metrics.

The listed requirements for the mathematical measure are strongly influenced by the Pareto plot. The Pareto plot is a bar chart and a line graph that is often used to display the importance of measures to a response. On the left vertical axis shows the frequency of occurrence. Along the horizontal axis is the listed contributors. These form the bar chart that is sorted by frequencies. Overlain on the bar chart is a line that plots the cumulative percentage of the previous contributors [211]. An example Pareto plot is shown in Figure 42. The Pareto plot helps to show what the major contributors are and the fraction they contribute to the output measure. Similarly, the impact relationships will identify which metrics have the greatest contribution, or impact, to a higher level metric as a fraction of the total.



**Figure 42:** Pareto Plot Example

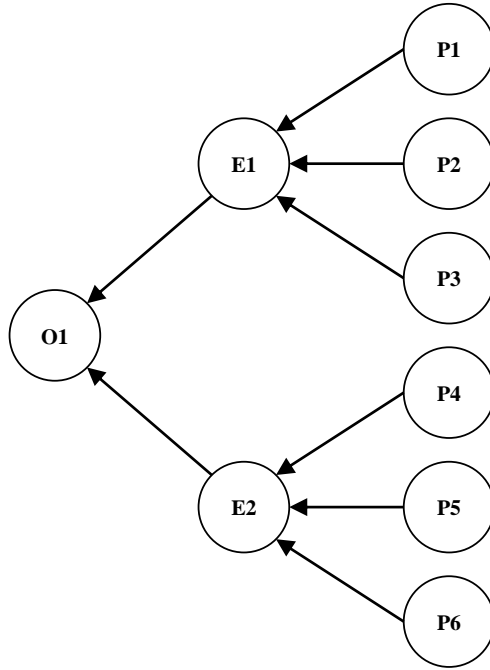
The impact relationships have been defined as relating the contribution of lower level metrics to higher level metric that when summed equal unity. The potential mathematical measures must now be tested to determine which measure best fits the requirement of an impact relationship. A set of candidate mathematical measures were identified and are mean, standard deviation, median, range, and correlation. There exists a countless number of measures that have the possibility of representing impact relationships. These measures were selected because they are some of the most common statistical measures.

A canonical example will be needed to test the potential measures. The canonical example is defined as follows. A factory is refining aluminum and copper ore to be sold on the market. The objective is to maximize the profit of the factory. Copper sells for three dollars per pound and aluminum sells for one dollar per pound. The objective can be broken into two effects: weight of copper produced and the weight of aluminum produced. These effects can be broken down into performance measures. There are trucks and trains that bring in raw ore. Additionally, there is an efficiency metric that determines the amount of the ore that can be turned into sellable metal per pound of raw material. A system decomposition of the canonical example can be seen in Figure 43. An explanation of each metric and the values that they take are shown in Table 20.

**Table 20:** Canonical Example Description

Name	Description	Value
O	Profit	$3 \times E1 + 1 \times E2$
E1	Weight of copper produced	$P2(P1 + P3)$
E2	Weight of aluminum produced	$P5(P4 + P6)$
P1	Weight of raw copper delivered by truck	10–100lbs
P2	Efficiency of copper production	30–50%
P3	Weight of raw copper delivered by train	500–1000lbs
P4	Weight of raw aluminum delivered by truck	10–100lbs
P5	Efficiency of aluminum production	30–50%
P6	Weight of raw aluminum delivered by train	500–1000lbs

The experiment is conducted as follows. Each performance metric is represented as a uniform distribution with a range defined in Table 20. There are a total of eight edges connecting the metrics. One at a time each edge is removed and the effects and objectives are



**Figure 43:** Canonical Example

recalculated with one million Monte Carlo runs. The mean, standard deviation, median, range, and correlation are then calculated for each case. The values for the objective, excluding correlation, can be seen in Table 21. The results for the correlation measure must be shown in a different form and will be shown subsequently. The first column result shows the calculated expected mathematical measure for profit of the full model, i.e. no removed edges. The remaining columns show the expected mathematical measure for profit when the listed edge is removed. As can be seen in Table 21, the removal of the P1–E1 link results in small changes for the value. The mean of the objective, i.e. profit, is reduced by 66, the standard deviation is reduced by 12, the median is reduced by 65, and the range is reduced by 83. In comparison, the P2–E1 link shows a significant loss for the measured values of the objective. A better view of this data is the percentage change from the baseline value shown in Table 22. Readdressing the comparison of the P1–E1 and P2–E1 links, it can be seen in Table 22 that each measure loses between 3 and 6% when the P1–E1 link is removed. Between 68% and 75% of each measure is lost when the P2–E1 link is removed.

**Table 21:** Canonical Example Metric Results for Objective

	Full	E1-O	E2-O	P1-E1	P2-E1	P3-E1	P4-E2	P5-E2	P6-E2
Mean	1288	322	966	1222	322	1266	966	388	988
Std	238	75	226	231	75	237	226	82	226
Median	1270	315	946	1205	315	1248	946	382	968
Range	1494	394	1182	1411	394	1471	1182	514	1210

**Table 22:** Canonical Example Metric Percentage Loss for Objective

	Full	E1-O	E2-O	P1-E1	P2-E1	P3-E1	P4-E2	P5-E2	P6-E2
Mean	-	75%	25%	5%	75%	70%	2%	25%	23%
Std	-	68%	5%	3%	68%	65%	0%	5%	5%
Median	-	75%	26%	5%	75%	70%	2%	26%	24%
Range	-	74%	21%	6%	74%	66%	2%	21%	19%

The values for effects 1 and 2, excluding correlation, can be seen in Table 23 and Table 25, respectively. The percentage change from the baseline values for effects 1 and 2 are shown in tables 24 and 26, respectively. Similar observations can be made for the effects as were made for the objective in these tables.

**Table 23:** Canonical Example Metric Results for Effect 1

	Full	P1-E1	P2-E1	P3-E1
Mean	322	300	0	22
Std	75	73	0	11
Median	315	294	0	22
Range	394	349	0	47

**Table 24:** Canonical Example Metric Percentage Loss for Effect 1

	Full	P1-E1	P2-E1	P3-E1
Mean	-	7%	100%	93%
Std	-	4%	100%	85%
Median	-	7%	100%	93%
Range	-	11%	100%	88%



**Table 25:** Canonical Example Metric Results for Effect 2

	Full	P4-E2	P5-E2	P6-E2
Mean	322	300	0	22
Std	75	73	0	11
Median	315	294	0	22
Range	393	350	0	47

**Table 26:** Canonical Example Metric Percentage Loss for Effect 2

	Full	P4-E2	P5-E2	P6-E2
Mean	-	7%	100%	93%
Std	-	4%	100%	85%
Median	-	7%	100%	93%
Range	-	11%	100%	88%

When comparing these measures against the measure requirement of summing to 100% and providing a clear understanding, the measures of standard deviation, median, and range all fail due to not summing to 100%. On first look of the results, the mean measure seems to be a sufficient measure for an impact relationship; however, the mean measure also would not sum to 100% in all cases. Only in this case, does the measure contribution sum to 100%. This is because the objective and effects are summations and multiplications. With this example when a link is removed the value is decreased. If the example had a situation where the removal of a link increased the value, then the mean measure would not sum to 100%. An example of this would be a throughput capacity limit placed on the effects. By removing the throughput capacity limit the total metal produced would increase. For these reasons the use of mean as a measure of the impact relationship is not sufficient.

The only remaining measure is correlation. The most common correlation used is the Pearson's Product-Moment Correlation. The equation for Pearson's Product-Moment Correlation is shown in Equation 1 where  $n$  is the sample size,  $x$  and  $y$  are the two sets of data [142]. When calculating the correlations between the objective and the lower level metrics, see Table 27, it is quickly seen that the summation is not zero. Additionally, since Pearson's Correlation is bounded between negative one and one, the summation could result in

a negative number as well. This result, if unaltered, would forbid correlation as a useful measure for impact relationships; however, by taking the normalized absolute values of the correlation, a useful measure can be found. The normalized correlation values offer a useful measure for impact relationships. This measure sums to 100% and has a clear meaning. Thus, a hypothesis to research question 3.B is offered below, suggesting the normalized correlation. This hypothesis now leads to another research question. Given that correlations should be used, which correlation is best suited? Research question 3.B.1 is presented below.

$$\rho = \frac{\sum_1^n (x_i - \bar{x})(y_i - \bar{y})}{\sqrt{\sum_1^n (x_i - \bar{x})^2 (y_i - \bar{y})^2}} \quad (1)$$

**Table 27:** Canonical Example Pearson Correlations

	E1	E2	P1	P2	P3	P4	P5	P6
Correlation	0.95	0.32	0.13	0.59	0.73	0.04	0.19	0.24
Normalized Correlation	75%	25%	6.8%	30.5%	37.9%	2.2%	10.1%	12.6%

**Research Question 3.B Hypothesis:** Normalized correlations should be used as the mathematical relation to the SME impact relationships.

**Research Question 3.B.1:** Which correlation should be used for measured values relating to impact relationships?

#### 4.1.2.1 Comparing Different Correlation Measures

After a review of the literature four correlations were identified as potentially useful for this application. These correlations are Pearson's Product-Moment Correlation, Spearman's Rank Correlation, Kendall tau Rank Correlation, Brownian Distance Correlation, and Copulas. Pearson's Correlation was introduced earlier in Equation 1. This correlation measure is the most common and is often simply referred to as correlation. The Pearson's Correlation has a few shortcomings. First, the measure is sensitive to outliers. A

few outliers in the data can significantly change its value. Most importantly the measure is limited to finding only linear relations. If the relationship between two sets of data is governed by a exponential relationship, a trigonometric relationship, or a step function, then the measure may result in a very low measure, despite a strong relationship between the two. Stated in a more mathematical terms, a low Pearson Correlation does not indicate statistical independence.

The two rank correlations attempt to address the sensitivity to outliers of the Pearson Correlation. Spearman's Rank Correlation is shown in Equation 2. At first glance the equations look the same. The primary difference is that Pearson's Correlation calculates on the raw value of the data, whereas Spearman's Correlation calculates on the rank value. The ranked value of a data is described as follows. Given a data set of  $N$ , the smallest value is given the rank value of 1. The largest value is given the rank value of  $N$ . If there are replications of the raw data, then both values are given the average of the two rank positions.

In his 1904 paper, Spearman lists two main advantages of using a rank correlation. The first is that the rank correlation reduces sensitivity to outliers, which he refers to as accidental error. The second benefit is that the rank correlation allows the comparisons between two datasets that are governed by different underlying distributions [184]. The drawback of Spearman's Correlation is that it captures primarily linear relationships.

$$\rho = \frac{\sum_1^n (S_i - \bar{S})(Y_i - \bar{Y})}{\sqrt{\sum_1^n (X_i - \bar{X})^2 (Y_i - \bar{Y})^2}} \quad (2)$$

Another rank correlation is provided by the Kendall tau Rank Correlation and is shown in Equation 3. The calculation of the Kendall tau Rank Correlation is similar to the Spearman Correlation. The process begins with finding the rank values of the data within the two datasets. From there, each pair of observations is checked to be concordant or discordant. A concordant pair is one in which the ranks of both elements agree, i.e. the pair  $(x_i, y_i), (x_j, y_j)$  are concordant if  $x_i > x_j$  and  $y_i > y_j$  or  $x_i < x_j$  and  $y_i < y_j$ . A discordant

pair is one in which the ranks of both elements disagree, i.e. the pair  $(x_i, y_i), (x_j, y_j)$  are discordant if  $x_i > x_j$  and  $y_i < y_j$  or  $x_i < x_j$  and  $y_i > y_j$ . The number of concordant and discordant pairs are then calculated and used in the correlation measure, shown as  $n_c$  and  $n_D$  in Equation 3 [85]. Unfortunately, the Kendall tau Rank Correlation suffers from the same fate as the Spearman and Pearson Correlations: the Kendall tau Correlation is primarily a measure of linear relationship; therefore, the correlation can calculate a value of zero for statistically dependent datasets.

$$\tau = \frac{n_C - n_D}{\frac{1}{2}n(n-1)} \quad (3)$$

The final correlation measure presented here is the Brownian Distance Correlation developed by Szekely. The primary motivation behind the Brownian Distance Correlation is to address the deficiency of the Pearson Correlation's ability to calculate a value of zero for statistically dependent variables [190, 189]. The calculation of this measure is much more involved than the previous measures. The equation used to calculate the Brownian Distance Correlation is shown in Equation 4. The parts of the correlation equation are defined in Equations 5 through 9. This correlation has a very strong benefit over the others discussed. This measure is able to capture correlations between linear and non-linear relationships. When the measured value is zero, then there is statistical independence between the two datasets. The drawback of this approach is that it is computationally intensive.

$$dCorr(x, y) = \sqrt{\frac{dCov_n^2(x, y)}{\sqrt{dCov_n^2(x, x)dCov_n^2(y, y)}}} \quad (4)$$

$$dCov_n^2(x, y) = \frac{1}{n^2} \sum_{j,k}^n A_{j,k} B_{j,k} \quad (5)$$

$$B_{j,k} = b_{j,k} - \bar{b}_{j,\bullet} - \bar{b}_{\bullet,k} - \bar{b} \quad (6)$$

$$A_{j,k} = a_{j,k} - \bar{a}_{j,\bullet} - \bar{a}_{\bullet,k} - \bar{a} \quad (7)$$

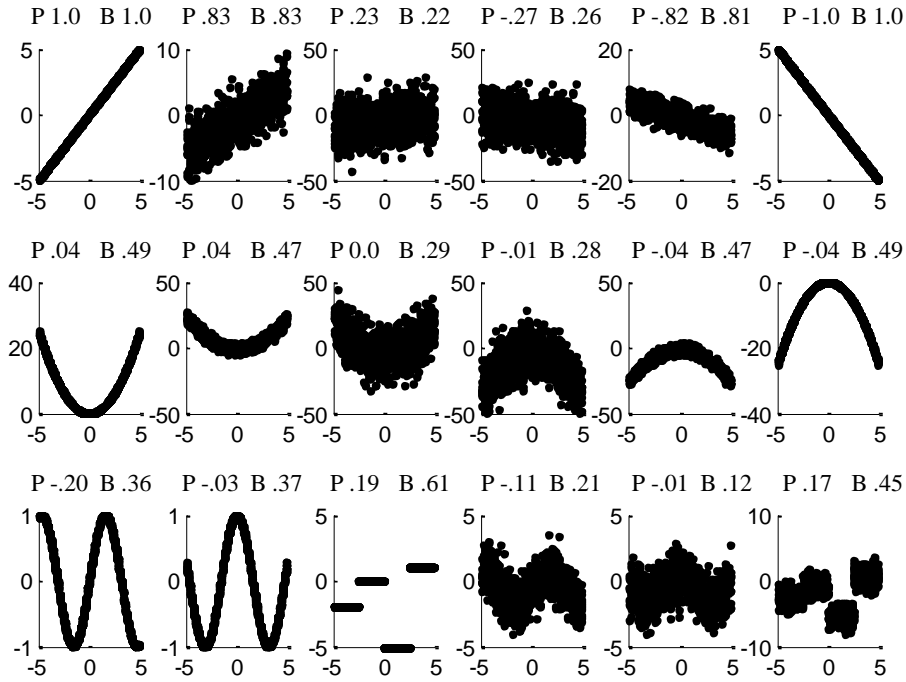
$$b_{j,k} = \left\| y_i - y_k \right\| j, k = 1, 2, \dots, n \quad (8)$$

$$a_{j,k} = \left\| x_i - x_k \right\| j, k = 1, 2, \dots, n \quad (9)$$

Brownian Distance Correlation differs many ways from the Pearson Correlation. For this reason, an example is given showing the correlation values for of various functions for both correlation measures in Figure 44. The first line of functions shown are liner functions with different amounts of noise added. The far left function has no noise added and shows a positive relationship. The far right also shows no noise and shows a negative relationship. The middle functions have the greatest noise added. The second row follows the same construct; however, the functions used are to the second power. The third row shows a sine function and a cosine function on the far left and second to the left, respectively. The next function is a step functions. The next three functions are the same functions listed but with added noise. Above each plot, the correlations for Pearson Correlation and Brownian Correlation are shown, denoted with a P and B, respectively.

As can be seen in the first row, the Pearson and Brownian Correlations are very similar. The major difference is that the Pearson Correlation values are bounded between one and negative one, whereas the Brownian Correlation values are bounded between zero and one. The second and third rows show a stark contrast between the two measures. For many of the functions there is an obvious dependence; however, the Pearson Correlation does not represent that. The Brownian Correlation measure is able to capture these trends. It is expected that a SME would rate these non-linear functions as having a large impact on the output.

Though not exactly a correlation copulas are considered as a possibility to capture the impact relationship. Copulas are commonly used to define multivariate probability distributions from a set of uniform marginal probability distributions. The marginal probability distribution is a description of the probability of the values that a variable may take without reference to the values of the other variables. Copulas are used to defined the dependence structure between the variables. Nelsen provides two simple definitions for copulas. The



**Figure 44:** Scatter Plots of Various Functions

first is ‘copulas are functions that join or “couple” multivariate distribution functions to their one-dimensional marginal distribution functions.’ The second is ‘copulas are multivariate distribution functions whose one-dimensional margins are uniform on the interval (0,1).’ [124]

The basis of the copulas comes from Sklar’s Theorem, which is as follows. Let  $H$  be a two-dimensional distribution. The marginal distribution functions are  $F$  and  $G$ . Then there exists a copula  $C$  such that  $H(x, y) = C(F(x), G(y))$ .

The copula can take many forms which can result in very complicated multivariate probability distributions. This is the allure of copulas as a measure of impact; however defining a copula and discovering a copula from experimental data are very different tasks. The former is the primary use of copulas found in the literature. The later will use common correlation measures to define the copula and thus are linear in nature. Since a method of using copulas to define the relationship between the two functions that define the joint distribution was found in the literature to be similar to the previously discussed correlations,

it will not be considered further.

#### *4.1.2.2 Conclusion*

The criterion for the selection of a measure of correlation were defined as capable of capturing non-linear behavior, being insensitive to outliers, and being computationally light. Every correlation measure with the exception of Brownian Distance Correlation failed to capture non-linear behavior. Of the measures discussed Pearson Correlation is the most sensitive to outliers. The other measures are mostly insensitive to outliers. The Pearson and Spearman Correlation are computationally light measures, where the Kendall tau Correlation measure requires a moderate amount of computation. The Brownian Distance Correlation is a computationally intensive measure.

The importance ratings of the three criterion depends on the situation; primarily, how many samples need to be measured. If a complex model is developed that requires many input variables, then a large number of cases would be required to analyze. Assuming the model is stochastic, replications would be required for each case. This would result in calculating the correlations with very large sample sizes. A moderate sample size may result if the stochastic measures do not require many replications. This expanded upon further in chapter five. Conversely, if a simple model is developed with few input variables, then a small number of cases would be required. Assuming the model is stochastic, replications would be required for each case. This would result in calculating the correlations with a moderate sample size. A small sample size may result if the stochastic measures do not require many replications.

Under these three scenarios, large, moderate, and small sample sizes, the importance of the criterion differ. A summary of the importance of the criterion for the three cases are shown in Table 28. For every sample size case, the non-linear criteria is important. As stated earlier, the SMEs will determine that a non-linear behavior is important as well as a linear behavior. For large and moderate sample sizes, the sensitivity to outliers of the measure is not important, because the outliers have less of an impact on the measures with sufficient sample sizes. However, the computation time becomes very important for large sample

sizes. As the sample size increases the computation time and resources required increase non-linearly. The Brownian Distance Correlation is particularly affected by this, because it measures the distance between every point. The computation time is considered important for moderate sample sizes and not important for small sample sizes. The calculations will be calculated a few number of times for this methodology. Generally, if the sample size is small enough such that the computation time is reasonable and the memory required does not exceed what is available, then the measure is sufficient for this criteria.

For the large sample size cases the Pearson Correlation is preferred. Since computation time is very important, Brownian Correlation is not preferred. Every other measure cannot account for non-linear behaviors and outliers are not important. This places the Pearson and Spearman Correlation as equally useful. The argument is made that when two approaches are equivalent then the most common one should be used; therefore, Pearson's Correlation will be used for very large sample sizes. For moderate sample sizes Brownian Distance Correlation is preferred. If the sample size is too large to compute the Brownian Correlation in a reasonable time then the Pearson Correlation should be used; otherwise, the Brownian Correlation should be used. This is because the change between the moderate sample size and large sample size case is the importance of computation; therefore, if it is possible Brownian Correlation should be used. Finally, for small sample sizes the Brownian Correlation should be used. When the computation criteria becomes unimportant, then the Brownian Correlation is the optimum measure.

**Table 28:** Importance of the Correlation Measure Criterion

Criteria	Large N	Moderate N	Small N
Non-Linear	Important	Important	Important
Outliers	Not Important	Not Important	Important
Computation	Very Important	Important	Not Important

**Research Question 3.B.1 Hypothesis:** Normalized Brownian Distance Correlations should be used as the mathematical relation to the SME impact relationships.



#### *4.1.2.3 A Modification of the Methodology for the use of Pearson's Correlation for Large Sample Sizes*

The inability of the Pearson Correlation to measure non-linear dependence is a significant shortcoming for its use as a measure of impact. SMEs may assign a large impact that is due to a non-linear relationship. This would create a conflict when the simulation outputs are compared to the subjective model developed by the SMEs. To address this issue, it is suggested that the Pearson Correlation be used the initial comparison. The relationships that prove to be unimportant for both the simulation outputs and the SME subjective model should be removed from the analysis. A new set of experiments would then be executed, which will reduce the sample size for the correlation measure. The Brownian Correlation could then be used with decreased sample size.

#### **4.1.3 Translation of Impact Relationships**

At this point the visual framework of the system decomposition has been discussed and decided upon. Additionally, the meaning and mathematical correspondence of the SME impact relationships have been addressed. Next, how the impact relationships relate to each other must be addressed. This brings about research question 3.C shown below.

**Research Question 3.C:** How do the impact relationships translate between the levels of the hierarchy?

Referring back to the original system decomposition of objectives into effects, effects into performance, and performance into technical performance parameters, if the SMEs provide weightings to the relationships between each level, then how do these relationships translate between the levels? An updated picture with fictional SME weightings can be seen in Figure 45. The mathematical graph of the decomposition can be seen in Equation 10. Logically, one would assume that if metric T1 contributed to 60% of the behavior of P1 and P1 contributes to 50% of the behavior of E1, then T1 contributes to 30% of the behavior of E1. With this reasoning, it is hypothesized that the multiple levels of the system decomposition can be translated to each other through matrix multiplication. This hypothesis says that the impact of the technical performance parameters on the effects can

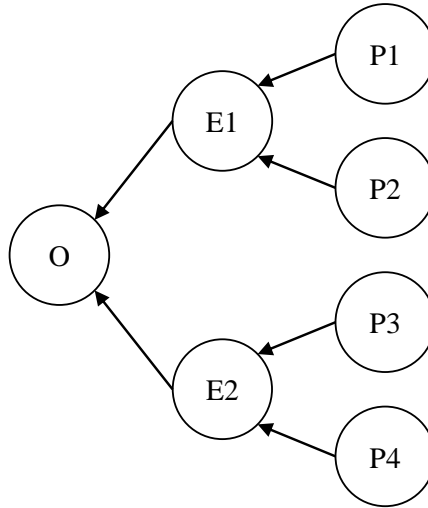


#### 4.1.3.1 Hypothesis Testing Against Linear and Non-Linear Systems

It is known immediately that there exists some error when applying hypothesis 3.C, because the multiplication of the direct normalized correlations do not carry through to the indirect normalized correlations correctly. To illustrate this, a canonical example of a impact graph is shown in Figure 46. Equation 11 shows a graph of the correlations, where  $e$  and  $f$  represent the correlation between the objective,  $O$ , and the two input metrics  $E_1$  and  $E_2$ , respectively,  $a$  and  $b$  represent the correlation between  $E_1$  and the two input metrics  $P_1$  and  $P_2$ , respectively, and  $c$  and  $d$  represent the correlation between  $E_2$  and the two input metrics  $P_3$  and  $P_4$ , respectively. The correlation between the objective and the performance metric  $i$  is given by  $y_i$  and must be estimated. An example of the calculation of  $y_i$  by multiplying the direct correlations is shown in Equation 12. The indirect correlation  $y_i$  can be estimated in this manner for Pearson and Brownian Correlations under some circumstances, which will be expanded upon later. The normalized indirect correlations are then found in Equation 13.

Equation 14 shows the impact matrix, where the correlations are normalized. Applying hypothesis 3.C, the indirect impacts are found by multiplying the direct impact values, i.e. the direct normalized correlations. The indirect impacts, i.e. normalized indirect correlations, are then calculated in Equation 15. It is immediately seen that these results are not equal. A comparison is shown in Equation 16. Each row of inequalities represent a different indirect normalized correlation estimate indicated by the label to the left of the parentheses. Each column represents the value achieved for the two different estimation methods, indicated by the label above the parentheses. The label 'Norm' refers to the estimate using hypothesis 3.C method, which was calculated in Equation 15. The label 'Non-Norm' refers to the estimate using the multiplication of correlations, which was calculated in Equation 13. Given the assumption that the 'Non-Norm' estimate is correct, the 'Norm' estimate will contain some error. Some error can be acceptable, because the conceptual model is based on a subjective estimate about the real system. Error will exit in the subjective estimate of the direct impacts. Therefore, if sufficient error is not found between the estimated indirect impacts and the measured direct impacts then, hypothesis 3.C is a

sufficient method for estimating indirect impacts. It must be determined how much error occurs between the estimated indirect impacts and the observed indirect impacts.



**Figure 46:** Canonical Example for Indirect Impact Testing

$$\begin{bmatrix} Corr & e & f & y_1 & y_2 & y_3 & y_4 \\ - & & & a & b & 0 & 0 \\ - & Corr & & 0 & 0 & c & d \end{bmatrix} \quad (11)$$

$$y = \begin{bmatrix} e & f \end{bmatrix} \begin{bmatrix} a & b & 0 & 0 \\ 0 & 0 & c & d \end{bmatrix} = \begin{bmatrix} ae & be & cf & df \end{bmatrix} \quad (12)$$

$$\bar{y} = \frac{1}{ae + be + cf + df} \begin{bmatrix} ae & be & cf & df \end{bmatrix} \quad (13)$$

$$\begin{bmatrix} Corr & e & f & y_1 & y_2 & y_3 & y_4 \\ - & & & \frac{a}{a+b} & \frac{b}{a+b} & 0 & 0 \\ - & Corr & & 0 & 0 & \frac{c}{c+d} & \frac{d}{c+d} \end{bmatrix} \quad (14)$$

$$\bar{y} = \begin{bmatrix} \frac{ae}{e(a+b)+f(a+b)} & \frac{be}{e(a+b)+f(a+b)} & \frac{cf}{e(c+d)+f(c+d)} & \frac{df}{e(c+d)+f(c+d)} \end{bmatrix} \quad (15)$$

$$\begin{array}{cc}
\textit{Norm} & \textit{Non - Norm} \\
\bar{y}_1 \left( \begin{array}{c} \frac{ae}{e(a+b)+f(a+b)} \neq \frac{ae}{e(a+b)+f(c+d)} \\ \frac{be}{e(a+b)+f(a+b)} \neq \frac{be}{e(a+b)+f(c+d)} \\ \frac{cf}{e(c+d)+f(c+d)} \neq \frac{cf}{e(a+b)+f(c+d)} \\ \frac{df}{e(c+d)+f(c+d)} \neq \frac{df}{e(a+b)+f(c+d)} \end{array} \right) & (16)
\end{array}$$

A series of three functions will be defined to observe the accuracy of the indirect impact estimates for the both Pearson and Brownian Correlations. Three levels of functions will be used. Each level will contain a set of functions that test hypothesis 3.C at different levels of non-linearity. There is no increasing scale of non-linearity. A non-linear function can take many forms, e.g. polynomial, exponential, piecewise. Therefore, a non-linear scale must be defined. The scale selected is based on the 2nd Order Model that is commonly used in Response Surface Methodology [113]. The three levels of functions are shown in Table 29 where  $a$  represents coefficients and  $x$  represents input variables. Level 1 is a purely linear set of equations. Level 2 includes linear terms and cross terms between the input variables. Level 3 includes linear terms, cross terms, and second order terms. Level 3 is the 2nd Order Response Surface Equation. This set of equations are capable of representing a large number of functional responses that would be seen as an output of a simulation. This is why they are commonly used in the Response Surface Methodology [113]. The purpose of the testing is to determine if the mathematical measures of impact can be translated between the levels.

**Table 29:** Levels of Non-Linear Functions

Level 1	$y = \sum a_i x_i$
Level 2	$y = \sum a_i x_i + \sum \sum a_{ij} x_i x_j$
Level 3	$y = \sum a_i x_i + \sum \sum a_{ij} x_i x_j + \sum a_{ii} x_i^2$

The canonical example problem used in this section resembles the metal refinement factory used to define the impact measure. One objective will be defined with two effects and four performance metrics. Performance metrics 1 and 2 will impact Effect 1. Performance metrics 3 and 4 will impact Effect 2. Both effects will impact the objective. A graph of

the canonical example was introduced earlier in Figure 46. Each performance metric will be uniformly distributed between  $-10$  and  $10$ . The coefficients will be modified for each level to best achieve values for the indirect impacts between 95% and 5%, i.e. a case within the level will achieve an indirect impact value of 95%. The indirect impacts using Pearson Correlations will be calculated analytically by solving the correlation equation. The indirect impacts using Brownian Correlation will be calculated numerically using sample sizes of 3,000. The normalized correlation values are calculated for each of the direct impacts. The indirect impacts are then estimated from the calculated normalized correlation values by multiplying the calculated normalized correlation values. The error is then defined as the absolute difference between the estimated indirect impacts and the calculated indirect impacts.

For comparative purposes the normalized correlation will also be estimated by the two methods presented above: the normalized method and the non-normalized method. The normalized method will be applying hypothesis 3.C, which estimates the indirect impact by multiplying the direct impacts together. This method follows the approach of Equation 14 and Equation 15. The non-normalized method will calculate the indirect impact by first multiplying the correlations to estimate the indirect correlations. The indirect correlations will then be normalized to estimate the indirect impacts and compared to the observed indirect impacts. This method follows the approach of Equation 11 and Equation 12.

The experiments run for the Level 1 set of functions are shown in Equation 17. Notice that only the coefficients  $p_1, p_3$ , and  $e_1$  are varied. Due to the symmetric nature of the canonical example, only half of the coefficients must be varied. The Brownian Correlations were calculated numerically. The Pearson Correlations were calculated using Equations 18, 19, and 20. Equation 18 is the Pearson Correlation between the objective and an effect metric. Equation 19 is the Pearson Correlation between an effect metric and a performance metric. Equation 20 is the Pearson Correlation between the objective and a performance metric. The calculation of the components within Equations 18, 19, and 20 are shown in Equation 60, which can be found in the Appendices in Section A.1.

The results for the error between the estimated indirect impacts and the calculated

indirect impacts are shown in the top two plots in Figure 47. The error between the estimated indirect correlation and the calculated indirect correlation through correlation multiplication are shown in the bottom two plots in Figure 47. Each plot is labeled for what is being measured. The top left plot shows the indirect impact error for the normalized Pearson Correlations. The top right plot shows the indirect impact error for the normalized Brownian Correlations. The ordinate shows the absolute error measured. The abscissa represents the case number. A total of 125 cases were run. There are four lines in each plot. Each line represents the error observed between the estimated indirect measure and the calculated indirect measure for a specific performance metric. Performance metric 1, 2, 3, and 4 are represented with a blue, green, red, and teal line, respectively.

The first observation made is that the error for each of the four results is very low. The estimate using Pearson Correlation multiplication shows the smallest error. The error observed is zero. From this observation a property of the Pearson Correlation is presented. The Pearson Correlation between  $O$  and  $P_1$  can be calculated by the product of the Pearson Correlation between  $O$  and  $E_1$  and  $E_1$  and  $P_1$  if it is known that  $O$  is a linear combination of  $E_1$  and  $E_1$  is a linear combination of  $P_1$ .

The normalized Pearson and Brownian Correlation show a small amount of error. The error observed in normalized Pearson Correlation estimate is due to the issues addressed earlier in Equation 16. In addition to the normalization error the normalized Brownian Correlation estimate contains some sample error. An important takeaway is that both levels of error for the normalized estimate are small.

$$P_i = a_i U(0, 1) + b_i \forall i \in [1, 2, 3, 4]$$

$$E_1 = p_1 P_1 + p_2 P_2$$

$$E_2 = p_3 P_3 + p_4 P_4$$

$$O = e_1 E_1 + e_2 E_2$$

(17)

$$p_1, p_3, e_1 \in [1, 5, 9, 13, 17]$$

$$p_2, p_4, e_2 = 1$$

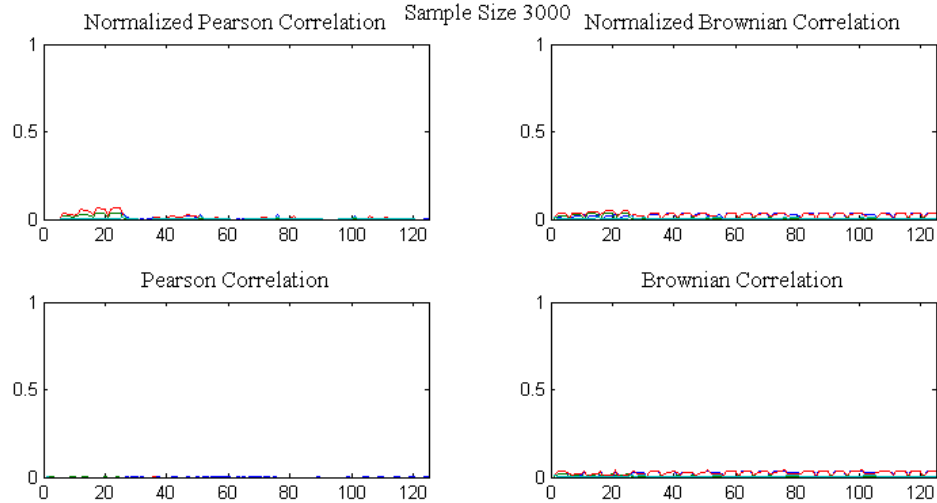
$$a_i = 20 \forall i \in [1, 2, 3, 4]$$

$$b_i = -10 \forall i \in [1, 2, 3, 4]$$

$$\text{Corr}(O, E_i) = \frac{E[OE_i] - E[O]E[E_i]}{\sqrt{E[O^2] - E[O]^2} + \sqrt{E[E_i^2] - E[E_i]^2}} \quad (18)$$

$$\text{Corr}(E_i P_i) = \frac{E[E_i P_i] - E[E_i]E[P_i]}{\sqrt{E[E_i^2] - E[E_i]^2} + \sqrt{E[P_i^2] - E[P_i]^2}} \quad (19)$$

$$\text{Corr}(O P_i) = \frac{E[O P_i] - E[O]E[P_i]}{\sqrt{E[O^2] - E[O]^2} + \sqrt{E[P_i^2] - E[P_i]^2}} \quad (20)$$



**Figure 47:** Results for Level 1

The set of experiments run for the Level 2 set of functions are shown in Equation 21. Notice that the coefficients  $p_1, p_3, p_{12}, p_{34}, e_1$ , and  $e_{12}$  are varied. Three additional



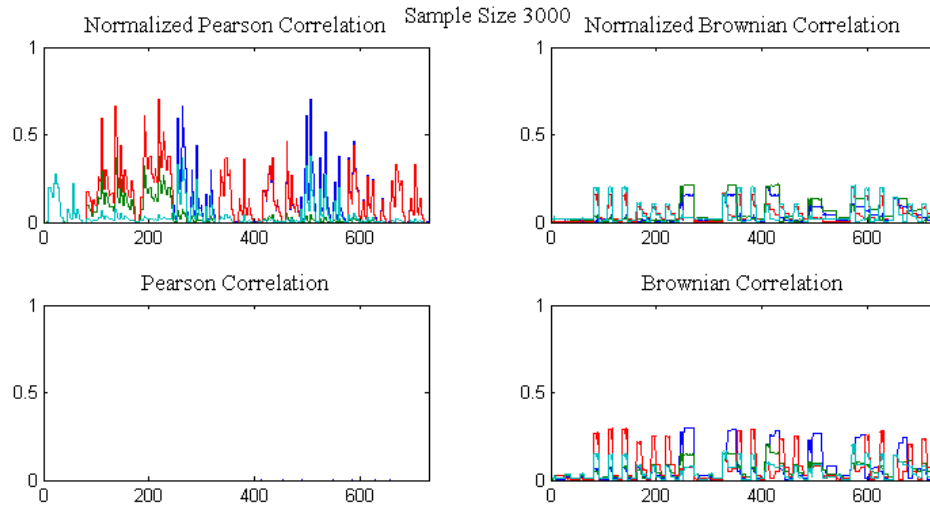
coefficients are added; therefore, the coefficient value set was reduced to 1, 9, and 17. This setup results in 729 cases. The Pearson Correlations were calculated using Equations 18, 19, and 20. The calculation of the components within Equations 18, 19, and 20 for Level 2 are shown in Equation 61, which can be found in the Appendices in Section A.2.

The results for the error between the estimated indirect impacts and the calculated indirect impacts using hypothesis 3.C are shown in the top two plots in Figure 48. The error between the estimated indirect impact and the calculated indirect impact using correlation multiplication are shown in the bottom two plots in Figure 48. The only difference between Figure 48 and Figure 47 is the number of cases and the axis range for the error. The ordinate range was changed from zero to one half to from zero to one.

The first observation made is that the errors in the estimates are larger for the Level 2 functions compared to the Level 1 functions. Particularly, the error is much larger for the indirect impact estimate using normalized Pearson Correlation multiplication. This is shown in the top left graph. The estimate to normalized Pearson Correlations using correlation multiplication is shown to be perfect having an error of zero. This is shown in the bottom left graph.

The error observed for the Brownian Correlation estimates may be due to other sources than sample error. The error for both Brownian estimates are larger than for Level 1. The maximum error observed is at about 0.25. This value is greater than desired. This is the maximum error that will be allowed for this estimate. Though the error is high, it does not present sufficient error to overturn hypothesis 3.C.

$$\begin{aligned}
P_i &= a_i U(0, 1) + b_i \forall i \in [1, 2, 3, 4] \\
E_1 &= p_1 P_1 + p_2 P_2 + p_{12} P_1 P_2 \\
E_2 &= p_3 P_3 + p_4 P_4 + p_{34} P_3 P_4 \\
O &= e_1 E_1 + e_2 E_2 + e_{12} E_1 E_2 \\
p_1, p_3, p_{12}, p_{34}, e_1, e_{12} &\in [1, 9, 17] \\
p_2, p_4, e_2 &= 1 \\
a_i &= 20 \forall i \in [1, 2, 3, 4] \\
b_i &= -10 \forall i \in [1, 2, 3, 4]
\end{aligned} \tag{21}$$



**Figure 48:** Results for Level 2

The set of experiments run for the Level 3 set of functions are shown in Equation 22. Notice that the coefficients  $p_1, p_3, p_{12}, p_{34}, p_{22}, p_{33}, e_1, e_{12}$ , and  $e_{22}$  are varied. Three additional coefficient are added; therefore, the coefficient value set was reduced to 1 and 17. This results in 512 cases. Using three coefficient values would result in 19,683 cases, a number of cases too large to calculate within a reasonable time frame.

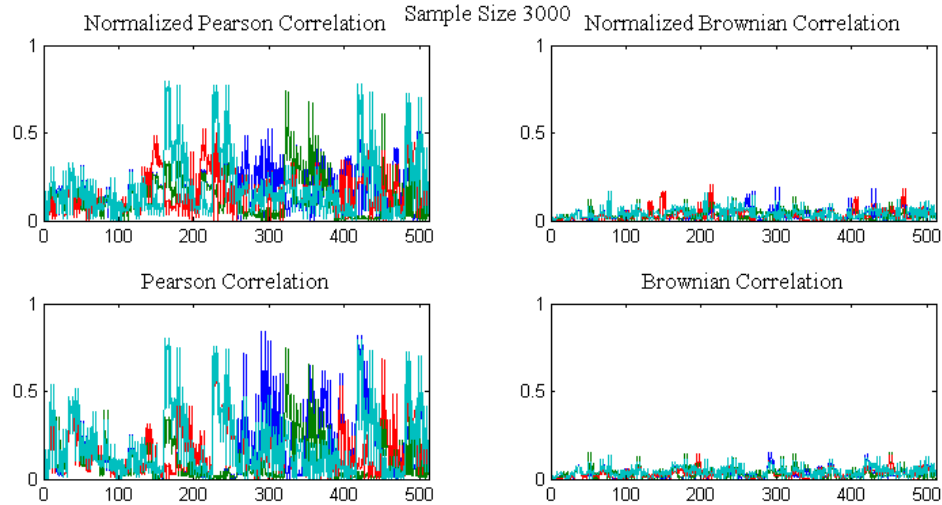
The Pearson Correlations were calculated using Equations 18, 19, and 20. The calculation of the components within Equations 18, 19, and 20 for Level 3 are shown in Equations 62 through 69, which can be found in the Appendices in Section A.3.

The results for the error between the estimated indirect impacts and the calculated

indirect impacts using hypothesis 3.C are shown in the top two plots in Figure 49. The error between the estimated indirect impact and the calculated indirect impact using correlation multiplication are shown in the bottom two plots in Figure 49. The only difference between Figure 49 and Figure 48 is the number of cases.

The first observation is that the error found for both Pearson Correlation estimates is much larger. In the lower level functions, the indirect impact estimate using Pearson Correlation multiplication was a perfect estimator. Significant error is observed for the Level 3 functions. The main difference between the Level 3 functions and the other lower level functions is that Level 3 includes squared performance metrics. Level 2 functions included only multiplication between the performance metrics, while Level 1 functions included only a linear combination of the performance metrics. Interestingly, the indirect Brownian Correlation estimates are found to have slightly smaller error than the Level 2 results.

$$\begin{aligned}
P_i &= a_i U(0, 1) + b_i \forall i \in [1, 2, 3, 4] \\
E_1 &= p_1 P_1 + p_2 P_2 + p_{12} P_1 P_2 + p_{11} P_1^2 + p_{22} P_2^2 \\
E_2 &= p_3 P_3 + p_4 P_4 + p_{34} P_3 P_4 + p_{33} P_3^2 + p_{44} P_4^2 \\
O &= e_1 E_1 + e_2 E_2 + e_{12} E_1 E_2 + e_{11} E_1^2 + e_{22} E_2^2 \\
p_1, p_3, p_{12}, p_{34}, p_{22}, p_{33}, e_1, e_{12}, e_{22} &\in [1, 17] \\
p_2, p_4, p_{11}, p_{33}, e_2, e_{11} &= 1 \\
a_i &= 20 \\
\forall i \in [1, 2, 3, 4] \\
b_i &= -10 \\
\forall i \in [1, 2, 3, 4]
\end{aligned} \tag{22}$$



**Figure 49:** Results for Level 3

#### 4.1.3.2 Conclusion of Hypothesis Tests Against Linear and Non-Linear Systems and Further Testing

Despite the existence of known errors due to the multiplication of normalized correlations, the Brownian indirect impact estimates using normalized Brownian Correlation multiplication is surprisingly accurate for all three levels of functions. The Pearson indirect impact estimates using normalized Pearson Correlation multiplication is accurate for only the linear set of functions, i.e. Level 1. To understand this phenomenon the comparison between the normalized correlation multiplication and the non-normalized correlation multiplication, shown in Equation 16, must be investigated further. It is observed that the two estimates would be equivalent if the sum of correlations  $a$  and  $b$  were equivalent to the sum of correlations  $c$  and  $d$ . Therefore, as the difference between the two sums of the correlations increase the error in the indirect impact estimate would also increase. Investigating the results from the Level 1 experiments, shown in Figure 47, it is seen that the estimate using normalized Pearson Correlation multiplication has the greatest error in the earlier cases. In these cases the sum of correlations  $c$  and is highest compared to the sum of correlations  $a$  and  $b$ .

It is known that for a function composed of a linear combination of metrics the Euclidean norm of the correlations between the function and the metrics is equal to one, i.e.  $\sqrt{c^2 + d^2} = 1$ . From this information we know that the largest value that the sum that  $c$

and  $d$  can achieve is  $\sqrt{2}$  and the smallest is 1. This limits the error for the estimate using hypothesis 3.C. It is assumed that the same behavior for the Brownian measure would be observed, because for the Level 1 functions the Brownian and Pearson measures report very similar values.

Investigating the results from the Level 2 experiments, shown in Figure 48, it is seen that the error for the estimate using normalized Pearson Correlation multiplication is much higher than in the Level 1 experiments. This is because the function,  $0 \leq \sqrt{c^2 + d^2} \leq 1$ , is no longer a linear combination of the metrics. This allows the error for the hypothesis 3.C method to increase. The estimate using normalized Brownian Correlation multiplication does not result in the same increase in error. This is because Brownian Correlation captures non-linear behavior, thus the difference between the sum of correlations is not as great.

Investigating the results from the Level 3 experiments, shown in Figure 49, it is seen that the error for the estimate using non-normalized Pearson Correlation multiplication is unreliable; therefore, the normalized Pearson Correlation multiplication will not be reliable either. Again the estimate using Brownian Correlation shows a promising result due to its ability to capture non-linear behaviors.

In conclusion, we failed to reject hypothesis 3.C. Though error exists in the estimate, hypothesis 3.C has shown a capability to estimate the impact of the lower level metrics on the objective. Additionally, the results from this analysis strengthens the argument to use normalized Brownian Correlations as the measure of impacts to be defined by the SMEs. The transitive property of normalized Brownian Correlations is a major benefit to its use.

#### **4.1.4 Determining the Importance of Impact Relationships to the System**

The purpose of this system decomposition and the setting of impact relationships is to understand the parts of the system that are most critical so that they can be modeled with the highest fidelity. This leads to research question 3.D concerning how to determine which relationships are the most important to the system.

**Research Question 3.D:** How is the importance to the system of a specific relationship determined?

Observing the results of Equation 10 from the previous section, it is easy to assign importance of what should be modeled on the variables, e.g.  $P_1$ ,  $P_2$ ,  $P_3$ . However, when modeling the primary focus is on the relationship between entities or variables. Thus, when determining the importance of what should be modeled the focus is on the relationships between metrics. In the above example one would predict that the relationship between  $E_1$  and  $O$  is of greatest importance and the relationship between  $P_4$  and  $E_2$  is of the least importance. As a means to determine how important each relationship is, the relationship is removed holding all other values constant, e.g. setting the value of the  $P_4-E_2$  relationship to  $O$  does not change the values of the other two performance metrics impacting  $E_2$ . The calculations are then carried through. As a result, the upper right matrix showing the impacts of the performance metrics on the objective would not sum to 100%. This missing percentage will act as the importance of the impact relationship. Since there is only one path between each performance metric and the objective, the percentage contribution of the performance metrics to the objective will also be the importance of the performance to effect relationships.

**Research Question 3.D Hypothesis:** The importance to the system of a specific relationship is determined by removing the impact relationship and calculating the missing percentage of the objective.

This hypothesis is tested by estimating the impact of each relationship first. Then, the canonical example is changed for each impact relationship by setting the input to the above metric to the mean of the lower metric. This approach will avoid the issue of removing the  $P_2-E_1$  relationship would also remove the  $P_1-E_1$  and  $P_3-E_1$  relationship. The new objective calculation will be compared to the original objective. The observed value will then be  $1 - dCorr(O, O_{new})$ . The values are then normalized. For example, it is estimated that the  $P_2-E_1$  relationship contributes to 30% of the objective output. The example problem will then be created where  $P_2$  is then held constant to its mean, 0.40. The new model will be simulated. The correlation between the new and original output is found and subtracted from one. This value will be the observed impact. The results can be found in Table 30.

There are some very noticeable discrepancies. What can be seen is that real impacts are greater for the impacts that were predicted to be large and smaller for the ones that were predicted to be small. It would be preferred that the estimates be more accurate; however, the modeler will be leaving out the relationships with small impacts. If the small impacts in actuality have even smaller impacts, then the method proposed in hypothesis 3.D can still be used.

**Table 30:** Canonical Example for Importance of Relations

	O-E1	O-E2	E1-P1	E1-P2	E1-P3	E2-P4	E2-P5	E2-P6
Estimate	0.760	0.240	0.067	0.300	0.394	0.017	0.098	0.125
Actual	0.914	0.086	0.018	0.326	0.555	0.002	0.049	0.060

#### 4.1.5 How Modelers Decide What and How to Model

The previous section addressed how to determine which relationships were the most important. The next logical step is to determine what should be modeled and how it should be modeled. This leads to research question 3.E given below.

**Research Question 3.E:** How should a modeler decide what relationships to model and how to model them?

Estimating the impact on the objective of specific relationships, as shown previously, will only aid in determining the level of fidelity in the modeling of the relationship. The decision on whether to model said relationship or not is to the discretion of the modeler and is based on the nature of the relationship. For example, in modeling the patrol of a naval ship the modeling of the radar must be addressed. If the range of detection were small, e.g. 100 nmi to 101 nmi, then the impact of the range of the radar is most likely of little importance to the objective. This does not mean that the radar range could be ignored. This only indicates that the fidelity of the representation of radar range can be low. Conversely, if there were stray civilian fishing vessels in the area that have little impact on the objective, then it would be reasonable to neglect modeling these entities. Obviously, the only relationships that would not be represented would be the ones with little impact

on the objectives. A relationship that has a low impact is only a candidate for not being modeled.

For each relationship, there exist infinite possibilities for representing the relationship in a model. The modeler will be able to identify a subset of these possibilities. They will then need a way to list which possibilities they have identified and a way to aids in making a selection. This approach must provide a mechanism for traceability and defensibility. Additionally, the approach must be flexible enough to accommodate a wide variety of possible relationships. Two possibilites are found in the literature: morphological analysis and Pugh charts.

Morphological analysis aid in design by decomposing the system into its different functional needs and offering a set of options that can fill the need [57]. Often a morphological matrix is used that lists the categories on the far left column with each row addressing a different category. Across the columns the identified options will be listed. An example is shown in Figure 50. In this example there exist 192 possible combinations. For real morphological matrices it is possible to achieve total possible combinations on the order of the number of atoms in the universe. The morphological matrix offers the decision makers a good understanding of the vastness of the problem and it provides a flexibility and traceability needed in this process. The shortcoming of the method is that it does not aid the modeler in making the final selection.

Category 1	Option 1	Option 2	Option 3	Option 4
Category 2	Option 1	Option 2		
Category 3	Option 1	Option 2	Option 3	Option 4
Category 4	Option 1	Option 2	Option 3	
Category 5	Option 1	Option 2		

**Figure 50:** Morphological Matrix Example



The Pugh Concept Selection Method works by comparing a set of options relative to a datum option against a set of criteria. Each option is evaluated against the datum to determine if it is better than, worse than, or about the same. This is done for each criteria and for each option. The counts for better and worse are then tallied. The best options is then listed as the new datum and often the worse option is removed and the process is started again [57]. An example, reproduced from Dieter [57], is shown in Figure 51. The Pugh Chart offers the decision makers a good understanding of the problem and it provides a flexibility and traceability needed in this process. Additionally, it aids in the decision making on how to model. The shortcoming of this method is that it is very work intensive. A chart would have to be created for each relationship present. Additionally, the structure of the Pugh Chart is constraining compared to the morphological matrix. For these reasons, it is concluded that the morphological matrix should be used with added comments to explain each selection to add traceability of the conceptual model development.

**Research Question 3.E Hypothesis:** A morphological matrix of model representations will be used accompanied with justification for why the specific model representation was selected.

#### 4.1.6 How SMEs Assign Impact Relationships

To this point, it has been assumed that the SME impacts have been assigned. No discussion has been had on exactly how the SMEs will apply the impact relationships. This leads to research question 3.G.

**Research Question 3.G:** How do SMEs assign the impact relationship values given between two metrics?

There are three possibilities that have been identified: direct assignment, interval assignment, and pairwise comparisons. The direct assignment approach is where the SMEs assign each impact relationship a value between 0% and 100%. This method is a possible approach; however, this could be very difficult for the SMEs to accurately guess. The SME would be required to not only identify the importance of a metric but also its importance

Criteria	Concepts			
	D	A	C	B
Cost	+	<b>Datum</b>	+	+
Added Functionality	=		=	-
Simplicity of Design	+		=	=
Availability of Materials	+		-	-
Ease of Manufacturing	+		+	=
Ease of Assembly	+		+	-
Ability to Prototype	+		=	+
Comfort	-		+	-
Weight	+		=	-
Aesthetics	-		=	-
Pluses	<b>7</b>		<b>0</b>	<b>4</b>
Minuses	<b>2</b>	<b>0</b>	<b>1</b>	<b>6</b>

**Figure 51:** Pugh Chart Example

compared to the other metrics. The next possibility is the use of interval assignment. In this approach SMEs will assign a value from a set of values on each relationship. The interval set can be linear, e.g. 0, 1, 2, 3, or non-linear, e.g. 0, 1, 3, 9. Once the values are assigned the values would then be normalized. Two issues arise with this approach. The first is the selection of the interval scale. The decision on which scale is used will have a big impact on the importance measures. The second issue is that the interval scale may not have enough resolution to capture the true relationship. The final possibility is to use pairwise comparison. This approach is similar to the one used by the Analytic Hierarchy Process. In this approach the SMEs will compare the impact for each contributing lower

level metric to the affected higher level metric. Stated in another way, each edge that feeds into a node will be compared against each other. For example, SMEs will state that edge one is twice as important as edge two. The percentage of importance will then be computed from the pairwise comparisons. The primary issue with this approach is that it will require a large number of comparisons.

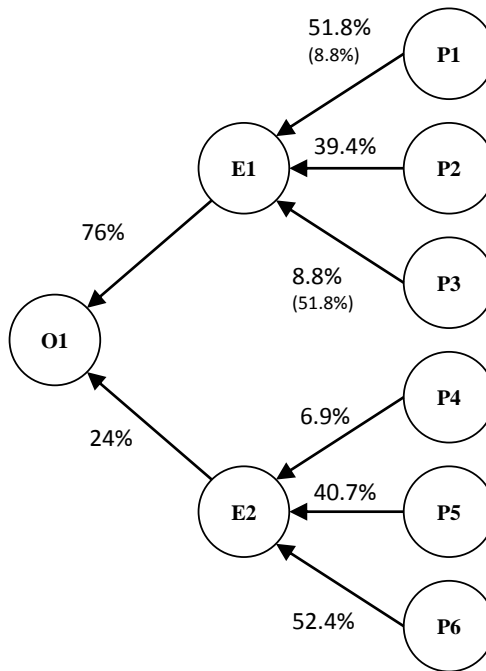
Each approach provides its benefits and shortcomings; however, the pairwise comparisons is considered to be the best approach. One supporting reason for this is that relative values will already be used in the impact matrix; therefore, it naturally follows that the pairwise comparisons would be best suited. Additionally, it is argued that pairwise comparisons are the best means for extracting information from SMEs. “The most effective way to concentrate judgement is to take a pair of elements and compare them on a single property without concern for the properties or other elements.” (T. L. Saaty 1990) [163].

**Research Question 3.G Hypothesis:** SMEs will assign the impact relationship values using pairwise comparisons?

#### 4.1.7 When the SMEs are Wrong

This method for conceptual modeling relies on the SME impact relations to be accurate. In reality, the SMEs will be inaccurate on some estimates. For this reason it is important to understand how sensitive this method is to the SME impact relations. Is it possible for this method to identify mistakes that the SMEs made in the initial impact relationship estimates once the model is built and analyzed? To answer this question, the previous canonical example of the ore refinery is addressed again. In this new experiment, a single incorrect assignment will be made and analyzed to determine if the method can identify it. The new SME impact relationships can be see in Figure 52. As can be seen the estimated values for the  $P_1-E_{-1}$  and  $P_3-E_1$  relationship has been reversed. With this incorrect assessment the two least important relationships are  $P_4-E_2$  and  $P_3-E_1$ . These two are estimated to represent about 8.4% of the behavior of the objective. In actuality the two represent about 41.1% of the objective behavior. When the two relationships are not modeled the expected values to be seen are shown on the top row in Table 31. Here the impact relationships have

been normalized to sum to 100% to estimate what would be seen. After running the model while defaulting the  $P_3-E_1$  and  $P_4-E_2$  relationships to their mean, a comparison can be made. The results are shown on the second row in Table 31. The relationships between the effects and the objective seem reasonable. Additionally, the performance relationships to effect two also seem to be within an acceptable tolerance. The major issues arise under effect one. Therefore, this method can aid in identifying where the SME have made mistakes. It should be noted though that if the SMEs are wrong it is very difficult to correct that mistake if the relationship was not modeled. Knowledge about the system relationships cannot be studied if they are not modeled.



**Figure 52:** Canonical Example with Incorrect Values

**Table 31:** Canonical Example for Incorrect Importance of Relations

	O-E1	O-E2	E1-P1	E1-P2	E1-P3	E2-P4	E2-P5	E2-P6
Estimate	0.757	0.243	0.430	0.327	-	-	0.107	0.136
Actual	0.678	0.322	0.097	0.541	-	-	0.152	0.211

#### 4.1.8 Calculating Same Level Correlations in the Impact Matrix

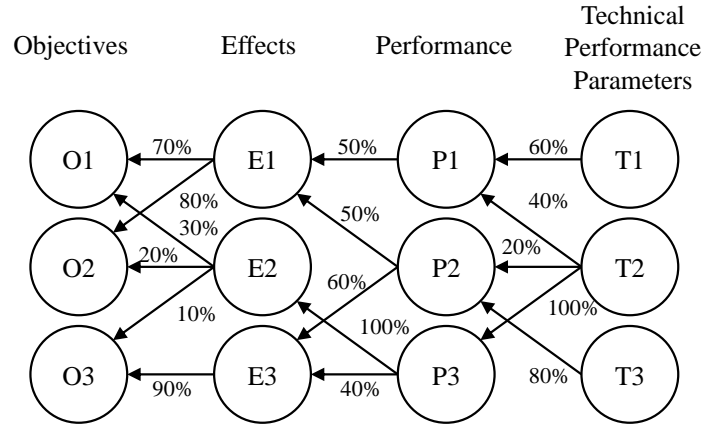
One part of the impact matrix that has not been addressed yet is same level correlations. This should not be confused with the calculated normalized Brownian correlations that are used as the mathematical representation of the SME impact relationships. These correlations are the part of the impact matrix that map the same level metrics against each other. For example, the correlation values that exist between the performance metrics are the same level correlations. The SMEs do not input these values, nor are they used in model development. They can, however, be used in model validation. Once the model is built, this can act as another method to check agreement between the SME system decomposition and the model results. This leads to research question 3.F.

**Research Question 3.F:** How should impact matrix correlations be calculated?

A set of requirements for the correlation calculations are as follows. First, the estimates should be as close to the actual values as possible. Second, self correlations, i.e. correlation of  $P_1$  to  $P_1$ , should be calculated to be one, and the absence of correlation should be calculated to zero. Finally, the calculation should be able to provide accurate estimates for linear and non-linear relationships.

To create some initial hypotheses a canonical example will be defined. This canonical example is based on the example system decomposition defined earlier and reproduced in Figure 53. Based on the system decomposition, a linear set of equations are defined based on the system decomposition in Equation 23. The impact matrix can be seen in Equation 24. The correlation matrix can be seen in Equation 25. The same level correlations are shown in Equation 26. Left most matrix shows the result from applying hypothesis 3.F to calculate the objective correlations. The second matrix from the left shows the result from applying hypothesis 3.F to calculate the effect correlations. The third matrix from the left shows the result from applying hypothesis 3.F to calculate the performance correlations. Finally, the right most matrix shows the TTP correlations which are known. The correlations were estimated with one million Monte Carlo Samples. The T metrics were represented with

a uniform distribution bounded between zero and one. The correlations for both Pearson and Brownian Correlations are equivalent since the equations in Equation 25 are linear. Finally, it should be noted that the correlation matrix captures correlation not dependence. This is why the correlation between objective one and effect three is very high, even though objective one does not depend on effect three. The correlation is based on the fact that both metrics are based on similar TPP metrics.



**Figure 53:** Example System Decomposition Graph with Values

$$\begin{aligned}
 O_1 &= 7E_1 + 3E_2 \\
 O_2 &= 8E_1 + 2E_2 \\
 O_3 &= E_2 + 9E_3 \\
 E_1 &= 5P_1 + 5P_2 \\
 E_2 &= 10P_3 \\
 E_3 &= 6P_2 + 4P_3 \\
 P_1 &= 6T_1 + 4T_2 \\
 P_2 &= 2T_2 + 8T_3 \\
 P_3 &= 10T_2
 \end{aligned} \tag{23}$$

$$\begin{bmatrix}
& & & & & & & & & & \\
& & & & & & & & & & \\
& & & & & & & & & & \\
\text{Corr} & 0.70 & 0.30 & 0.00 & 0.35 & 0.35 & 0.30 & 0.21 & 0.51 & 0.28 \\
& 0.80 & 0.20 & 0.00 & 0.40 & 0.40 & 0.20 & 0.24 & 0.44 & 0.32 \\
& 0.00 & 0.10 & 0.90 & 0.00 & 0.54 & 0.46 & 0.00 & 0.27 & 0.43 \\
- & - & - & & 0.50 & 0.50 & 0.00 & 0.30 & 0.30 & 0.40 \\
- & - & - & & \text{Corr} & 0.00 & 0.00 & 1.00 & 0.00 & 1.00 & 0.00 \\
- & - & - & & & 0.00 & 0.60 & 0.40 & 0.00 & 0.52 & 0.48 \\
- & - & - & - & - & - & - & - & 0.60 & 0.40 & 0.00 \\
- & - & - & - & - & \text{Corr} & 0.00 & 0.20 & 0.80 & & \\
- & - & - & - & - & & 0.00 & 1.00 & 0.00 & & \\
- & - & - & - & - & - & - & - & & & \\
- & - & - & - & - & - & - & - & \text{Corr} & & \\
- & - & - & - & - & - & - & - & & & 
\end{bmatrix} \tag{24}$$

$$\begin{bmatrix}
& & & & & & & & & & \\
& & & & & & & & & & \\
& & & & & & & & & & \\
\text{Corr} & 0.91 & 0.82 & 0.91 & 0.74 & 0.64 & 0.82 & 0.34 & 0.82 & 0.45 \\
& 0.96 & 0.74 & 0.91 & 0.75 & 0.70 & 0.74 & 0.40 & 0.74 & 0.54 \\
& 0.82 & 0.80 & 1.00 & 0.44 & 0.78 & 0.80 & 0.00 & 0.80 & 0.60 \\
- & - & - & & 0.71 & 0.79 & 0.51 & 0.51 & 0.51 & 0.67 \\
- & - & - & & \text{Corr} & 0.55 & 0.24 & 1.00 & 0.00 & 1.00 & 0.00 \\
- & - & - & & & 0.41 & 0.84 & 0.74 & 0.00 & 0.74 & 0.68 \\
- & - & - & - & - & - & - & - & 0.83 & 0.55 & 0.00 \\
- & - & - & - & - & \text{Corr} & 0.00 & 0.24 & 0.97 & & \\
- & - & - & - & - & & 0.00 & 1.00 & 0.00 & & \\
- & - & - & - & - & - & - & - & & & \\
- & - & - & - & - & - & - & - & \text{Corr} & & \\
- & - & - & - & - & - & - & - & & & 
\end{bmatrix} \tag{25}$$

An observation is made about the correlation matrix shown in Equation 25. The dot product of the of the correlations to the technical performance parameters are good predictors for the correlation between the objectives, effects, and performance metrics. For

example, the correlation between objective one and objective two can be approximated by  $[corr(O1, T1), corr(O1, T2), corr(O1, T3)]^T [corr(O2, T1), corr(O2, T2), corr(O2, T3)]$ . To demonstrate this, Equation 27 shows the result from applying hypothesis 3.F where the dot products of the impact measures of the TTP metrics were used to calculate the same level correlations. The layout of the matrices matches that of Equation 26. This leads to an initial hypothesis to research question 3.F.

**Research Question 3.F Initial Hypothesis:** Impact matrix correlations can be estimated by multiplying across base level impact relationships.

Equation 28 shows the result from applying hypothesis 3.F where the dot products of the impact measures of the TTP metrics were used to calculate the same level correlations. The layout of the matrices matches that of Equation 26 and Equation 27. Observing these results it is obvious that the initial hypothesis 3.F is overturned. Multiplying across base level impact relationships is not a good method for estimating the same level correlations.

Another option for estimating the same level correlations is to sum the minimum normalized correlations of the TTP metrics. For example, to estimate the correlation between objective one and objective two the minimum normalized correlation for each TTP with respect to the objectives would be summed, e.g.  $CorrEst = min(0.21, 0.24) + min(0.51, 0.44) + min(0.28, 0.32)$ . Equation 28 shows the result from applying this approach. As can be seen there is some error in this approach. However, the estimate is close to the actual value. The result presents another hypothesis for research question 3.F. Further analysis is required to make a determination on hypothesis 3.F.

**Research Question 3.F Revised Hypothesis:** Impact matrix correlations can be estimated by summing minimum normalized correlations across base level impact relationships.

$$\begin{bmatrix} 1.00 & 0.99 & 0.93 \\ - & 1.00 & 0.91 \\ - & - & 1.00 \end{bmatrix} \begin{bmatrix} 1.00 & 0.51 & 0.84 \\ - & 1.00 & 0.74 \\ - & - & 1.00 \end{bmatrix} \begin{bmatrix} 1.00 & 0.14 & 0.55 \\ - & 1.00 & 0.24 \\ - & - & 1.00 \end{bmatrix} \begin{bmatrix} 1.00 & 0.00 & 0.00 \\ - & 1.00 & 0.00 \\ - & - & 1.00 \end{bmatrix} \quad (26)$$

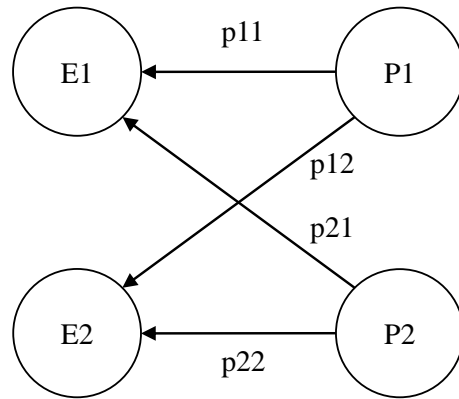


$$\begin{bmatrix} 1.00 & 0.99 & 0.93 \\ - & 1.00 & 0.91 \\ - & - & 1.00 \end{bmatrix} \begin{bmatrix} 1.00 & 0.51 & 0.84 \\ - & 1.00 & 0.74 \\ - & - & 1.00 \end{bmatrix} \begin{bmatrix} 1.00 & 0.14 & 0.55 \\ - & 1.00 & 0.24 \\ - & - & 1.00 \end{bmatrix} \begin{bmatrix} 1.00 & 0.00 & 0.00 \\ - & 1.00 & 0.00 \\ - & - & 1.00 \end{bmatrix} \quad (27)$$

$$\begin{bmatrix} 0.38 & 0.36 & 0.41 \\ - & 0.35 & 0.39 \\ - & - & 0.51 \end{bmatrix} \begin{bmatrix} 0.34 & 0.30 & 0.35 \\ - & 1.00 & 0.52 \\ - & - & 0.50 \end{bmatrix} \begin{bmatrix} 0.52 & 0.08 & 0.40 \\ - & 0.68 & 0.20 \\ - & - & 1.00 \end{bmatrix} \begin{bmatrix} 1.00 & 0.00 & 0.00 \\ - & 1.00 & 0.00 \\ - & - & 1.00 \end{bmatrix} \quad (28)$$

$$\begin{bmatrix} 1.00 & 0.93 & 0.79 \\ - & 1.00 & 0.76 \\ - & - & 1.00 \end{bmatrix} \begin{bmatrix} 1.00 & 0.30 & 0.70 \\ - & 1.00 & 0.52 \\ - & - & 1.00 \end{bmatrix} \begin{bmatrix} 1.00 & 0.2 & 0.40 \\ - & 1.00 & 0.20 \\ - & - & 1.00 \end{bmatrix} \begin{bmatrix} 1.00 & 0.00 & 0.00 \\ - & 1.00 & 0.00 \\ - & - & 1.00 \end{bmatrix} \quad (29)$$

A canonical example is developed to test the revised hypothesis to research question 3.F. The system decomposition of the canonical example is shown in Figure 54. The system of equations that define the relationships between the effect metrics and the performance metrics is shown in Equation 30. Each effect metric is dependent upon both performance metric one and performance metric two. The relationships vary on their linearity. The factor  $k$  is varied between one and five. Additionally, each term contains a coefficient, which take the values 1, 9, and 17. This results in a total of 80 cases for each power  $k$ . The performance metrics are normally distributed with bounds of  $[-10, 10]$ .



**Figure 54:** Canonical Example for Hypothesis 3.F Testing

$$\begin{aligned}
E_1 &= p_{11}P_1^k + p_{21}P_2^k \\
E_2 &= p_{12}P_1^k + p_{22}P_2^k \\
p_{11}, p_{21}, p_{12}, p_{22} &\in [1, 9, 17]
\end{aligned} \tag{30}$$

$$\begin{bmatrix}
1 & \bar{y} & \frac{a}{a+b} & \frac{b}{a+b} \\
& 1 & \frac{c}{c+d} & \frac{d}{c+d} \\
& & 1 & 0 \\
& & & 1
\end{bmatrix} \tag{31}$$

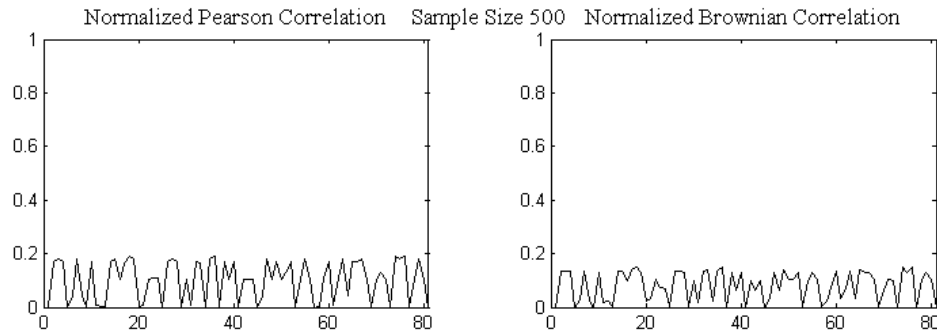
$$\bar{y} = \min\left(\frac{a}{a+b}, \frac{c}{c+d}\right) + \min\left(\frac{b}{a+b}, \frac{d}{c+d}\right) \tag{32}$$

The results from the experiments are shown in Figures 55 through 59. For each figure the left graph shows the results for using normalized Pearson Correlation. The right graph shows the results for using normalized Brownian Correlation. The absolute error that is observed between the correlation estimate and the calculated correlation is plotted on the vertical axes. Each case is plotted along the horizontal axes.

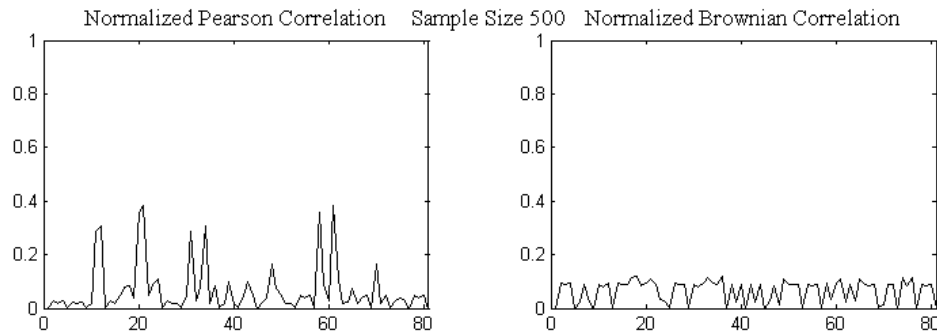
Figure 55 shows the 80 cases when the performance metrics are raised to the first power. As can be seen, the error does not exceed 0.2. It is also observed that the Brownian Correlations perform better than the Pearson Correlations. Figure 56 shows the results when the performance metrics are raised to the second power. It is observed that the error for the Pearson Correlation graph has increased by a factor of two. The Brownian Correlation error, however, remains at the same level. Figure 57 shows the results when the performance metrics are raised to the third power. It is observed that the error for the Pearson Correlation graph has dropped to the level observed in Figure 55. Brownian Correlation error remains unchanged. Figure 58 shows the results when the performance metrics are raised to the fourth power. It is observed that the error for the Pearson Correlation graph has increased again. The error is similar to the Figure 56 error. The Brownian Correlation error remains unchanged. Finally, Figure 59 shows the results when the performance metrics are raised to the fifth power. It is observed that the error for the Pearson Correlation

graph has returned to its lowest observed error. The Brownian Correlation error remains unchanged.

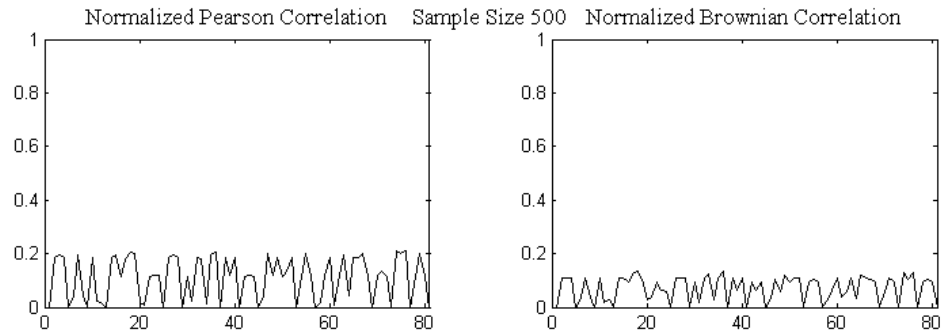
The trend discovered from the results is that the use of the hypothesis 3.F method while using Pearson Correlations tends to have lower error when the driving factor is raised to an odd power. When the driving factor is raised to an even power greater error is observed. This is most likely due to the fact that a straight line can better represent a variable that ranges between  $-10$  and  $10$  to an odd power than a variable to an even power. The Pearson Correlations of the even power variables is zero; therefore, significant error arises. The primary source of the error is from sample error. In conclusion, hypothesis 3.F has failed to be rejected. Hypothesis 3.F performs better for Brownian Correlations than for Pearson Correlations. Since the impact values are normalized Brownian Correlations, hypothesis 3.F should perform well for providing an estimate of the same level correlation that will be used later in model validation.



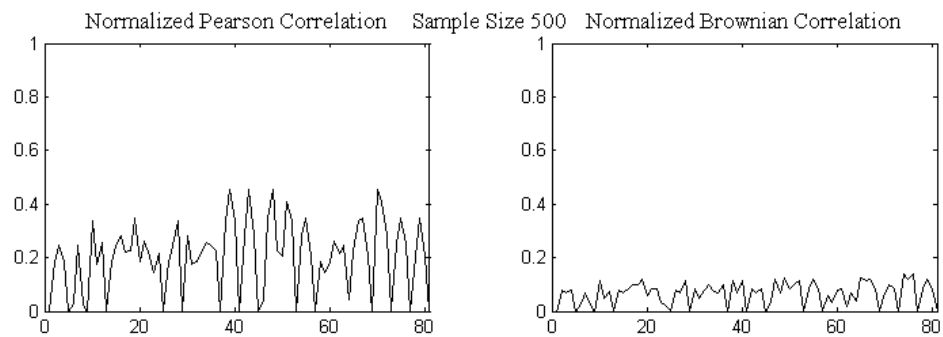
**Figure 55:** Canonical Example Results for Level 1



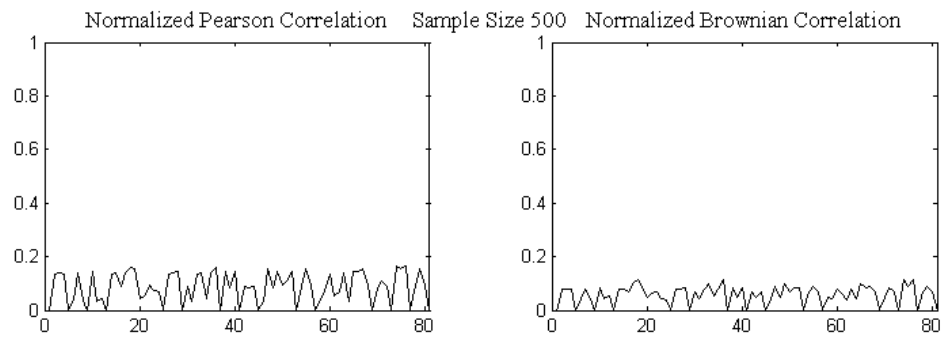
**Figure 56:** Canonical Example Results for Level 2



**Figure 57:** Canonical Example Results for Level 3



**Figure 58:** Canonical Example Results for Level 4



**Figure 59:** Canonical Example Results for Level 5

It must be noted that the experiments performed in this section fail to reject Hypothesis 3.F. These results lend support for the hypothesis. However, later observations made while applying this methodology to an example problem show that Hypothesis 3.F is a poor predictor of the true values. It will be concluded that this methodology should not include

the same level correlation estimates as a means for validation.

## **4.2 Communicative Models**

Once a conceptual model is defined, a communicative model must be produced. A communicative model is a model representation that can be communicated to both model and non-model developers. The purpose is to let the model be judged and compared against the system that is to be modeled with respect to the study objectives. Balci offers six functions of communicative models. A communicative model should be capable of describing the system, capable of communicating to a wide variety of technical backgrounds, enable analysis and verification, support model documentation, and enable translatability into the programmed model. This leads to research question 4 shown below.

**Research Question 4:** Which frameworks available in the literature should be used for communicative models?

Additionally, Balci provides six optional forms for the communicative model: structured computer-assisted graphs, flowcharts, structured English and pseudocode, entity-cycle diagrams, condition specification, and diagramming techniques [20]. The techniques that were covered in system and objective definition can be applied to the communicative model with the exception of the Soft Systems Methodology. Soft Systems Methodology is primarily used for describing a real system and is not well suited for defining a conceptual model. These tools cover many of the forms of communicative models that Balci suggests. The techniques are SysML, UML, and DoDAF. Details into each of the possible techniques were covered under system and objective definition; therefore, only the comparative analysis will be covered in this section.

### **4.2.1 Selection of a Method for Communicative Models**

All of the identified methods were discussed in the previous chapter. From this discussion a selection will be made in the same manner in which Research Question One and Research Question Two were answered, using AHP. The comparisons will be based on the Comparison Scale shown in Table 2. Before creating comparisons of the criterion for the

communicative model, the criteria are placed in order from most desirable to least. The criteria for Research Question 4 are based on Balci's communicative model functions. The ranking of the criteria is as follows: capable of describing the system, enable translatability into the programmed model, enable analysis and verification, support model documentation, and enable communication to a wide variety of technical backgrounds. The primary purpose of the communicative model is to describe the programmed system and translate it into a programmed model, thus it is natural for the capability of describing the system to be the most important and the translation into a programmed model to follow a close second. The next most important task beyond describing the programmed system and translating it into a programmed model is making sure the tasks were completed accurately; therefore, the allowing of analysis and verification is the third most important criterion. The reason as to why this criterion falls below the first two is that it cannot be completed without the first two. Model documentation is of primary interest to the research objective. Model documentation enables model traceability thus creating a better argument for the manner in which the model was created. This is the most important criterion whose task is not a critical function of the communicative model. Finally, the capability of communicating to a number of users with various technical backgrounds is important for assuring the model is developed correctly, providing traceability, and for potential model reuse. The comparisons are then made between the criteria. This is detailed in Table 32. The results of this comparison show that the ability to describe the programmed system comprises of 47% of the priority and enabling the translation into a programmed model comprises of 23% of the priority. They constitute the bulk of the impact of the criteria. The next two, enabling analysis and verification and supporting documentation, both represent 12% of the priority. Finally, the capability of communicating to other is 7% of the priority.

The next step is to compare the three methods for the criterion of descriptive. The three methods Systems Modeling Language, Unified Modeling Language, and Department of Defense Architecture Framework were rated for descriptive using four levels: very good, good, moderate, and poor. SysML is seen as having a good ability to describe a programmed system. This is mainly due to its origin based in UML. UML is seen as having a very good

**Table 32:** Pairwise Comparison of Criteria

<i>Criterion</i>	Desc.	Trans.	Enable A&V	Support Doc	Comm.	<i>Priority</i>
Descriptive	1	2	4	4	6	0.47
Translatable	1/2	1	2	2	3	0.23
Enable A&V	1/4	1/2	1	1	2	0.12
Support Doc	1/4	1/2	1	1	2	0.12
Communicable	1/6	1/3	1/2	1/2	1	0.07

ability to describe a programmed system, since this is the primary purpose of UML. Finally, the ability to describe for DoDAF is considered moderate. The comparisons are then made between the methods and is shown in Table 33. UML, SysML, and DoDAF were rated at 65%, 23%, and 12%, respectively.

**Table 33:** Pairwise Comparison of Descriptive

<i>Descriptive</i>	UML	SysML	DoDAF	<i>Priority</i>
SysML	1	1/3	2	0.23
UML	3	1	5	0.65
DoDAF	1/2	1/5	1	0.12

The next criterion is translatable, how well the method will enable the creation of a programmed model. The three methods were rated for translatable using the four levels. SysML and DoDAF were seen to perform equally well for this task. Given that neither method is primarily used to define a programmed model, but both have views that would enable this task. Therefore, it is assumed that both would perform equally well. The UML method is found to perform the best and is rated as very good. The comparisons are then made between the methods and is shown in Table 34. UML, SysML, and DoDAF were rated at 51%, 17%, and 17%, respectively.

**Table 34:** Pairwise Comparison of Translatable

<i>Translatable</i>	UML	SysML	DoDAF	<i>Priority</i>
SysML	1	1/3	1	0.20
UML	3	1	3	0.60
DoDAF	1	1/3	1	0.20

The next criterion is enable analysis and verification. The three methods were rated for translatable using the four levels. SysML and DoDAF were seen to perform equally well for this task. Given that neither method is primarily used to define a programmed model but both have views that would enable this task. Comparisons can still be made between the views and the programmed model for verification; therefore, it is assumed that both would perform equally well. Since the task of A&V and translation are so similar, often the tasks are performed simultaneously, the results are the same for enable A&V and translatable. The UML method is found to perform the best and is rated as very good. The comparisons are then made between the methods and is shown in Table 35. UML, SysML, and DoDAF were rated at 51%, 17%, and 17%, respectively.

**Table 35:** Pairwise Comparison of Enable Analysis and Verification

<i>Enable A&amp;V</i>	UML	SysML	DoDAF	<i>Priority</i>
SysML	1	1/3	1	0.20
UML	3	1	3	0.60
DoDAF	1	1/3	1	0.20

The next criterion is support documentation. Interestingly, all methods should be able to support the documentation of the communicative model into the programmed model equally well. There is nothing unique about one of these methods over another that would enable it to be better documented. For this reason the three methods are equally weighted. The comparisons are shown in Table 36. UML, SysML, and DoDAF were rated at 33% each.

**Table 36:** Pairwise Comparison of Support Doc

<i>Support Doc</i>	UML	SysML	DoDAF	<i>Priority</i>
SysML	1	1	1	0.33
UML	1	1	1	0.33
DoDAF	1	1	1	0.33

The last criterion is communicable. The criterion communicable defines how well the method will enable the communication of what form the programmed model will take to



the interested parties. The three methods were rated for translatable using the four levels. SysML is rated as good. The SysML diagrams are made to be easily interpreted; however, one does have to be familiar with SysML diagrams to fully understand the views. For this reason SysML was not given a rating of very good. UML is very similar to SysML and suffers from the same issue as SysML with regards to the communicable criterion, thus it is also weighted as good. DoDAF contains numerous views and the standards for the views are more relaxed than are for UML or SysML. This causes issues for its ability to communicate to future users of the model. For this reason DoDAF is rated as a moderate. The comparisons are then made between the methods and is shown in Table 37. UML, SysML, and DoDAF were rated at 42%, 42%, and 17%, respectively.

**Table 37:** Pairwise Comparison of Communicable

<i>Communicable</i>	UML	SysML	DoDAF	<i>Priority</i>
SysML	1	1	3	0.43
UML	3	1	3	0.43
DoDAF	1/3	1/3	1	0.14

The results of the AHP method are shown in Table 38. As can be seen the Unified Modeling Language performed the best with a rating of 57%. The combination of SysML and UML within the model methodology presented in this thesis will make model development easier on the modeler, given that dissimilar methods will not have to be used. Many of the views created in system definition using SysML can be converted into UML views for the communicative model fairly easily.

**Table 38:** AHP Score RQ4

	SysML	UML	DoDAF
Score	0.24	0.58	0.18

**Research Question 4 Hypothesis:** The Unified Modeling Language is the best suited method for the communicative model phase of model development

## CHAPTER V

### EXPERIMENTATION AND RESULTS

After the model has been programmed, producing the programmed model, the experimental model must be developed. The experimental model includes both the plan to gather desired information and a programmed model that enables the execution of these experiments. If many experiments are required then the model must be modified to enable numerous experiments, e.g. the use of a batch file. Balci suggests several techniques for experimentation: response surface methodologies, variance reduction techniques, ranking and selection, and general statistical analysis. The experiments are then executed and analyzed for a specific purpose.

There exist a significant number of experimental methods in the literature; however, a gap was found. The area that seems to be neglected is the analysis of stochastic simulations that are not industrial systems. In this field two questions are presented: which measures should be used when faced with a stochastic simulation, and how many replications are required for accurate confidence interval estimation of the stochastic measures? The first research question addresses the fact that many stochastic simulations primarily use the mean as the final measure. At times the variance will also be included. The question addresses which of the stochastic measures, e.g. mean, variance, percentiles, and quantiles, should be used. The second question addresses the fact that the number of replications needed for an accurate stochastic measure varies depending on the measure used and the distribution applied to it.

**Research Question 5:** Which stochastic measures should be used when faced with a stochastic simulation?

**Research Question 6:** How many replications are required for accurate confidence interval estimation?

## ***5.1 Investigation into the Measures of Stochastic Simulation***

Stochastic outputs are common in military operational models and other complex models [208, 198, 7, 183, 50]. This stochasticity introduces added complexity to the analysis of the simulation output. This complexity is often simplified to the sample mean [196] or is modeled deterministically [11, 13]. Some simulation practitioners expand their analysis to include an investigation of the variance [12, 198, 7, 183, 50]. Unfortunately, these stochastic measures do not fully capture the output distributions. In fact, the only legitimate measure of the stochastic output of a simulation is the full observed distribution; however, this is not practical. The simulation output must be simplified for a human to grasp the behavior of the output. Conversely, over simplification can mislead the analysis. Therefore, the output must be simplified, but it can not be overly simplified. This conclusion returns back to Research Question 5, “Which stochastic measures should be used when faced with a stochastic simulation?”. To answer this, a set of potential stochastic measures will be compared in their application of a canonical problem. A candidate list of common measures of stochastic outputs includes mean, variance, binomial proportions, and quantiles. This list is based on the measures provided by common text books [100, 77]. It should be noted that skewness and kurtosis are found to be common descriptors of distributions; however, they have not been found to be used in output analysis. The test problem that will be used to compare these measures is based on a simple Mine Counter Measures (MCM) model that was developed as a student project. A basic overview of the model will be given in the next section.

### **5.1.1 Mine Counter Measures Test Problem**

A simple MCM model was developed as part of a student project. The results of the simulation were stochastic and multi-modal. Additionally, the space of the output was heteroscedastic, i.e. the sub-populations of the output have different variabilities. This output makes it an ideal candidate for comparing different stochastic measures.

The MCM model was developed to estimate the time required to clear an area of mines and the percentage of the mines cleared. In addition, the model was used to estimate the

sensitivities of the output measures to various vehicle performance metrics. The simulated MCM activities occurred in two phases. The first phase consisted of using a set of surveillance vehicles to identify potential mine locations. The second phase consisted of using a set of vehicles that would travel to the potential mine locations and neutralize the mine, if found. The second phase vehicles are not released until the surveillance vehicles have returned. These vehicles are stored and released from a Littoral Combat Ship (LCS) away from the suspected mine locations. The vehicles used for the first phase are the Remote Environmental Monitoring Units (REMUS). The REMUS is a small torpedo shaped autonomous underwater vehicle (AUV) used to detect underwater objects. This platform was successfully used in 2003 during Operation Iraqi Freedom [177] and in recovering the black box of Flight 447 [79]. The vehicles used for mine neutralization are diver teams and helicopters. The diver teams travel to the mine in rigid-hulled inflatable boats, dive to the mine, and neutralize the mine. The helicopter used is the Sikorsky MH-60 Knighthawk. The helicopter neutralizes the mine using a Rapid Airborne Mine Clearance System (RAM-ICS) or similar system. Each of the vehicles used to neutralize the mine have a limited store of neutralizing equipment. Additionally, each of the crews can only operate up to a set amount of time. An image of the model is shown in Figure 60 during the first phase of the simulation. The yellow dots represent mines that have been identified. The red dots are mines that have not been identified. The yellow arrowheads represent the REMUS vehicles surveying the waters. The seemingly grey circle in the upper right is the collection of the LCS along with the dive teams and helicopters.

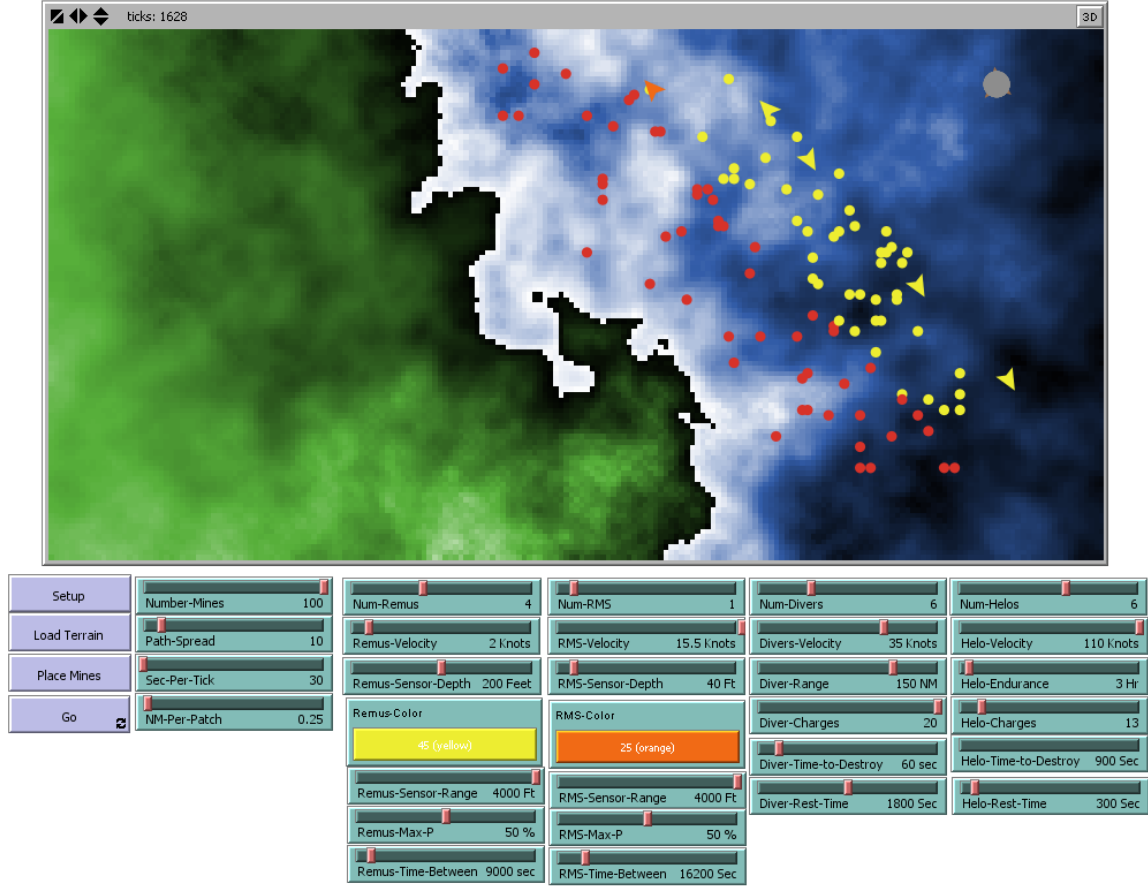


Figure 60: Mine Counter Measures Canonical Example

### 5.1.2 Stochastic Measures Definition

The candidate list of stochastic measures are mean, variance, binomial proportions, and quantiles. The equation for the sample mean is shown in Equation 33, where  $n$  represents the sample size, and  $x_i$  is the  $i^{th}$  sample. The sample variance, a measure of statistical dispersion, is shown in Equation 34, where  $S$  is the standard deviation.

$$\bar{X}(n) = \frac{1}{n} \sum_{i=1}^n x_i \quad (33)$$

$$\hat{S}^2(n) = \frac{\sum_{i=1}^n [x_i - \bar{X}(n)]^2}{n - 1} \quad (34)$$

The bimodal proportion is a measure of the proportions of occurrences that are observed within some bounds, e.g.  $[10, \infty]$ . The equation for the bimodal proportion estimate is

shown in Equation 35, where Z is the number of occurrences within a defined bound. The quantile is the opposite measure to the bimodal proportion. Instead of defining a number and determining the portion of the observation that occurs above or below said number, the quantile defines a proportion and finds the value below which the specified proportion of the observation occurs. The equation of the quantile estimate can be seen in Equation 36, where q is the defined proportion, e.g. 0.25, and X represents the order statistic of the samples. The order statistics of a sample is the definition of the sample from smallest to largest. For the order statistics,  $X_{(1)}, X_{(2)}, \dots, X_{(n)}$ ,  $X_{(1)}$  would represent the smallest sample and  $X_{(n)}$  would represent the largest sample.

$$\hat{p} = \frac{Z}{n} \quad (35)$$

$$\hat{x}_q = X_{\lceil nq \rceil} \quad (36)$$

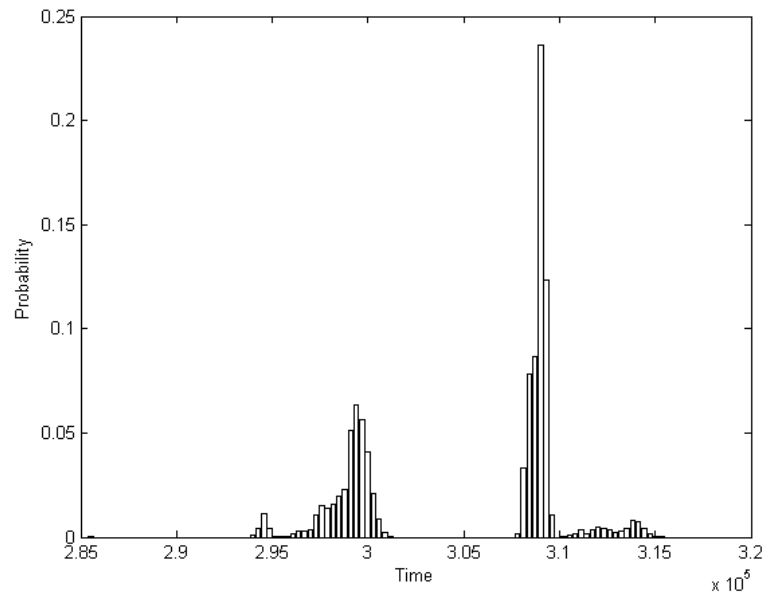
### 5.1.3 Experimentation and Results

Comparisons of the statistical measures will be conducted through two analyses. The first will be a single point analysis. For a given set of inputs the model will be replicated in order to form a sufficient distribution. The four different measures will then be taken and compared for their ability to describe the distribution. The second analysis will be a single variable analysis. One of the variables will be varied over its range with replications. All other variables will be held constant. The four measures will be made at each of these points and compared for their ability to describe the distribution.

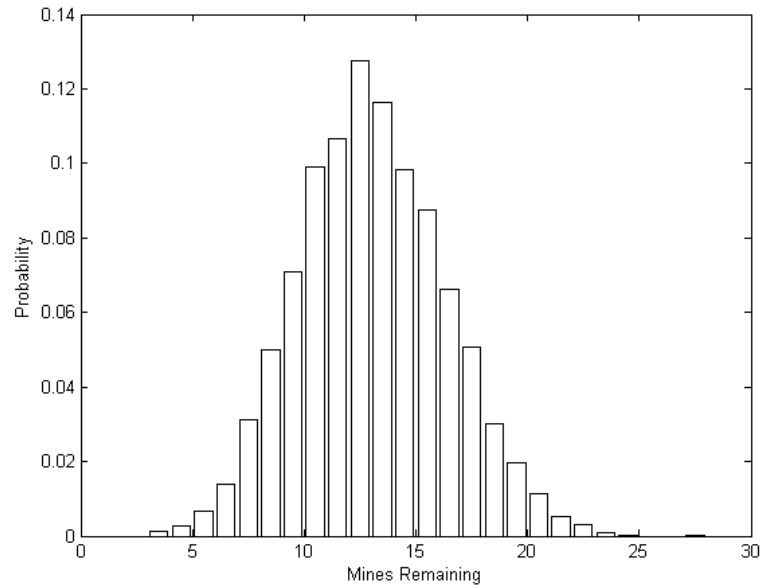
#### 5.1.3.1 Single Point Analysis

The test data for the single point analysis is shown in Figure 61 and in Figure 62. The first shows a histogram of the time required to complete the mission. The second shows the number of mines that were cleared. A total of 5,000 replications were made. The output distribution of the time variable is seen to be multi-modal. This was discovered to be due to the interaction between the number of mines discovered and the number of vehicles and charges available to neutralize the mine. For example, if 12 mines were identified, there

are three dive teams available, and each dive team is able to neutralize four mines without returning to the LCS, then each dive team will travel out and back once. However, if 13 mines were identified with the same number of dive teams and neutralization capabilities, then one of the dive teams would have to travel out and back from the LCS twice resulting in a jump on total time. The output distribution of the mines remaining variable shows a more traditional Gaussian shape.



**Figure 61:** Mine Counter Measures: Time to Clear



**Figure 62:** Mine Counter Measures: Mines Remaining

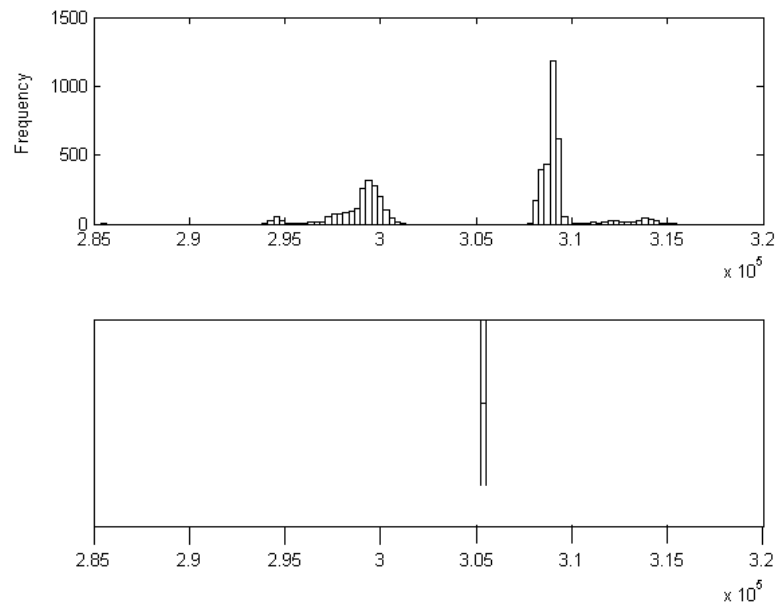
The sample mean and variance for each output is shown in Table 39. It is not reasonable to consider the mean or variance, individually, as a measure for the distribution. This would only be possible for a select few distributions, e.g. Exponential Distribution. Additionally, one would have to know the output follows a specific distribution for these measures to work. For this reason the mean and variance will be compared in conjunction. From the sample mean and variance some information is gathered about the distribution. This data can be used to provide the mean estimate along with confidence intervals. An example of this usage for the time output data is shown in Figure 63. The top histogram shows the probability distribution of the time output. The bottom histogram shows the mean estimate with 95% confidence interval bands. Upon observing this data it seen that the mean estimate with confidence intervals is an insufficient measure of the statistical distribution. Another shortcoming of the use of the mean and variance estimates is that the estimate suggests the data follows a Gaussian distribution. A comparison between the two distributions is shown in Figure 64. It is obvious that the observed output distribution for the time variable is not a Gaussian distribution. Conversely, this measure would work fairly well for the number of mines remaining. A comparison between the observed mines remaining distribution and the



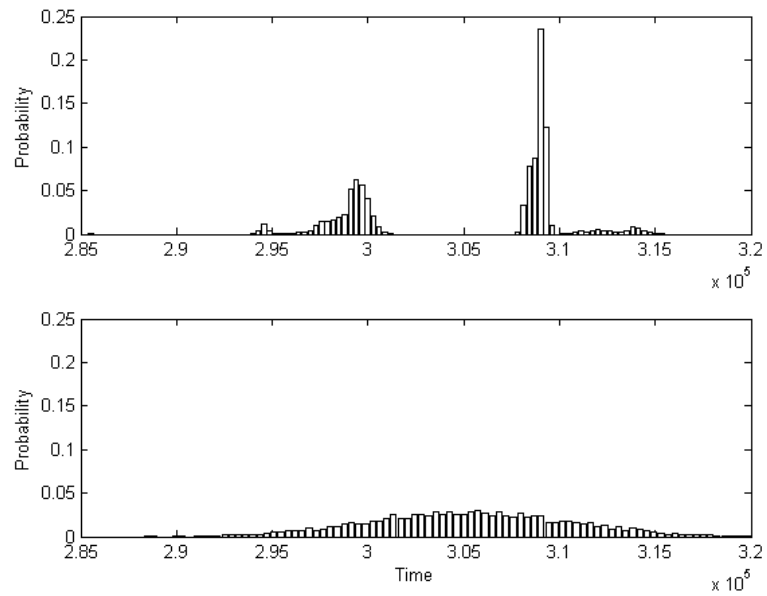
Gaussian distribution with the same mean and variance is shown in Figure 65. As can be seen the two distributions are very similar; however, the mode of the distributions appears to be different. From this discussion it is concluded that the mean and variance estimates are insufficient for the estimation of output distributions for stochastic simulations when the output distribution is not already known.

**Table 39:** Statistical Measures Mean and Variance

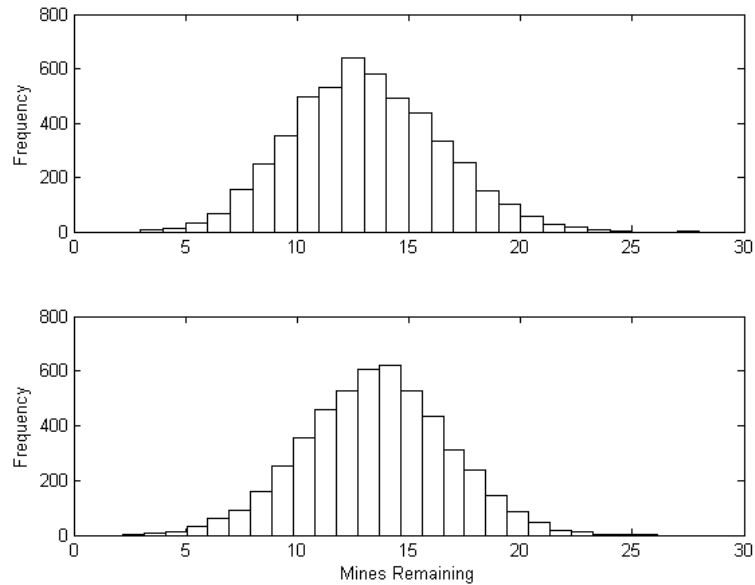
Metric	Mean	Variance
Time	3.0538e5	2.7156e7
Mines Remaining	13.559	10.879



**Figure 63:** Mine Counter Measures: Time Distribution Comparison to Mean with 95% Confidence Intervals



**Figure 64:** Mine Counter Measures: Time Distribution Comparison to Normal Distribution with Same Mean and Variance



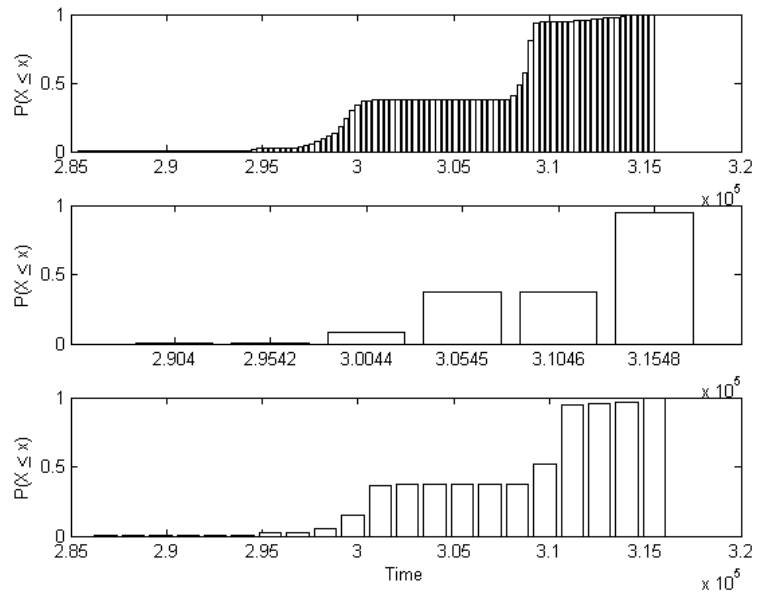
**Figure 65:** Mine Counter Measures: Mines Remaining Distribution Comparison to Normal Distribution with Same Mean and Variance

The binomial proportion estimates are shown in Table 40. The binomial proportion

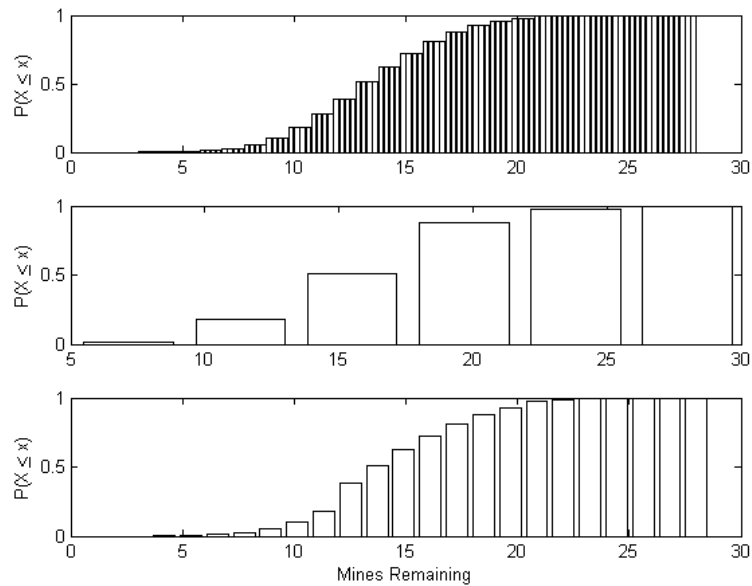
estimates were made by dividing the range of the output data into six parts. The first binomial proportion estimate determines the portion of the output that is observed below the minimum observed value plus one sixth of the range. The second binomial proportion estimate determines the portion of the output that is observed below the minimum observed value plus two sixth of the range. This pattern is continued up to the entire range, which would be unity. The estimate is similar to tracking the cumulative distribution of the output. The comparison between the cumulative distribution of the output and the binomial proportions are shown in Figure 66. The top image shows the observed data. The middle image shows the results from Table 40. Finally, the bottom image shows binomial proportions split into 20 measures. The middle image indicates that there is a multi-modal behavior occurring. This is shown by the flat bars in the middle. If this were uni-modal then continuously increasing bars should be seen. The more estimates that are measured the closer this will resemble the cumulative distribution. This is shown by the increased number of binomial proportion estimates in the bottom image. This figure is repeated for the mines remaining output in Figure 67. The middle figure shows gradually increasing values, which indicates a uni-modal distribution. This is more easily seen in the bottom figure.

**Table 40:** Statistical Measures Binomial Proportions

Metric	BP(1/6)	BP(2/6)	BP(3/6)	BP(4/6)	BP(5/6)	BP(6/6)
Time	0%	2.1%	36.3%	37.4%	94.4%	100%
Mines Remaining	0.1%	4.1%	27.5%	72.4%	97.5%	100%



**Figure 66:** Mine Counter Measures: Time Cumulative Distribution Comparison to Binomial Proportion Estimates



**Figure 67:** Mine Counter Measures: Mines Remaining Cumulative Distribution Comparison to Binomial Proportion Estimates

Finally, the quantile estimates are shown in Table 41. The quantile estimates for seven

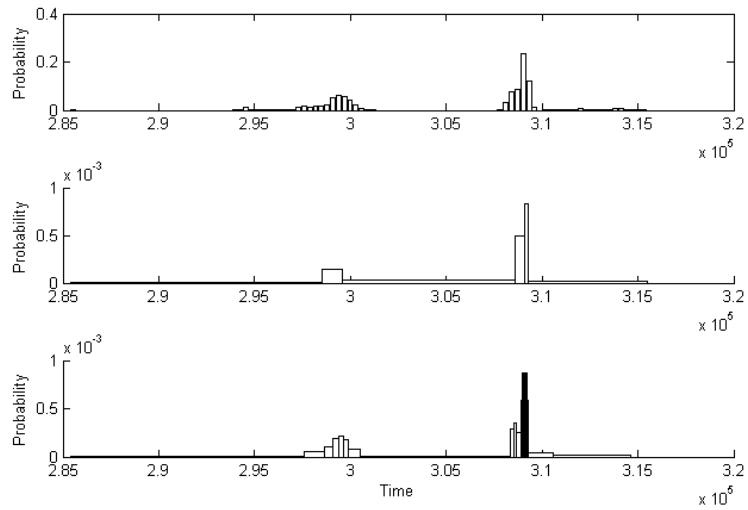
measures were selected to be 0%, 10%, 25%, 50%, 75%, 90%, and 100%. The quantile measures are one more than that of the binomial proportion. This is because the comparison of the quantiles to the observed distributions required the maximum and minimum of the distributions. The observed probability distribution for the time required is shown at the top of Figure 68. The quantile estimate for the seven measures is shown in the middle of the figure and another set is shown with 20 measures. The quantile distributions were made by representing each segment with the proportion it represents. For example, the first segment in Figure 68 ranges from  $2.8539e5$  seconds to  $2.9853e5$  and accounts for 10% of the observed values. The 10% is then divided by the x-value range. This process is repeated for each section.

The middle graph in Figure 68 shows that there exists a multi-modal distribution with one peak occurring slightly below 300000 seconds and another occurring slightly below 310000 seconds; the latter being the more prevalent one. When more quantile measures are taken, as is shown in the bottom graph, the distribution becomes more clear. Note that the quantile graphs are not quite the same as the histogram shown in the top of the figure. This is because the distance between the bins on the time axis are of different sizes. This also explains the smaller values on the ordinate, because by definition no value can be larger than 0.25. The values are smaller than that because they cover some distance on the time axis.

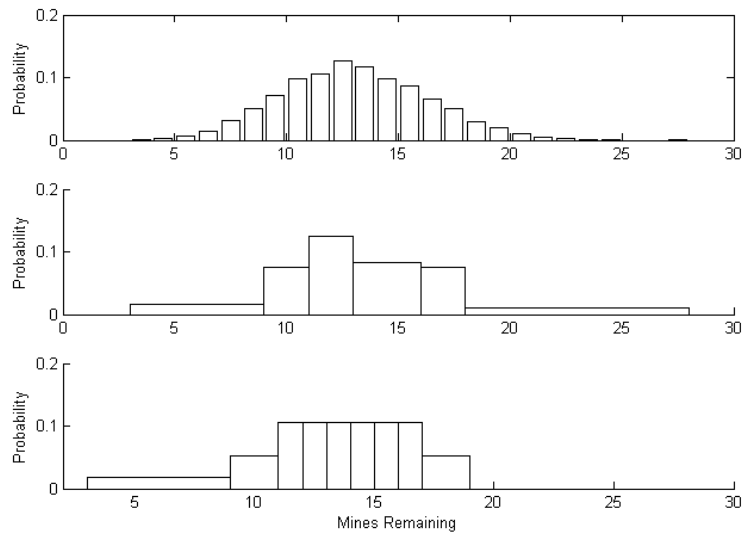
The same figure is reproduced for the mines remaining output and is shown in Figure 69. The middle figure shows that data from Table 41 and represents the observed data well. The middle figure also helps to illuminate the fact that the observed data is skewed and has a long right tail. The bottom graph in Figure 69 shows 10 quantile measures. A limitation of the quantile approach arises here. As can be seen, the quantile measures in the middle of the bottom graph have the same width and the same level of occurrence. This is due to the discrete nature and small range of the output. Therefore, the limitation of quantiles is on the measurement of outputs with discrete values and small ranges.

**Table 41:** Statistical Measures Quantiles

Metric	Q(0%)	Q(10%)	Q2(25%)	Q3(50%)	Q4(75%)	Q5(90%)	Q(100%)
Time	2.8539e5	2.9853e5	2.9955e5	3.0861e5	3.0912e5	3.0930e5	3.1548e5
Mines	3	9	11	13	16	18	28

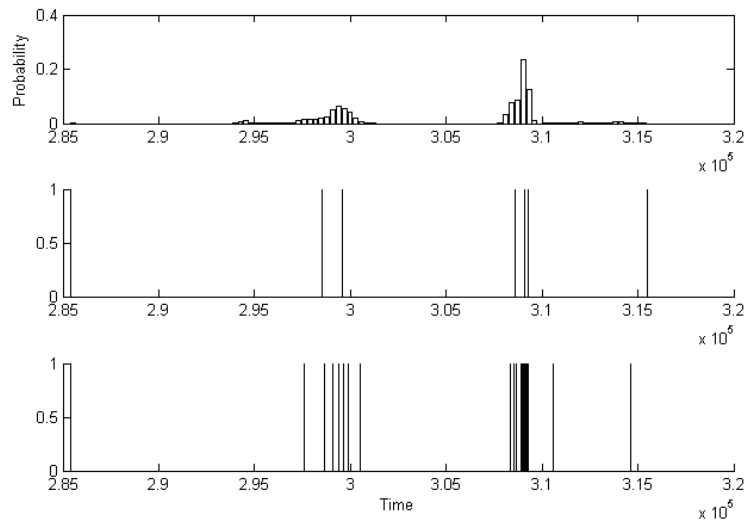


**Figure 68:** Mine Counter Measures: Time Probability Distribution Comparison to Quantile Estimates

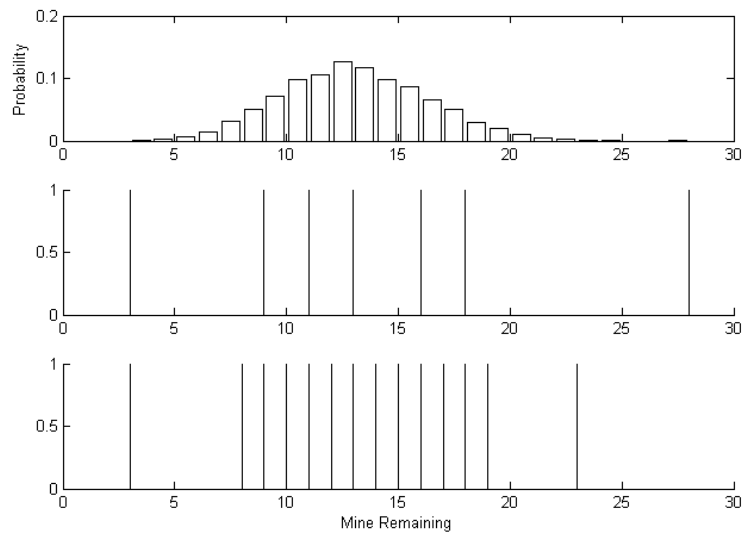


**Figure 69:** Mine Counter Measures: Mines Remaining Probability Distribution Comparison to Quantile Estimates

Since quantiles do not match histograms for probability density functions as well as the binomial proportions it is better to represent the information as lines. This is similar to a box plot view. Examples of this graph for the time and mines remaining outputs are shown in Figures 70 and 71, respectively. Figure 70 shows the observed distribution at the top, the seven quantile measures in the middle, and the 20 quantile measures at the bottom. As can be seen there is a line for the maximum and minimum. The other lines tend to cluster towards the peak(s) of the distribution. In this case two clusters are seen. In Figure 71 one cluster can be seen.



**Figure 70:** Mine Counter Measures: Time Probability Distribution Comparison to Quantile Estimates (Lines)



**Figure 71:** Mine Counter Measures: Mines Remaining Probability Distribution Comparison to Quantile Estimates (Lines)



### 5.1.3.2 Variable Selection for Multipoint Analysis

The next set of analyses of the stochastic measures is to measure with each stochastic measure one variable over a range while holding all other variables constant. The results will then be discussed, a comparison will be made, and a conclusion will be drawn.

The selection of which variable will be varied across its range is critical to the success of this analysis. A variable must be selected that has a sufficient impact of the measured outputs so that comparisons can be made. A Design of Experiments (DOE) was developed and executed in order to assess variable sensitivity. The DOE used was a 500 case Latin Hypercube Sampling (LHS) DOE developed for 17 variables. These variables and their ranges are shown in Table 42. A total of 45 replications were run for each case resulting in a total of 22,500 runs. The sample mean and sample variance were used as indicators for the variable that should be selected. The sample mean and sample variance act as indicators of central tendency and dispersion. The reasoning goes that a significant change in the variance would also cause a significant change in the binomial proportions and the quantiles, since these three measures are measures of statistical dispersion. Note that the distance between the binomial proportions and quantiles indicate the dispersion; not necessarily a measure in isolation. The binomial proportion and quantiles can also give information on central tendency; therefore, the sample mean will also provide useful information to the variable that should be selected.

**Table 42: DOE Variable Ranges**

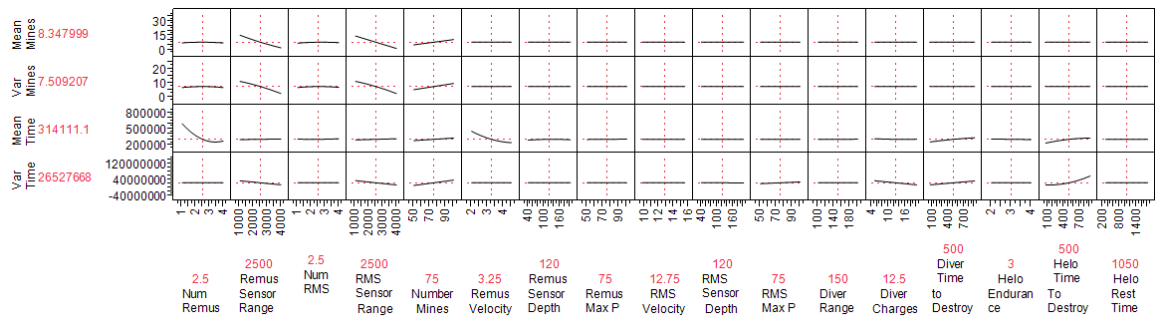
Variable	Minimum	Maximum
Number Remus	1	4
Remus Velocity	2 Knots	4.5 Knots
Remus Sensor Depth	40 ft	200 ft
Remus Sensor Range	1000 ft	4000 ft
Remus Max Prob Detection	50%	100%
Number RMS	1	4
RMS Velocity	9.5 Knots	15.5 Knots
RMS Sensor Depth	40 ft	200 ft
RMS Sensor Range	1000 ft	4000 ft
RMS Max Prob Detection	50%	100%
Diver Range	100 nmi	200 nmi
Diver Charges	5	20
Diver Time-to-Destroy	60 sec	900 sec
Helo Endurance	2hr	4hr
Helo Time-to-Destroy	60 sec	900 sec
Helo Rest Time	300 sec	1800 sec
Number Mines	50	100

The mathematical program MATLAB was used to filter the output data from the DOE and find the sample mean and sample variance for the time and mines remaining outputs. This data was then transferred into the statistical program JMP where the regressions and sensitivity analysis was performed.

The regressions were performed in a two step process. The first step was to use the Stepwise Regression feature available in JMP. This step helps to identify the correct mathematical model to use for the regression in the next step. Stepwise Regression is particularly useful when there is no guidance on the selection of terms for a model and the primary goal

is to achieve a good fit [175]. The stopping criterion used was the Minimum BIC which uses the minimum Bayesian Information Criterion to select the model. Once the model was selected the model was regressed using the JMP Fit Model to create a Response Surface Model using standard least squares. For the sample mean the inverse of the standard deviation was used for weightings to ensure a better fit.

Once the four regression models were developed, prediction profilers were used as a measure of local sensitivity analysis. Prediction profilers traces are displayed for each input variable of a response. The trace is a line that shows the predicted response to an input variable over its range while the other input variables are held constant. If another variable's value is changed, then the traces are updated to reflect that change. The prediction profiler is useful for, among other applications, identifying the response's sensitivity to the input factors. The prediction profiler for all four responses, statistical measures, can be seen in Figure 72. After interaction with the profiler a few conclusions as to which inputs have the greatest impact were drawn. These results are summarized in Table 43. Each quadrant displays in order of importance the three or four variables that have the greatest impact on the statistical measure. As can be seen the Remus Sensor Range variable appears in three of the four measures. For this reason this variable will be used for the multipoint analysis.



**Figure 72:** Mine Counter Measures: Prediction Profiler

**Table 43:** MCM Variable Sensitivity

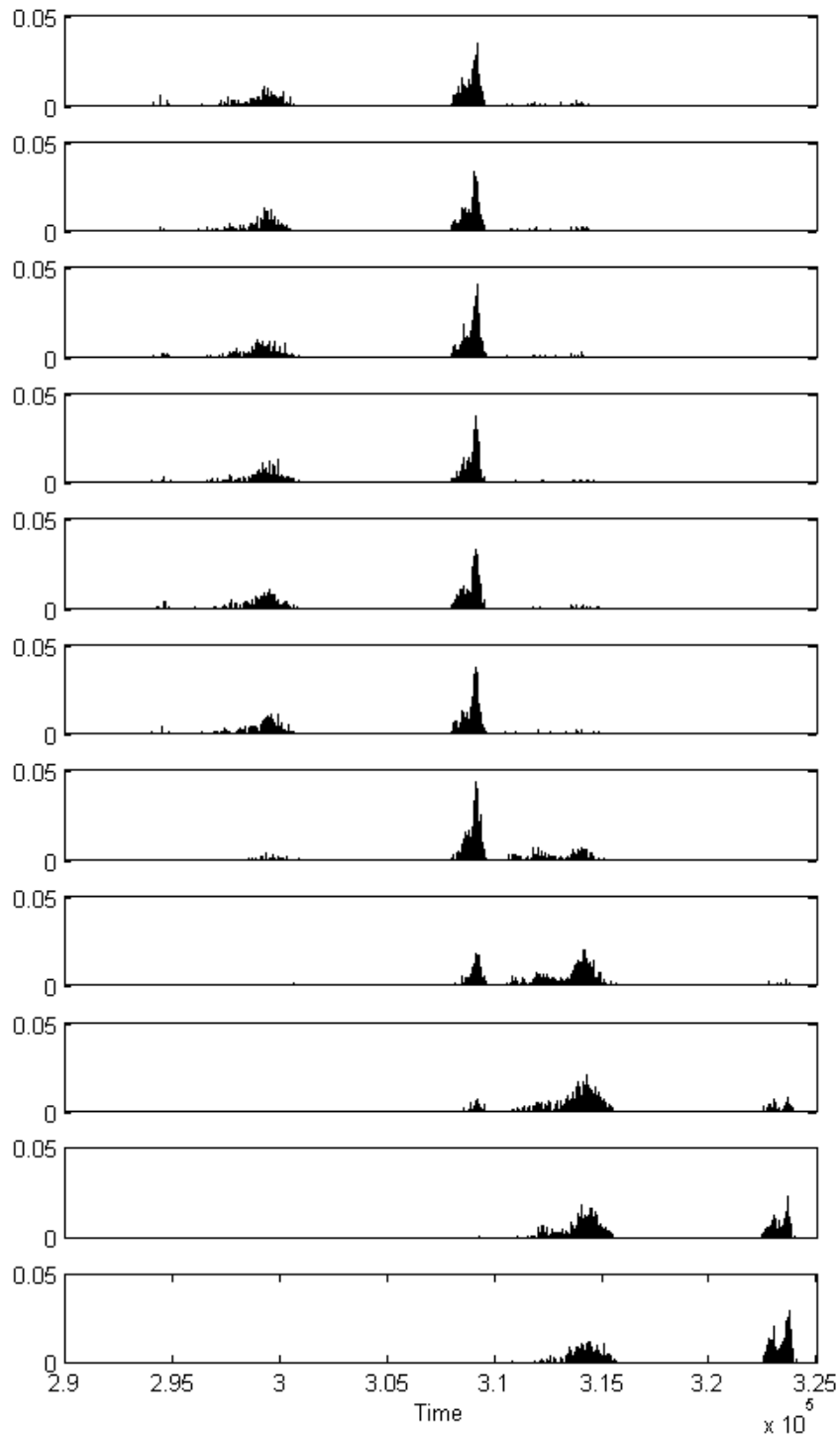
	Time	Mines Remaining
Mean	- Number Remus - Remus Velocity - Helo Time to Destroy	- Remus Sensor Range - RMS Sensor Range - Number Mines
Variance	- Helo Time to Destroy - Number Mines - Remus Sensor Range - Diver Time to Destroy	- Remus Sensor Range - RMS Snesor Range - Number Mines

### 5.1.3.3 Multipoint Analysis

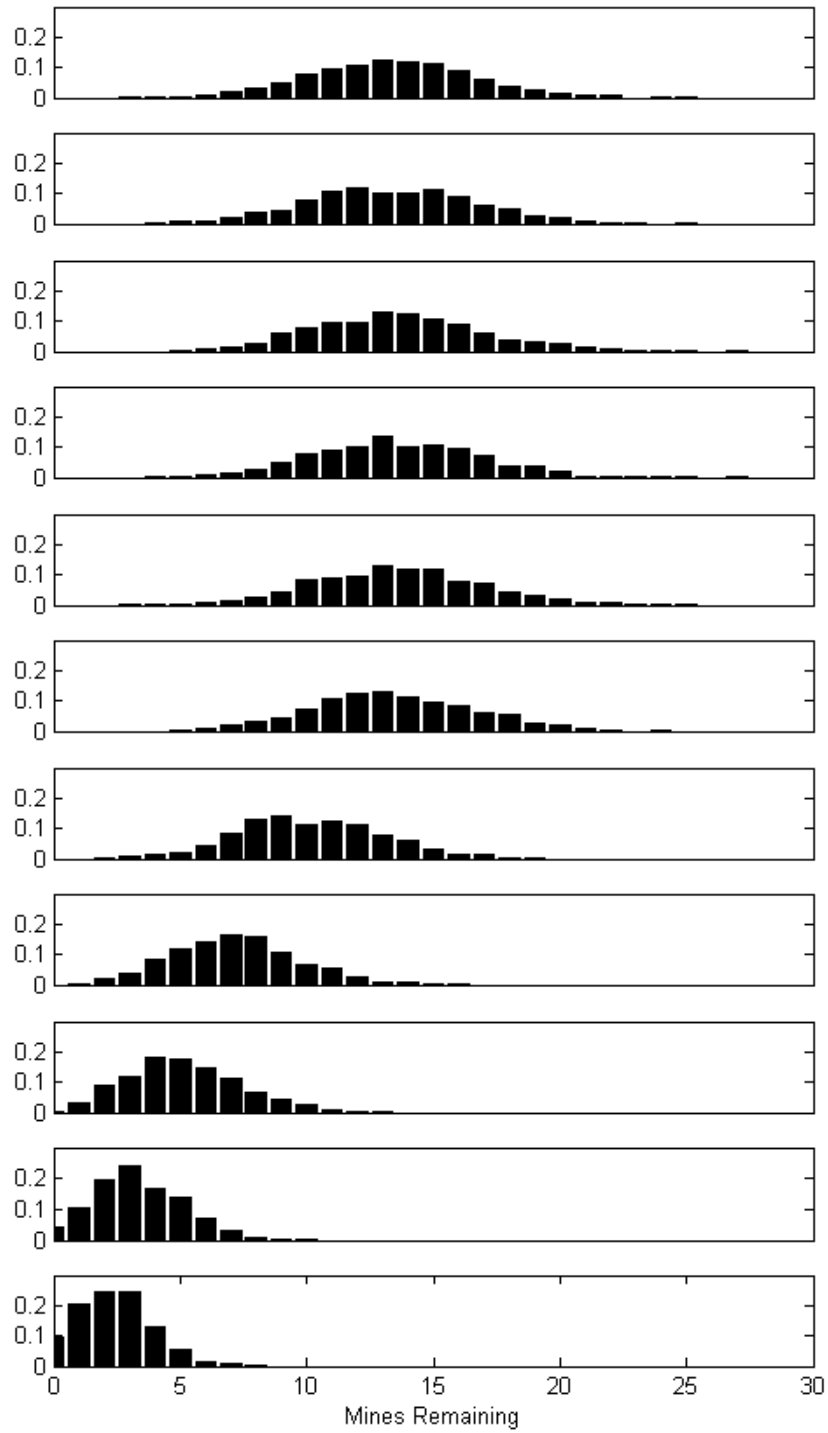
The Remus Sensor Range variable will be varied between 1000 ft and 4000 ft at 11 levels. Each case will then be replicated 1000 times resulting in a total of 11000 runs. The results for these runs are shown in Figures 73 and 74 for the time and mines remaining simulation output, respectively. The top histogram of the figures is the first run where the Remus Sensor Range is set at 1000 ft. The bottom histogram of the figures is the last run where the Remus Sensor Range is set at 4000 ft.

The histograms in Figure 73 show the same multi-modal behavior that was observed in the previous analysis section. An interesting observation is that as the Remus Sensor Range increases the smallest peak becomes less common of an occurrence. Then another peak begins to emerge at about  $3.15e5$  seconds and the peak at about  $3.1e5$  begins to soften. Finally, a peak forms at about  $3.25e5$ .

The histograms in Figure 74 show the same uni-modal behavior that was observed in the previous analysis section. As the Remus Sensor Range increases the number of mines continues to drops. Eventually, the distribution becomes skewed due to the limitation that the output cannot drop below zero.

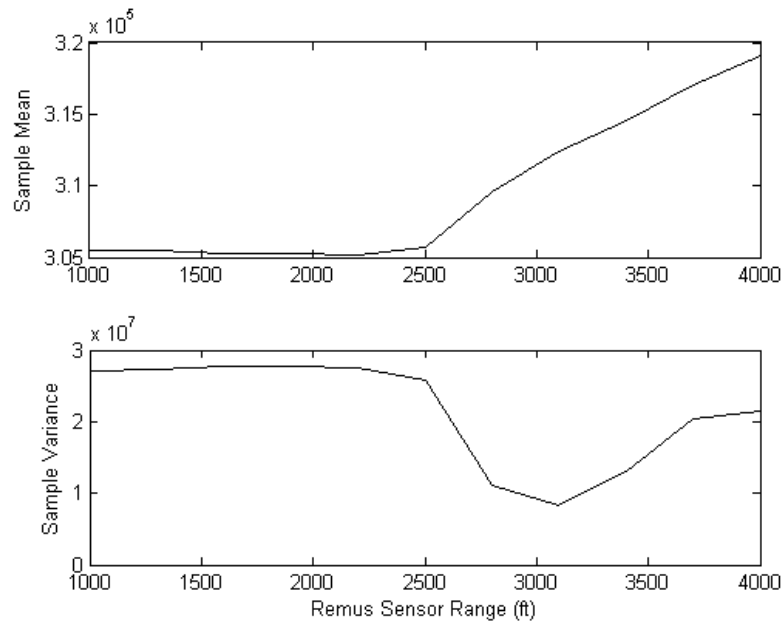


**Figure 73:** Mine Counter Measures: Histogram of Time for 11 Levels

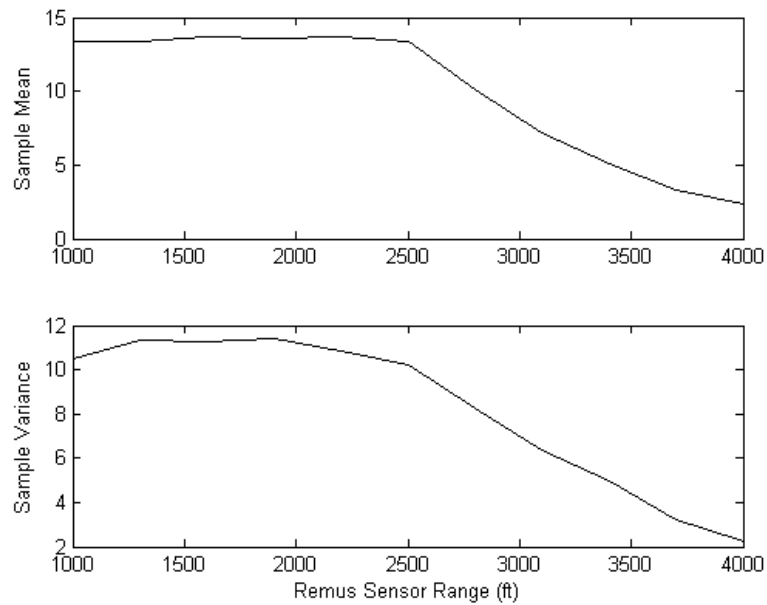


**Figure 74:** Mine Counter Measures: Histogram of Mines Remaining for 11 Levels

The sample mean and sample variance for the Time output and the Mines Remaining output can be seen in Figures 75 and 76, respectively. The sample mean measure for time is seen at the top of Figure 75. What is observed is that the Remus Sensor Range of 2500 is a tipping point in which the time increases linearly. The sample variance, below the mean graph, shows a more convoluted result. The variance remains mostly constant until the Remus Sensor Range of 3000 ft in which the variance dips and then begins to return to its previous level. The sample mean for the mines remaining output, shown in the top graph of Figure 75, indicates the same result as the output for the mean of the time. It is shown that the Remus Sensor Range of 2500 is a tipping point in which the mines remaining decreases linearly. Similarly, the sample variance shows the same behavior.



**Figure 75:** Mine Counter Measures: Sample Mean and Sample Variance of Time for 11 Levels



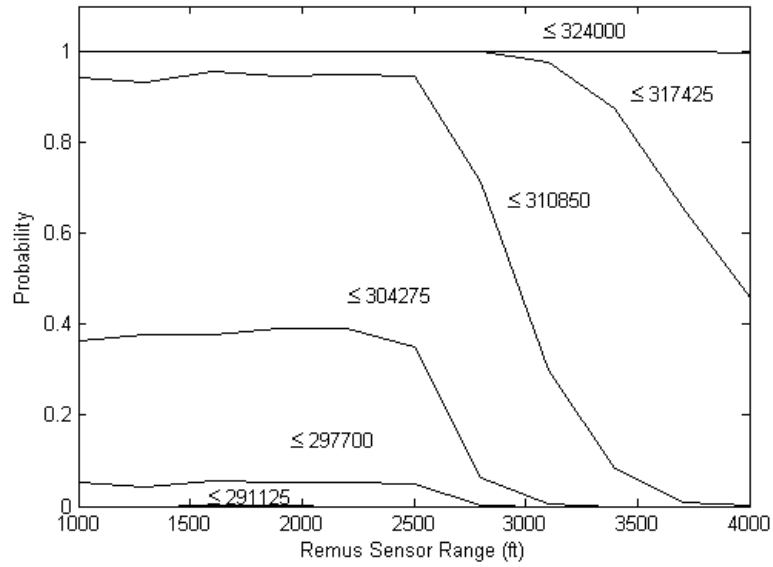
**Figure 76:** Mine Counter Measures: Sample Mean and Sample Variance of Mines Remaining for 11 Levels

The binomial proportion estimates for the time output is shown in Figure 77. Six binomial proportion values were selected in a similar manner as in the single point analysis section. The binomial proportion estimates were made by dividing the observed range of all output data into six parts. The first binomial proportion estimate determines the portion of the output that is observed below the minimum observed value plus one sixths of the range. The second binomial proportion estimate determines the portion of the output that is observed below the minimum observed value plus two sixth of the range. This pattern is continued up to the entire range, which would be unity. Figure 77 includes labels indicating the time used for the binomial proportion. Similar conclusions can be drawn from this figure as was drawn from the sample mean and variance measures, which is that the time output begins to change once the Remus Sensor Range rises above 2500 ft. This is indicated by the knee in the curve for three of the middle measures. The binomial proportions for the mines remaining output is shown in Figure 78. The same conclusion is drawn from this figure; however, the curves are different indicating fewer mines remaining.

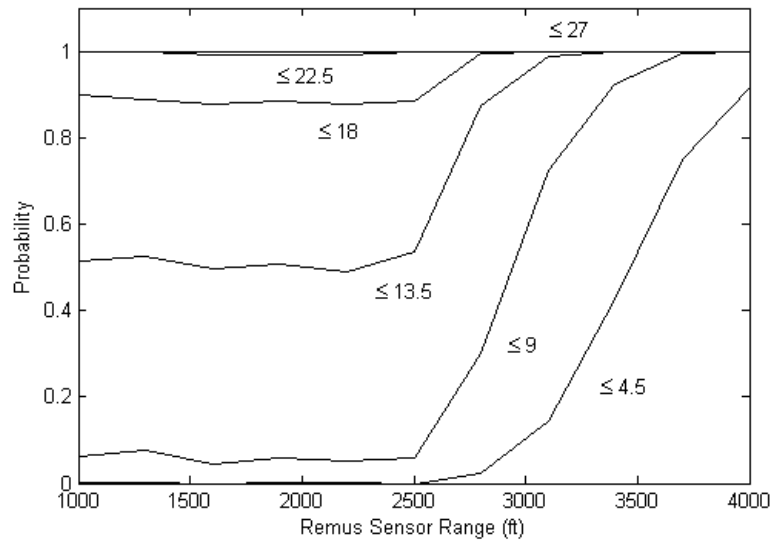
Another observation can be drawn from these figure; however, they are more subtle and



difficult to identify. It can be observed that there exist a multi-modal behavior in the time output and not in the mines remaining output. This observation is made by observing the difference between the lines in their probability. Note in Figure 77 the bottom two lines are very close together. Also the top three lines are very close together, but there is a large gap between them. This is indicative of a multi-modal distribution. Note in Figure 78 the difference in spacing is more uniform, which is indicative of a uni-modal distribution.



**Figure 77:** Mine Counter Measures: Binomial Proportions of Time for 11 Levels

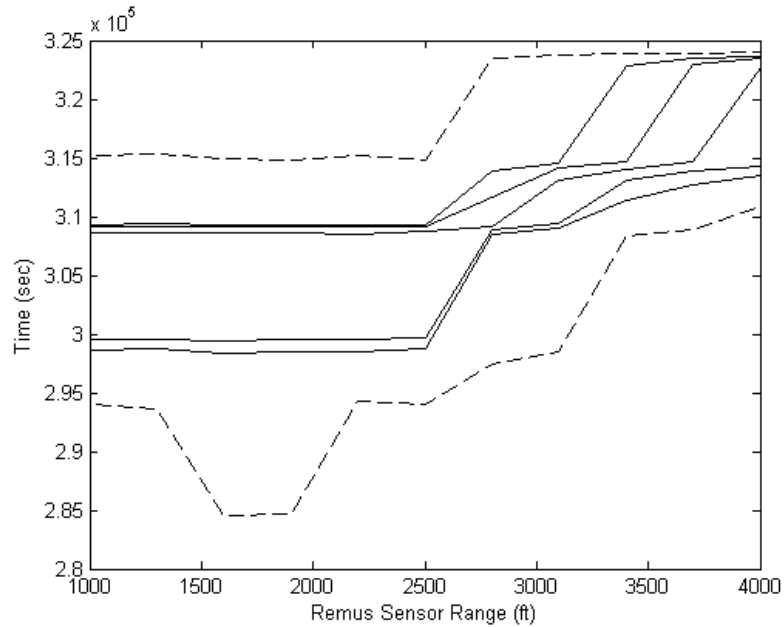


**Figure 78:** Mine Counter Measures: Binomial Proportions of Mines Remaining for 11 Levels

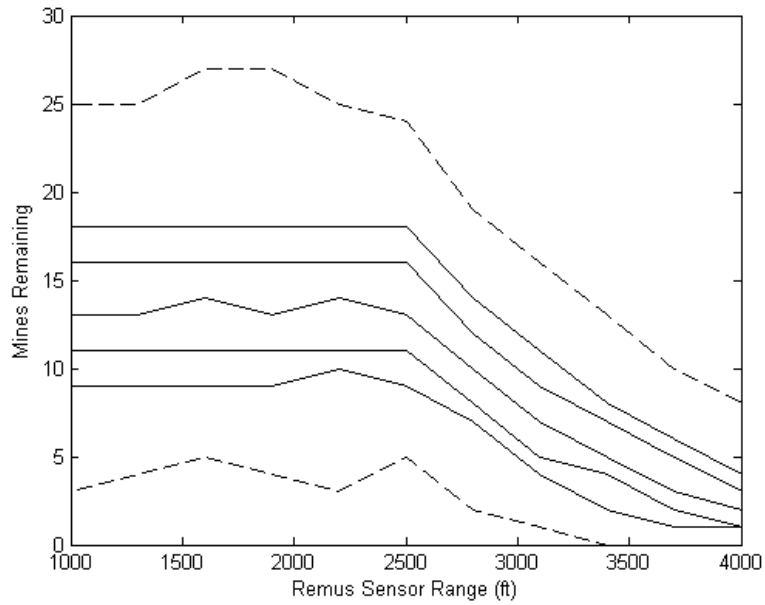
The quantile estimates for the time output is shown in Figure 79. Seven quantiles are tracked across the 11 Remus Sensor Range values. These quantiles are the same as were used in the single point analysis section, 0%, 10%, 25%, 50%, 75%, 90%, and 100%. The 0% and the 100% quantiles represent the observed minimum and maximum, respectively. These measures are plotted with dashed lines to indicate their difference for the other measures. The middle quantile estimates are plotted with solid lines. Again, the same observation that the Remus Sensor Range causes increases after 2500 ft can be made. Additionally, it is easily seen that there exist two peaks in the distribution. This is indicated by the closeness of the lines. The quantile measures of 10% and 25% remain close to each other across the cases. The same closeness is shown by the quantile measures of 50%, 75%, and 90%. Another observation can be made about the distributions. For a change in the Remus Sensor Range of 2500 ft to 2800 ft the five middle quantile measures become much closer together. This indicates that the distribution is more centrally located, losing its multi-modal shape. As the range is increased this centrality dissipates, which is indicated by the 90% quantile jumping in value at a range of 3400 ft. Finally, at a range of 4000 ft and new multi-modal distribution can clearly be seen.

The quantile estimates for the mines remaining output is shown in Figure 80. Again,

the same observation that the Remus Sensor Range causes increases after 2500 ft can be made. Additionally, it is easily seen that there exists a symmetric uni-modal distribution at the lower sensor ranges. This is indicated by the uniform distances between the quantile measures. After the sensor ranges increases past 2500 ft it can be seen that the distribution shifts lower and becomes more skewed. This is indicated by the 10% and 25% quantiles collapsing onto each other.



**Figure 79:** Mine Counter Measures: Quantiles of Time for 11 Levels



**Figure 80:** Mine Counter Measures: Quantiles of Mines Remaining for 11 Levels

#### 5.1.3.4 Conclusion

The sample mean and sample variance were shown to be insufficient for single point analysis, given that the output distribution is not known a priori. For output analysis over a range of a variable, the mean and variance was shown to be more useful. Some information was able to be derived, primarily the general trend. The variance estimate showed to be useful for only the mines remaining output. This was due to the uni-modal distribution of the mines remaining output. For the multi-modal distribution of the time output the variance performed poorly, as would be expected.

The binomial proportion estimates performed very well for the single point analysis. These measures were able to identify the multi-modal and uni-modal nature of the time and mines remaining outputs, respectively. The measures also acted as fairly good approximations of the distribution. Applied to multi-point analysis, the binomial proportions performed better than the mean and variance by identifying the general trends and the presence of multi-modal and uni-modal distributions.

Finally, the quantile estimates performed equally as well as the binomial proportion

for single point analysis. The major difference occurred during multi-point analysis. The quantile estimates were able to identify the general trends, the presence of a multi-modal and uni-modal distribution, and provide information on the shape of the distribution as it changed due to sensor range increasing. The binomial proportions would be able to make the same observations; however, more measures would be required. Each measure contained five useful measures. The quantile measure was found to provide more information than the binomial proportions. For this reason it is recommended that quantiles are used in the analysis of simulations with stochastic outputs.

**Research Question 5 Hypothesis:** Quantile estimates should be used for outputs of stochastic simulations when faced with an unknown or changing distribution.

## ***5.2 Investigation into the Required Replications for Accurate Confidence Interval Estimation***

The previous section addressed which stochastic measures should be used for the outputs of a stochastic simulation. During the analysis the number of replications required or available was not considered. It was assumed that the estimates were accurate. This was able to be done because a large number of replications were used giving considerable confidence in the estimate. Realistically, there is a limitation on the number of replications one can acquire from a simulation. This can be due to numerous reasons including time requirements on analysis combined with run time requirements of the simulation and required number of unique cases that must be investigated for the desired analysis. For example, if one has a weekend available to run the simulation (about 63 hrs) and each run requires an hour then only 63 runs can be accomplished. These 63 runs must be divided between unique cases and repetitions. Therefore, how many repetitions are required for the different measures? The following analysis and results to this question are based on a paper by the author published in late 2013 [197].

Research Question 6, presented at the beginning of this chapter, addresses the question of the number of replications needed for an accurate stochastic measure. Every measure will

contain some error, therefore it is common to calculate Confidence Intervals (CIs) about the estimate. CIs are used to define a range of possible values in which the true value of the estimate will lie at some confidence interval level, e.g. 95%. For example, if the 95% CI of an estimate is found, then the true value will lie within those bounds for 95% of the estimates. Not to be confused with the statement that the CI has a 95% probability of containing the true value. The CI either contains the true value or not, but 95% of the CIs calculated from the estimates will contain the true value.

There are two general methods for determining how many replications are required using CI. The first determines how many replications are required to achieve an absolute error tolerance of  $\beta$ . The error tolerance is the half-width of the CI. Law [100] presents a sequential procedure to solve for the number of repetitions, which is as follows:

1. Make  $n_0$  replications and set  $n = n_0$
2. Compute estimate and CI half-width of estimate
3. If the CI half-width is equal to or less than  $\beta$  then stop. Otherwise, increment  $n$  upwards by 1 and go to step 2.

This procedure is not recommended by Law due to its sensitivity of the selection of  $\beta$  on the coverage accuracy [97]. The coverage is defined as the portion of time the interval contains the true value. The same sequential procedure as listed above can be applied to relative errors. The relative error is defined as  $\lambda = |\bar{X} - \mu|/|\mu|$ , where the mean is the estimate and  $\mu$  is the true mean. The procedure is repeated until Equation 37 holds true. The same approach can be taken with other statistical measures. For the sample mean Law recommends an initial sample size of at least 10 and a relative error no greater than 0.15 [100].

$$\frac{t_{i-1, 1-\alpha/2} \sqrt{S^2(n)/n}}{|\bar{X}(n)|} \leq \frac{\lambda}{1-\lambda} \quad (37)$$

The procedure mentioned, along with numerous others in the literature, provide a good approach towards determining how many replications are required. This approach works well for numerous problems; however, when constructing a surrogate model, or meta-model,

this procedure does not apply. For surrogate models there does not exist an absolute or relative error requirement. The CI widths are not important. Instead the goal is to match the true underlying model. If the surrogate model perfectly captures the true statistical measure, then one does not care about the CI width of the sample data.

When faced with a heteroscedastic output, i.e. non-constant variance, there are two basic approaches for linearly regressing the equation coefficients to produce the surrogate model. The first is to use Ordinary Least Squares (OLS) while making sure that the confidence intervals are the same width. This can be accomplished by using the procedure suggested by Law for absolute error tolerances. The other approach is to use Weighted Least Squares (WLS) where the weightings take into account the different CI widths. An issue with the first method is that the modeler would not know how many total runs would be required before starting the runs. Given that the number of cases would need to be maximized in order to maximize the analysis, the modeler needs to know the number of replications needed before the execution of the runs. An issue with the second method is that there is no information available in the literature as to how many replications are required for accurate CIs without specifying an error tolerance. More specifically, the literature does not indicate the number of replications that are required for the CI estimate to hold true.

This question will be answered by studying the coverage accuracy of the CI estimates for sample mean, variance, binomial proportions, and quantiles. The coverage of the CI varies depending on the measure used and the distribution applied to it. For the sample mean and variance measure, the skewness, kurtosis, sample size, and desired coverage level will be varied using the Pearson Family of Distributions. The Pearson Family of Distributions was selected because it contains some of the most common distributions, e.g. Gaussian, Student's t, uniform, exponential, beta, gamma. The accuracy of the CI estimates for the binomial proportions and quantiles are tested by varying the sample size, coverage level, and quantile level using the Gaussian distribution. It was found that the distribution does not significantly impact the CI accuracy of binomial proportions and quantiles. Additionally, for each of the four measures the mean and variance of the Pearson Distributions were not varied because they were found not to impact the accuracy of the CI. Estimates on the

accuracy of the CIs are found by repeating every combination 20,000 times and calculating the coverage. An algorithm will be developed to run every case for the four different stochastic measures.

### 5.2.1 Required Replications for Accurate Sample Mean Confidence Intervals

The sample mean is an unbiased estimator of the true mean and is reproduced below in Equation 38. The equation for calculating the CI of the sample mean is shown in Equation 39. This equation is based on the Student's t distribution instead of the Gaussian because the Student's t performs better [100]. The sample mean CI is accurate for all distributions if n is sufficiently large. This is because the sample mean CI is based on the Gaussian Distribution and the sample mean follows a Gaussian Distribution which is a result of the Central Limit Theorem.

$$\bar{X}(n) = \frac{1}{n} \sum_{i=1}^n x_i \quad (38)$$

$$\bar{X}(n) \pm t_{n-1, 1-\alpha/2} \sqrt{\frac{S^2(n)}{n}} \quad (39)$$

The algorithm developed to study the CI coverage accuracy for the sample mean was developed in the mathematical program MATLAB. The Pearson function was used to create distributions from the Pearson Family of Distributions. The distributions were created by varying the skewness between -3 and 3 at 0.25 increments. The kurtosis was varied between 2 and 11 at 0.25 increments. The feasible Pearson Distributions were the ones with a kurtosis that is greater than the square of the skewness plus one. Each distribution contained two million data points. The samples were then taken from the distribution for sample sizes of 5 to 50 incrementing by 5. The desired CI coverage varied between 75% and 95% at increments of 10%. Each combination was then replicated 20,000 times.

The results from the sample mean confidence interval study are shown in Figure 81. There are a total of 15 contour plots. The first row shows contour plots of desired CIs of 95%. The second row show CIs of 85% and the third row shows CIs of 75%. Each column represents the sample size. The first column shows results for sample sizes of 5.



Each preceding column increases the sample size by 5 until the final column which has a sample size of 50. Each contour plot has the kurtosis on the ordinate and the skewness on the abscissa. The body of the plot contains contour regions of the error between the desired CI and the observed coverage. An example of the error is that if the desired CI level was 95% and the observed coverage was 85% then there would be an error of 0.1. The same error would be observed if the values were switched. The color scale ranges from 0 to 0.1 and is shown on the far right of the figure. Upon examination of Figure 39, it is seen that the estimate is very accurate. Distributions with low skewness perform the best. Skewness seems to be the primary driver for the accuracy of the sample mean CI accuracy. As the sample size is increased the error drops, which would be expected. Interestingly, as the desired CI level is lowered the error also decreases. The conclusion drawn is that the sample size should not fall below 10 for low skewness distributions. Increased sample sizes would be required for greater skewness; however, the sample size should not have to exceed 50. The error with extreme skewness and kurtosis with 50 samples and a CI level of 95% is only 0.05.

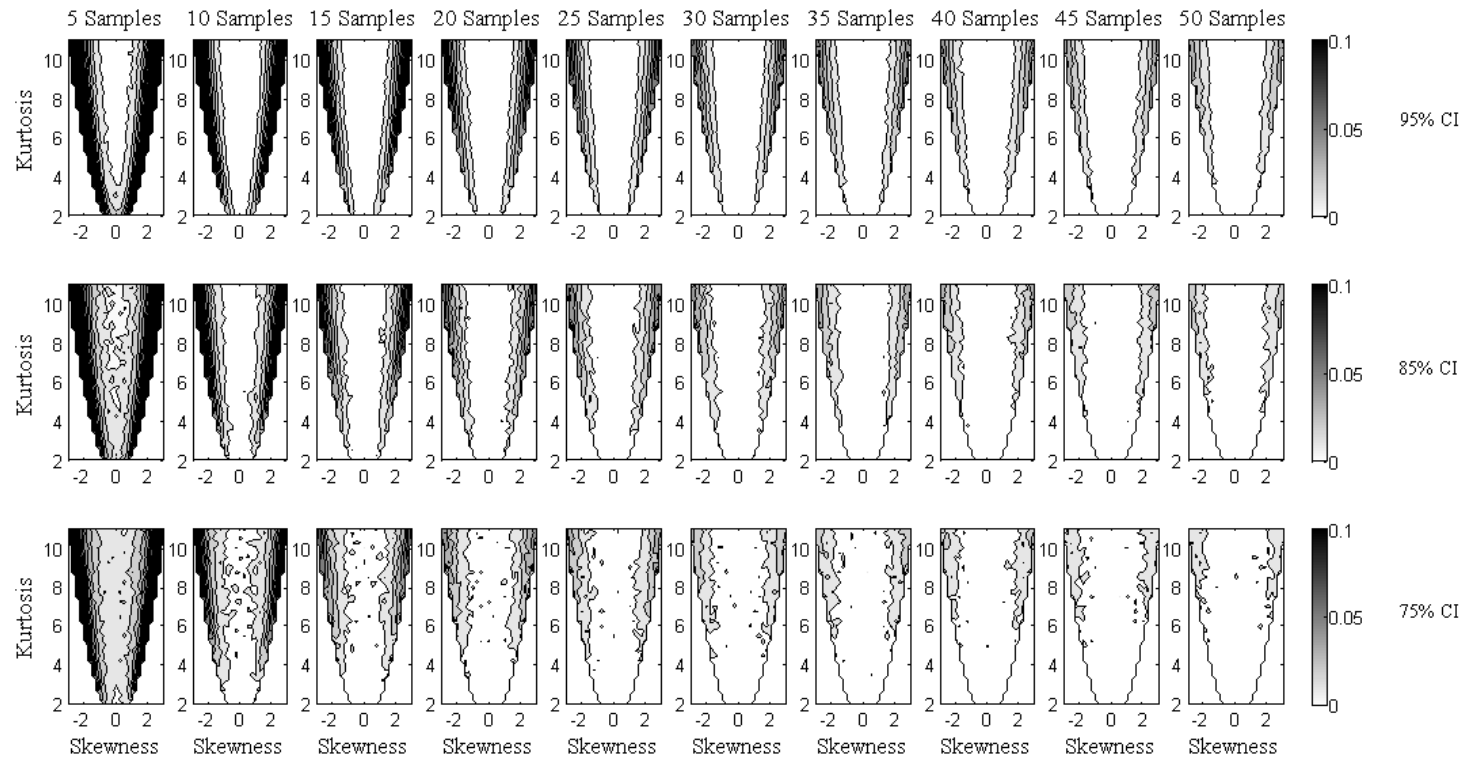


Figure 81: Coverage Error for Sample Mean Confidence Intervals

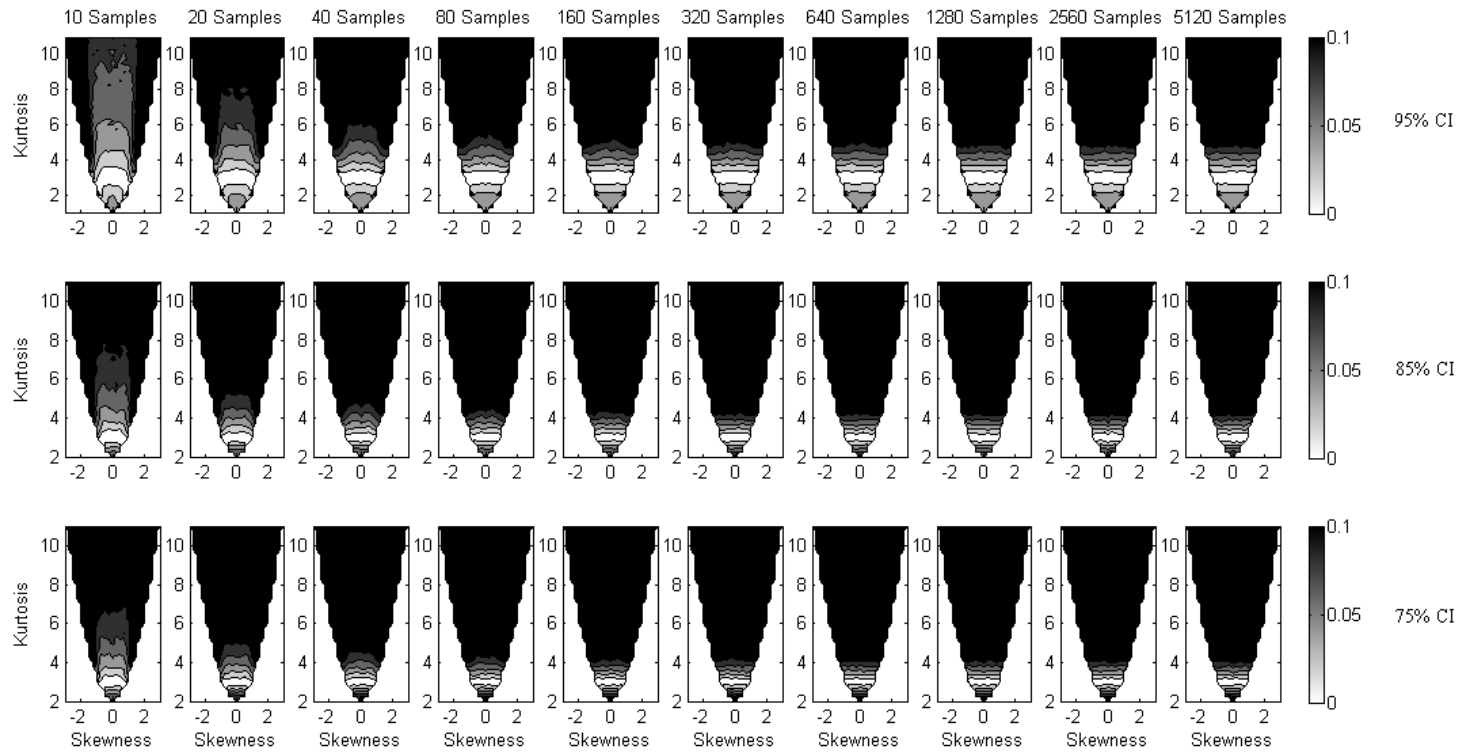
### 5.2.2 Required Replications for Accurate Sample Variance Confidence Intervals

The sample variance measure is an unbiased estimator for the variance. The equation for the sample variance is reproduced below in Equation 40. The CI for the sample variance is estimated with the use of the  $\chi^2$  Distribution and is shown below in Equation 41. The  $\chi^2$  Distribution can be shown to be the distribution of the sample variance from a Gaussian Distribution. From this it is known that the CI estimator for the sample variance assumes that the original data set is Gaussian distributed. Therefore, if the original distribution is not a Gaussian then the CI estimator will have some error no matter the sample size.

$$\hat{S}^2(n) = \frac{\sum_{i=1}^n [x_i - \bar{X}(n)]^2}{n-1} \quad (40)$$

$$\left[ \frac{(n-1)\hat{S}^2(n)}{\chi_{\alpha/2, n-1}^2}, \frac{(n-1)\hat{S}^2(n)}{\chi_{1-\alpha/2, n-1}^2} \right] \quad (41)$$

The results from the sample mean confidence interval study are shown in Figure 82. This figure is represented in the same format as the previous figure. The one difference between the two figures is that Figure 82 varies the sample size between 10 and 5120. The sample sizes were scaled as a geometric progression with a common ratio of 2. These samples sizes were used to show that the repetitions do not improve the accuracy of the CI estimate. What is observed from this result is that the sample variance CI estimator is a poor estimator for most Pearson Distributions. The further the distribution is from a Gaussian Distribution, indicated by a skewness of 0 and a kurtosis of 3, the more inaccurate the estimate. An interesting observation, however, is that the CI estimator is more sensitive to kurtosis than it is to skewness. Another trend observed is that as the CI desired level is decreased, the band of distributions that perform well with the CI estimator decreases in the kurtosis dimension. What is concluded is that the CI estimator, shown in Equation 41, performs well for distributions with a kurtosis of three within the Pearson Family of Distributions.

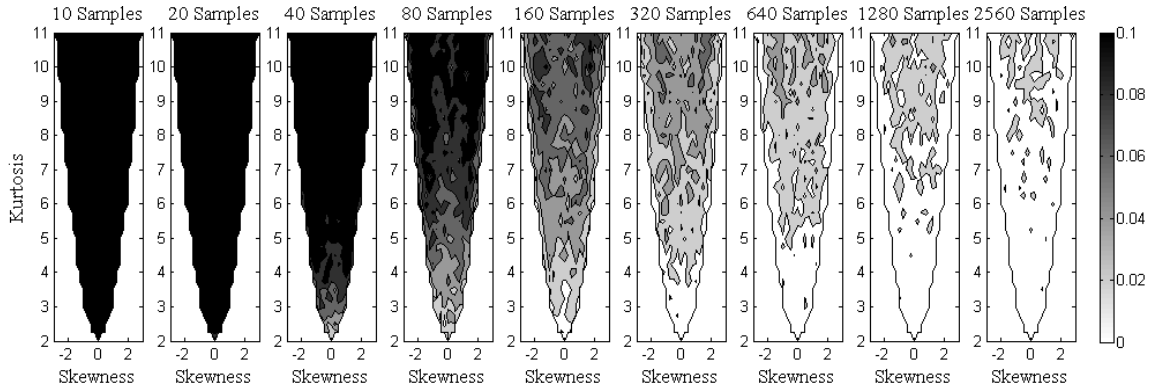


**Figure 82:** Coverage Error for Sample Variance Confidence Intervals

The CI for sample variance was found to be a poor estimator for most Pearson Distributions; therefore, another technique must be used for this estimate. A computational intensive, non-parametric technique called bootstrapping will be used to estimate the sample variance interval. The philosophy behind bootstrapping is that in the absence of a known distribution, the sample set itself acts as the best approximation of the distribution [111, 63]. There are four primary CIs for the bootstrap technique: Normal Approximation Method, Percentile Method, Bias Corrected Method, and Percentile t-Method [111]. The Bootstrap Percentile Method was selected for the sample variance CI estimation method. The Percentile Method estimates the CI of some metric,  $\theta$ , as follows:

1. Generate  $B$  bootstrap samples from the original sample set of size  $n$ . Each sample in the original set has a probability  $1/n$  of being in the bootstrap sample. Repeated samples are allowed in the bootstrap set.
2. Calculate  $\theta$  for each bootstrap sample set,  $\theta_b$ .
3. The  $(\alpha/2)B$  smallest sample estimates the lower interval and the  $(1-\alpha/2)B$  smallest sample estimates the upper interval.

The results from the sample mean confidence interval using the Bootstrap Percentile Method are shown in Figure 83. This figure is represented in the same format as the previous figure. Here only the CI level of 95% is shown. Additionally, the sample size of 5120 was removed due to computational limitations. The number of bootstrap samples was set to the sample size, i.e.  $B=n$ . The actual coverage was estimated by repeating each case 1000 times. This number was reduced from earlier studies due to the computational intensity of bootstrapping. The irregular pattern seen in the contour plots in Figure 83 are due to random fluctuations in the coverage estimate. It can be seen that as the number of samples increases, the coverage estimate becomes more reliable. Additionally, the larger the kurtosis of the distribution the larger the sample size needs to be. Despite the bootstrapping method offering a solution for estimating sample variance CI, the sample size required is still quite large. A sample size of at least 640 is suggested when using the Bootstrap Percentile Method for estimating sample variance CIs.



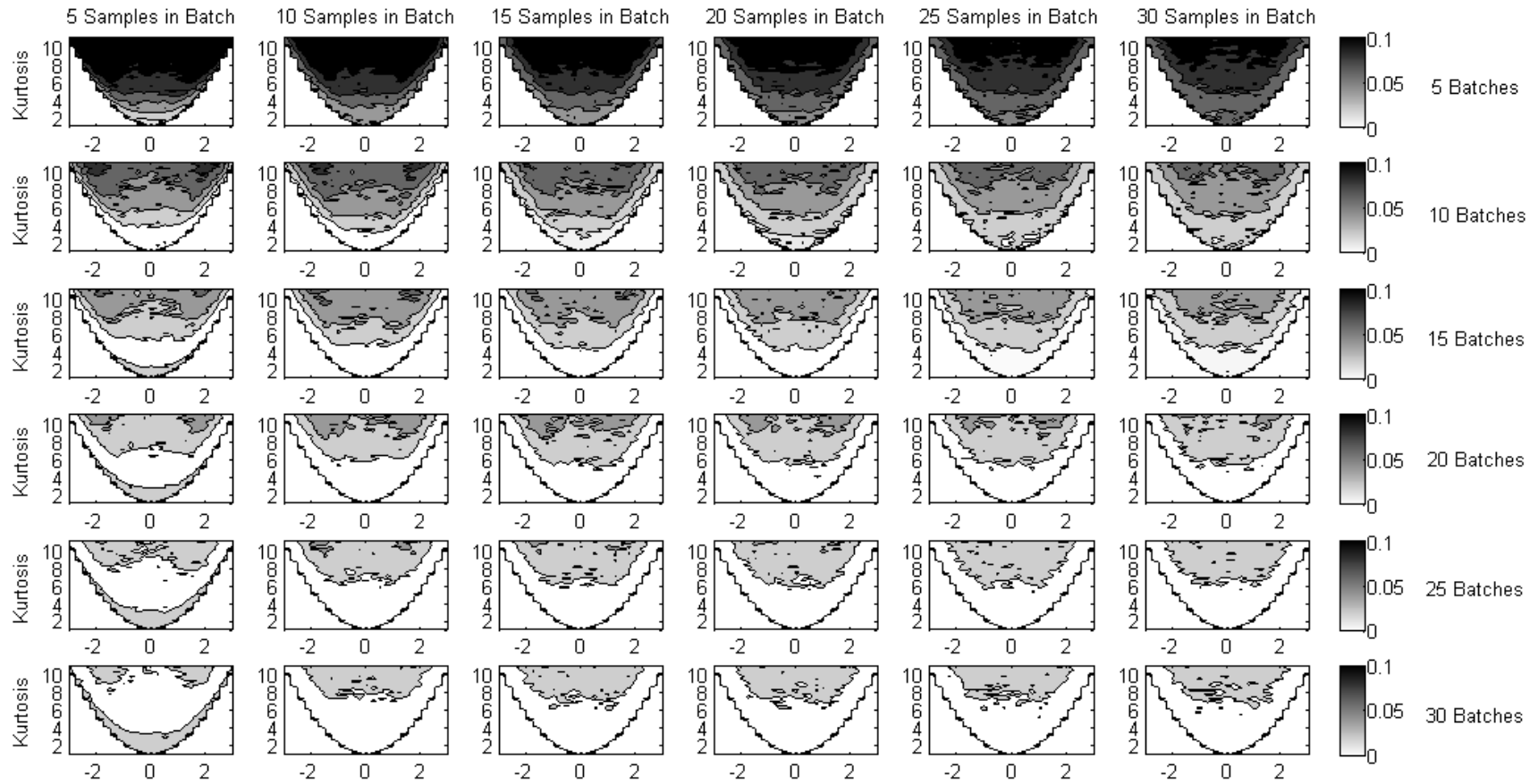
**Figure 83:** Coverage Error for Sample Variance 95% Confidence Interval Using Bootstrap Percentile Method

As shown the number of samples required for the Bootstrap Percentile Method is very large. With 640 samples the distribution is visible using a standard histogram. The number of samples required may be reduced by using Gaussian error bounds on the variance estimate. As the number of samples increases the distribution of the sample variance approaches a Gaussian Distribution. A given sample of size  $N$  can be discretized into  $B$  batches each having  $Bn$  samples. The sample variance is then calculated for each batch. The mean and variance of the batch variances are then used to create confidence intervals using Equation 39. An experiment was developed that varied the number of batches, the samples in each batch, the skewness, and the kurtosis. The results can be observed in Figure 84. The total number of samples used for each contour plot is shown in Equation 42.

$$Samples = \begin{bmatrix} 25 & 50 & 75 & 100 & 125 & 150 \\ 50 & 100 & 150 & 200 & 250 & 300 \\ 75 & 150 & 225 & 300 & 375 & 450 \\ 100 & 200 & 300 & 400 & 500 & 600 \\ 125 & 250 & 375 & 500 & 625 & 750 \\ 150 & 300 & 450 & 600 & 750 & 900 \end{bmatrix} \quad (42)$$

Observations from Figure 84 lead one to conclude that the number of batches contribute more to accurate confidence interval coverage than the number of samples in each batch, assuming the same number of total samples. If one compares the upper row with the left

most column it is easily seen that increasing the number of batches is preferred to increasing the number of samples in each batch. Finally, it is observed that the batch sample size of 5 results in some interesting error on the border of feasible distributions. It is recommended that 10 samples per batch are used and at least 25 batches. This results in a required sample size of 250 to accurately estimate the confidence interval of the sample variance. The sample size can be adjusted if the kurtosis of the distribution were known. Given this knowledge larger kurtosis values would require greater sample sizes and smaller kurtosis values would require smaller sample sizes.



**Figure 84:** Coverage Error for Sample Variance 95% Confidence Interval Using Batches for Normality



### 5.2.3 Required Replications for Accurate Binomial Proportion Confidence Intervals

The unbiased point estimator for a binomial proportion is reproduced below in Equation 43, where  $Z$  is the number of occurrences within the defined bounds, e.g.  $[10, \infty)$ . The most common CI for the binomial proportion is called the Wald Interval and is shown in Equation 44. In Equation 44  $z_{1-\alpha/2}$ , not to be confused with  $Z$ , refers to the critical points of a Gaussian Distribution, which can be found in data tables in the back of most text books on statistics.

Despite the Wald Interval being the most common CI estimator, the performance of the interval is erratic and often produces below-desired coverage [40]. Two better alternatives to the Wald Interval are the Wilson Interval [212] and the Agresti-Coull Interval [6] shown in Equations 45 and 46, respectively. Brown et al [40] provides an analysis of the interval estimations and suggests the use of the Wilson Interval for small sample sizes and the Agresti-Coull Interval for large sample sizes, e.g.  $n \geq 40$ . Given that the goal is to find the minimum number of repetitions required the Wilson Interval will be used for further analysis.

$$\hat{p} = \frac{Z}{n} \quad (43)$$

$$\hat{p} \pm z_{1-\alpha/2} \sqrt{\frac{\hat{p}(1-\hat{p})}{n}} \quad (44)$$

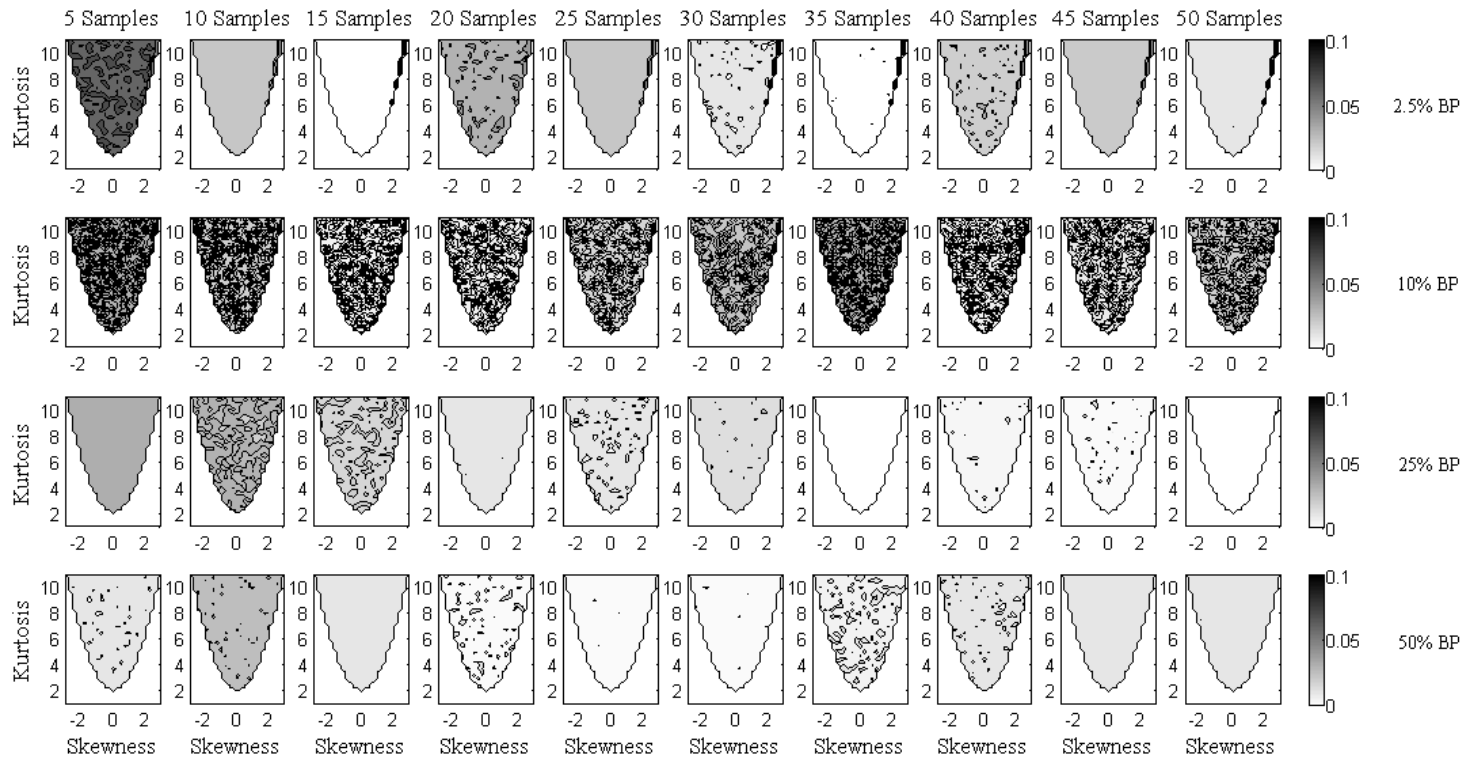
$$\frac{\hat{p} + \frac{1}{2n} z_{1-\alpha/2}^2 \pm z_{\alpha/2} \sqrt{\frac{\hat{p}(1-\hat{p})}{n} + \frac{z_{1-\alpha/2}^2}{4n^2}}}{1 + z_{1-\alpha/2}^2/n} \quad (45)$$

$$\frac{\hat{p} + z_{1-\alpha/2}^2/2}{n + z_{1-\alpha/2}^2} \pm z_{1-\alpha/2} \sqrt{\frac{\frac{\hat{p} + z_{1-\alpha/2}^2/2}{n + z_{1-\alpha/2}^2} \left( 1 - \frac{\hat{p} + z_{1-\alpha/2}^2/2}{n + z_{1-\alpha/2}^2} \right)}{n + z_{1-\alpha/2}^2}} \quad (46)$$

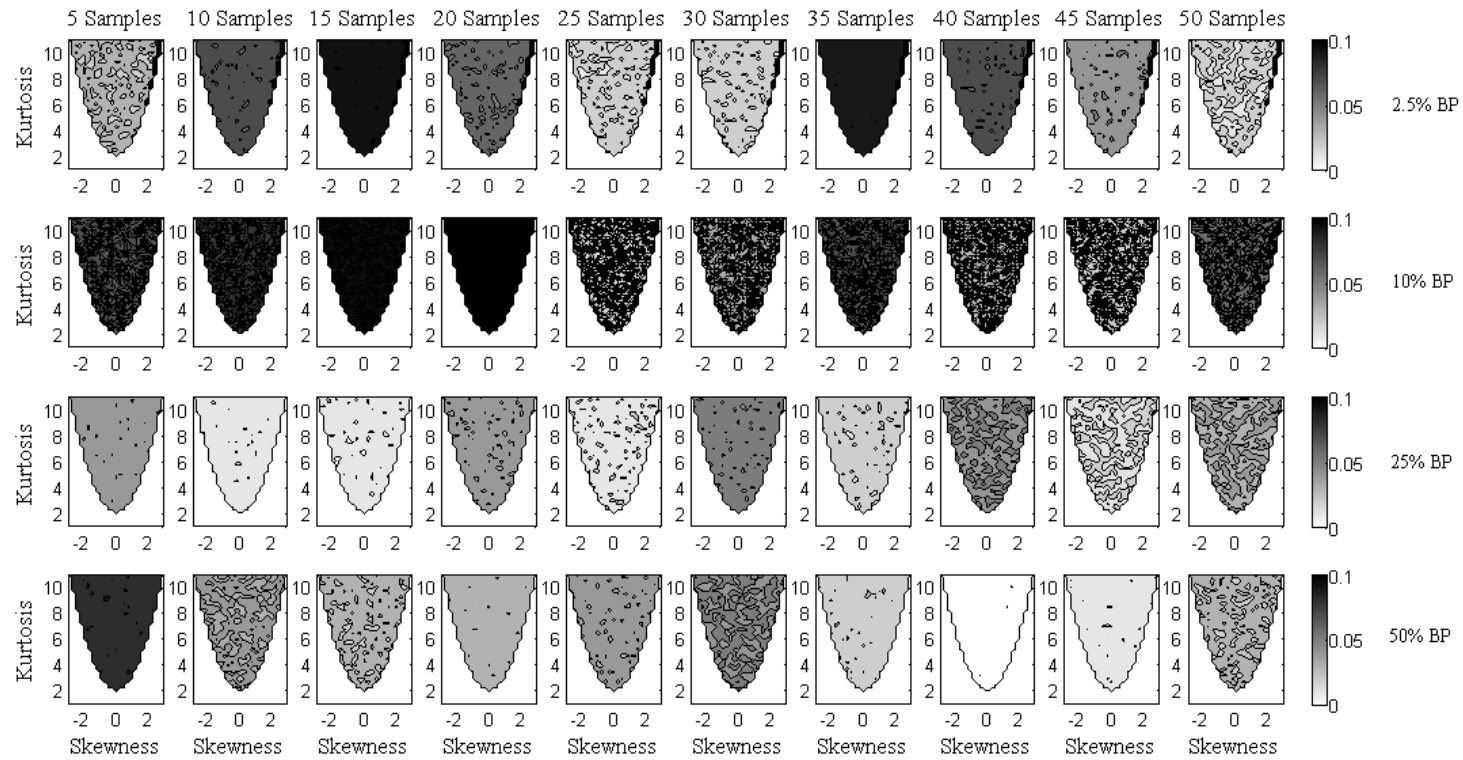
The analysis was performed in the same manner as the previous two sections. One difference in the binomial proportion analysis is that there exist numerous measures instead of a single measure. The binomial proportion measures used are the values of the distribution

that would result in 2.5%, 10%, 25%, 50%, 75%, 90%, and 97.5%. Given that there is an additional dimension to the analysis the results will be displayed in three figures. The results from the four binomial proportion confidence intervals at a 95% CI are shown for binomial proportions of 2.5%, 10%, 25%, and 50% in Figure 85. Only half of the values are displayed because the results are mostly symmetric about the 50% binomial proportion. The results from the four binomial proportion confidence intervals at a 85% CI are shown for binomial proportions of 2.5%, 10%, 25%, and 50% in Figure 86. The results from the four binomial proportion confidence intervals at a 75% CI are shown for binomial proportions of 2.5%, 10%, 25%, and 50% in Figure 87.

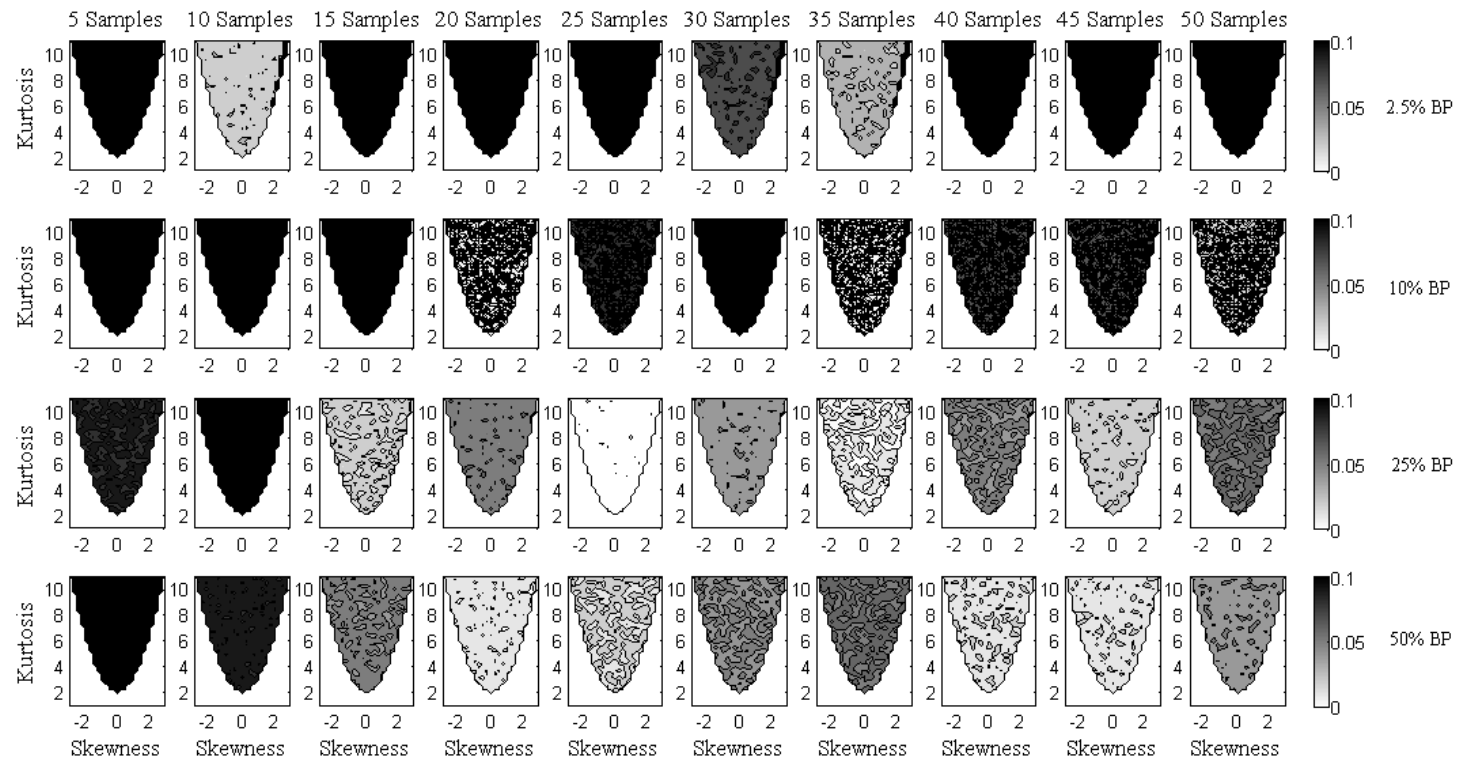
What is observed from these figures is that there is no discernible pattern. As the sample sizes are increased the coverage accuracy may increase or decrease. The same behavior is observed when the CI levels are changed. The skewness and kurtosis show no change at all in the coverage accuracy. This indicates that the Wilson Interval accuracy is not dependent on the distribution. The only indication contrary to this is seen for 2.5% binomial proportion at extreme kurtosis and positive skewness where the coverage drops significantly. The same observation is made for extreme kurtosis and positive skewness for the 97.5% binomial proportion, which is not shown. Finally, the only reliable pattern observed is increasing coverage accuracy as the binomial proportion approaches 50%. From these observations, no heuristic can be drawn for the number of replications required. Another set of analyses must be conducted.



**Figure 85:** Coverage Error for Binomial Proportions at 95% Confidence Interval using Wilson Intervals



**Figure 86:** Coverage Error for Binomial Proportions at 85% Confidence Interval using Wilson Intervals

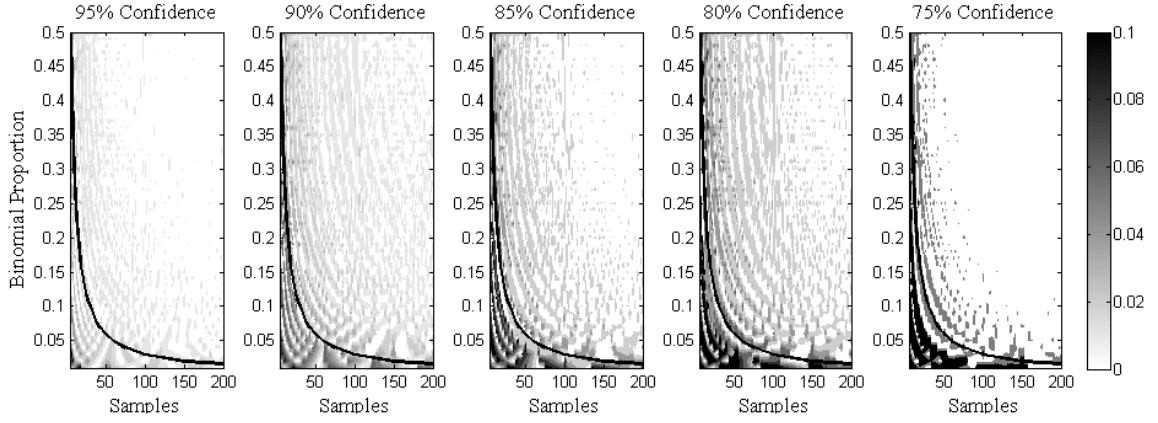


**Figure 87:** Coverage Error for Binomial Proportions at 75% Confidence Interval using Wilson Intervals

The previous analysis was unable to draw conclusions on the number of replications needed for accurate CI estimates for binomial proportions. A new series of experiments were run based on different variables. It was concluded from the previous analysis that the skewness and kurtosis are not significant contributors to the accuracy of the Wilson Interval; therefore, a Gaussian Distribution with a mean of zero and a variance of one was used for all cases. The CI level was varied between 75% and 95% by increments of 5%. The sample size was varied between 3 and 200 at increments of 1. The binomial proportion ranged from 0.01 to 0.50 at increments of 0.01. Each combination was repeated 20,000 times to calculate the expected coverage.

The results are shown in Figure 88. The layout of this figure is similar to the previous two; however, the axes have changed. Each column represents a different CI level with the leftmost column representing 95% CI and the rightmost column representing 75% CI. For each contour plot the sample size is on the abscissa and the binomial proportion value is on the ordinate. The body of the contour plot, like the previous figures, shows the contours of the error of the coverage.

The first observation is that the coverage accuracy of the Wilson Interval decreases as the CI level drops. For a given CI level, the error decreases with both increasing sample size and as the binomial proportion value approaches 0.50. Another observation is that the error follows an inverse relationship, i.e.  $n = 1/\hat{p}$ . Finally, when observing the body of the contours, the error has an erratic nature to it. Therefore, increasing the sample size or binomial proportion value does not always improve the coverage accuracy. It will only improve the accuracy on the macro scale. The conclusion that is drawn from this analysis is that the heuristic  $n \geq 3/\min(\hat{p}, 1 - \hat{p})$  is sufficient for the number of replications needed for a given binomial proportion. The heuristic is plotted as a line in each contour in Figure 88. This value may need to be increased with lower levels of CI.



**Figure 88:** Coverage Error of Confidence Intervals of Binomial Proportions using Wilson Intervals and  $n \geq 3/\min(p, 1 - p)$

#### 5.2.4 Required Replications for Accurate Quantile Confidence Intervals

The equation of the quantile estimate is reproduced in Equation 47, where  $q$  is the defined proportion, and  $X$  represents the order statistic of the samples. The equation that approximates the CI for the quantiles is shown in Equation 48 where  $X_r$  represents the  $r^{th}$  smallest value and  $X_s$  represents the  $s^{th}$  smallest value.  $X_r$  and  $X_s$  represent the lower and upper bound, respectively, of the interval. For example, if the sample size were 100,  $n = 100$ , the 50% quantile is being measured,  $q = 0.50$ , and the confidence interval level is 95%,  $z_{1-\alpha/2} = 1.96$ , then  $r = \left\lceil 100(0.5) - 1.96\sqrt{100(0.5)(1 - 0.5)} \right\rceil$  and  $s = \left\lceil 100(0.5) + 1.96\sqrt{100(0.5)(1 - 0.5)} \right\rceil$ . The lower and upper bound of the interval will be the 41<sup>st</sup> smallest value and the 60<sup>th</sup> smallest value, respectively.

$$\hat{x}_q = X_{\lceil nq \rceil} \quad (47)$$

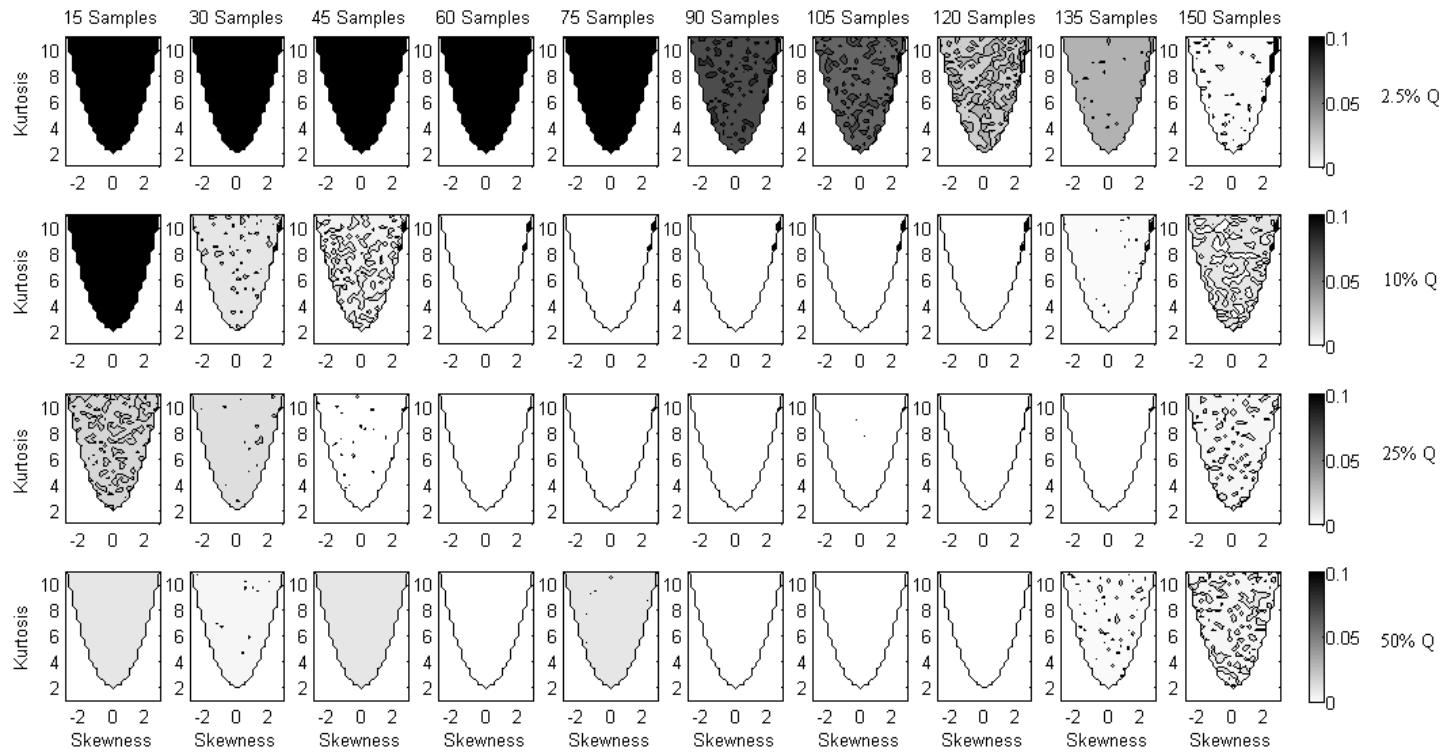
$$\begin{aligned} P(X_r \leq \hat{x}_q \leq X_s) &\geq 1 - \alpha \\ r &= \left\lceil nq - z_{1-\alpha/2} \sqrt{nq(1 - q)} \right\rceil \\ s &= \left\lceil nq + z_{1-\alpha/2} \sqrt{nq(1 - q)} \right\rceil \end{aligned} \quad (48)$$

The analysis was performed in the same manner as the sample mean and sample variance CI analysis. The range for sample size was changed to range from 15 to 150 at increments of 15. Similar to the binomial proportion, there are numerous measures possible for the

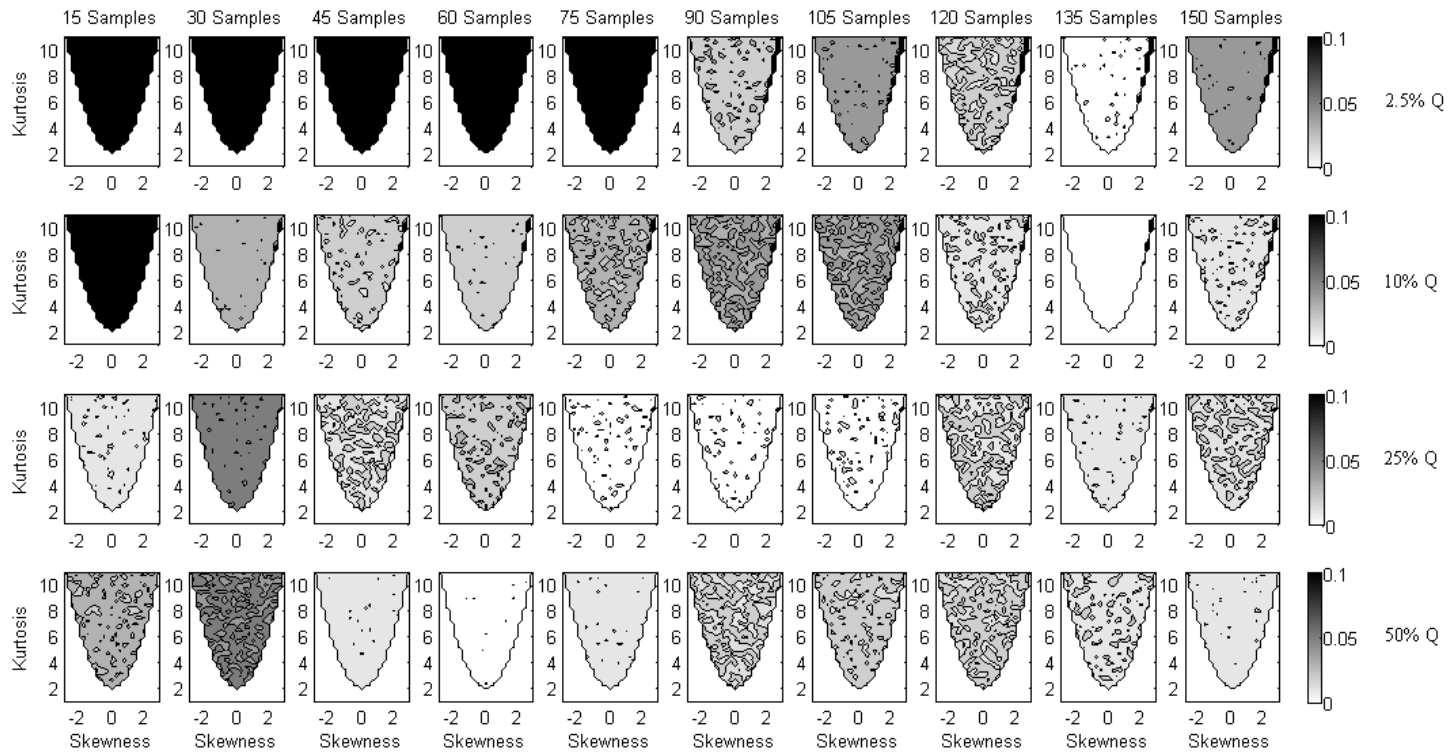
quantile analysis. The quantile measures used are 2.5%, 10%, 25%, 50%, 75%, 90%, and 97.5%. Given that there is an additional dimension to the analysis the results will be displayed in three figures. The results for quantiles of 2.5%, 10%, 25%, and 50% at the 95% CI are shown in Figure 89. Only half of the values are displayed because the results are mostly symmetric about the 50% quantile. The results for quantiles of 2.5%, 10%, 25%, and 50% at the 85% CI are shown in Figure 90. The results for quantiles of 2.5%, 10%, 25%, and 50% at the 75% CI are shown in Figure 91.

The first observation made is that the quantile CI estimator performs much better than the binomial proportion CI estimator. An observation made from the three figures is that as the sample size increases the coverage accuracy improves. Another observation made is as the quantile measured approaches 50%, the coverage accuracy improves. There is some evidence that the coverage accuracy fluctuates with sample size and quantile level, but this is not to the degree that was observed with binomial proportions. Finally, as is with the binomial proportion, the coverage accuracy does not appear to be strongly influenced by the distribution. Despite performing better than binomial proportions no heuristic can be drawn for the number of replications required from these observations. Another set of analyses must be conducted.





**Figure 89:** Coverage Error for Quantiles at 95% Confidence Interval



**Figure 90:** Coverage Error for Quantiles at 85% Confidence Interval

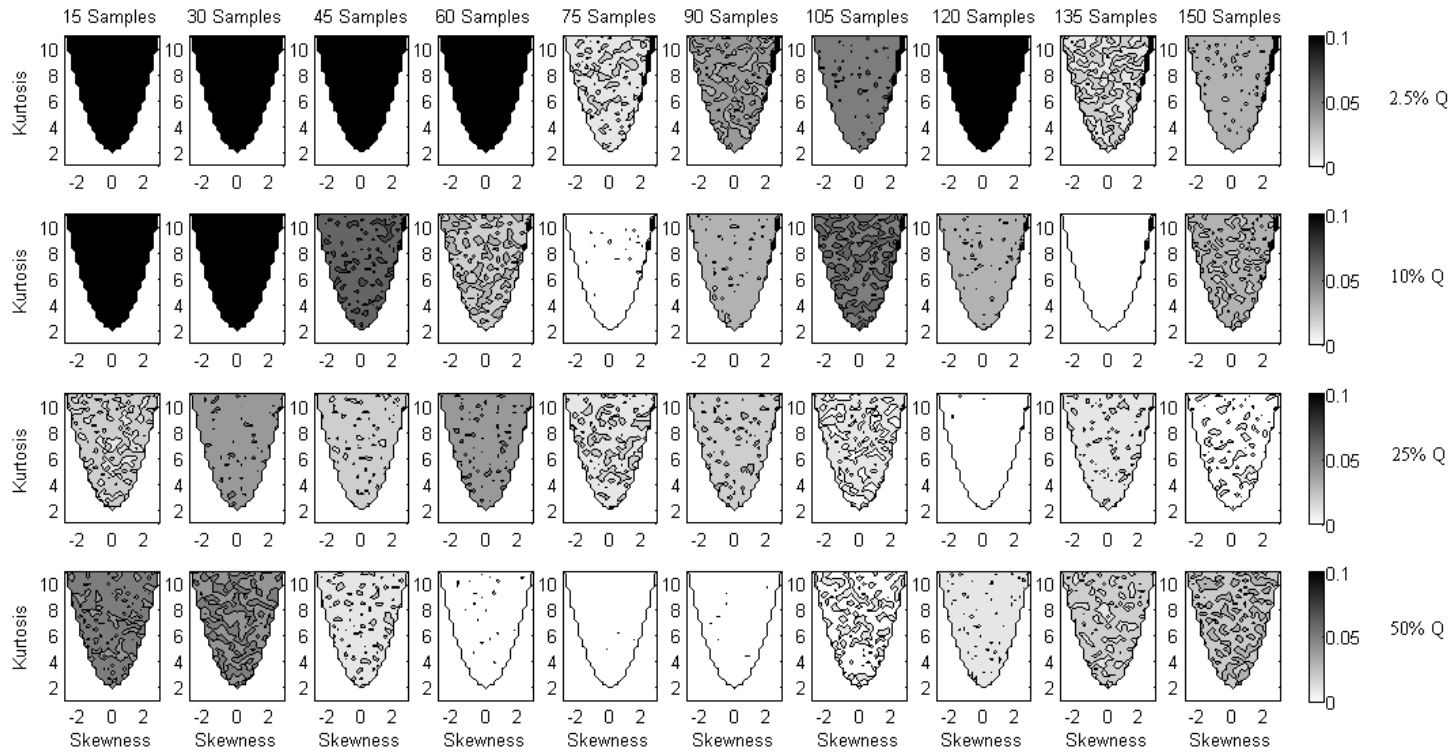
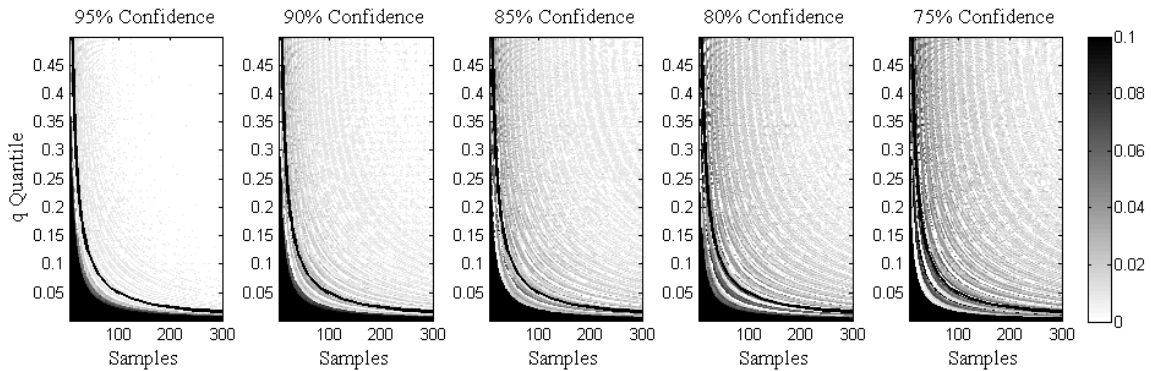


Figure 91: Coverage Error for Quantiles at 75% Confidence Interval

The previous analysis was unable to draw conclusions on the number of replications needed for accurate CI estimates for quantiles. A new series of experiments were run based on different variables. It was concluded from the previous analysis that the skewness and kurtosis are not significant contributors to the accuracy of the interval; therefore, a Gaussian Distribution with a mean of zero and a variance of one was used for all cases. The CI level was varied between 75% and 95% by increments of 5%. The sample size was varied between 3 and 300 at increments of 1. The binomial proportion ranged from 0.01 to 0.50 at increments of 0.01. Each combination was repeated 20,000 times to calculate the expected coverage.

The results are shown in Figure 92. The layout of this figure is the same as Figure 88, which was used to find the heuristic for binomial proportions. The first observation made is that the coverage accuracy decreases as the CI level decreases. It is observed that the error follows an inverse relationship, i.e.  $n = 1/\hat{q}$ . Following this inverse relationship there appears to be ‘waves’ of increased and decreased error. Finally, a conclusion is drawn on a heuristic for quantiles. This heuristic is  $n \geq 5/\min(\hat{p}, 1 - \hat{p})$  and is plotted on each contour graph in Figure 92.



**Figure 92:** Coverage Error of Confidence Intervals of Quantiles and  $n \geq 5/\min(p, 1 - p)$

### 5.2.5 Conclusion

Note that these heuristics represent the minimum required sample sizes for accurate CI estimates. More repetitions may be required based on the needs of the analysis. The analysis on the coverage accuracy of the sample mean CI showed that the estimate was

very accurate. A sample size of at least 10 is suggested for the heuristic of the sample mean CI. For distributions with a skewness of above one, larger sample sizes are required. The kurtosis of the distribution was not seen to be a significant factor in the required sample size.

The CI estimator for sample variance was found to be a very poor measure. Interestingly, the skewness was not found to be a significant factor in the coverage accuracy of the CI estimate. Kurtosis was found to be critical. For distributions that do not have a kurtosis of three this measure should not be used. The Bootstrap Percentile Method was found to accurately estimate the bounds at sample sizes of 640 and larger. For kurtosis above five more repetitions would be required. In general sample variance was not found to have a reliable confidence interval estimate for reasonable sample sizes.

The Wilson Interval was selected to estimate the binomial proportion confidence interval. It was found that the skewness and kurtosis were not significant contributors to the accuracy of the interval. A Gaussian Distribution with a mean of zero and a variance of one was used for the analysis. A heuristic for the required sample size was found to be  $n \geq 3 / \min(\hat{p}, 1 - \hat{p})$ . The CI level was found to have an impact on the accuracy of the interval; therefore, it is suggested that for CIs of 75% or lower that larger sample sizes are used.

The quantile confidence interval estimator was found to perform much better than the binomial proportion CI estimators. Similarly, it was found that the skewness and kurtosis were not significant contributors to the accuracy of the interval. A Gaussian Distribution with a mean of zero and a variance of one was used for the analysis. A heuristic for the required sample size was found to be  $n \geq 5 / \min(\hat{p}, 1 - \hat{p})$ . The CI level was found to have an impact on the accuracy of the interval; therefore, it is suggested that for CIs of 75% or lower that larger sample sizes are used. As a result of this analysis heuristics can be presented to answer Research Question 6 shown below.

**Research Question 6 Hypothesis:** Sample mean confidence interval estimation requires at least a sample size of ten. For distributions with a skewness above one use a sample size of 50 or more.

Sample variance confidence interval estimation requires the use of batching. A sample size of at least 10 is required and 25 batches are suggested.

Binomial proportion confidence interval estimation requires that at least  $3/\min(\hat{p}, 1-\hat{p})$  samples are used. For a CI of 75% or lower use a larger sample size.

Quantile confidence interval estimation requires that at least  $5/\min(\hat{q}, 1-\hat{q})$  samples are used. For a CI of 75% or lower use a larger sample size.

### 5.3 *Heuristics on Regressions of Stochastic Outputs*

The heuristics presented in the previous section addressed the questions of “how many replications are required for confidence interval estimates to hold true?” This does not help if one were attempting to create a surrogate model of the simulation output data. Therefore, the next logical question is “which method should be used when creating surrogate models of stochastic measures?”, which is Research Question 7. Before answering this question, three explanations need to be given. The first is to define a surrogate model. The second is to define what makes a good surrogate model. The third, and final, explanation is on how stochastic data presents challenges to surrogate modeling.

**Research Question 7** Which method should be used when creating surrogate models of stochastic measures?

A surrogate model is a mathematical representation of a data set that relates a set of input variables to a set of output variables. There are several different types of surrogate models that can be created. The focus here will be placed on response surfaces. Typically, data is gathered by executing a series of experiments on either a computer simulation or a physical test using a Design of Experiments (DoE). A DoE is a series of experiments that have been designed in such a way as to maximize the information that can be gathered for a given set of test cases. Once the data is gathered, a polynomial equation is created that approximates the computational or physical environment. This equation commonly takes the form of the  $2^{nd}$  Order Response Surface Equation shown in Equation 49 [113], where  $x$  represents the input variable values and  $b$  represents the regressors. The regressors are

then found using least squares method.

$$R = b_0 + \sum_{i=1}^k b_i x_i + \sum_{i=1}^k b_{ii} x_i^2 + \sum_{i=1}^{k-1} \sum_{j=i+1}^k b_{ij} x_i x_j + \epsilon \quad (49)$$

A good surrogate model is one that captures the behavior of the true model with sufficient accuracy. This is tested by determining the Goodness of Fit. The Goodness of Fit contains five tests [54, 51], which are enumerated below. The Coefficient of Determination, denoted  $R^2$ , is an indicator of how well a set of data points fit a line or curve. This measure will be used extensively in this section. It is calculated by Equation 50.  $SS_{res}$  represents the Residual Sum of Squares and is shown in Equation 51.  $SS_{tot}$  represents the Total Sum of Squares and is shown in Equation 52. The remaining steps for assessing the Goodness of Fit are not needed for the analysis in this section; therefore, the details of the steps will not be expanded upon.

1. Determine if  $R^2$  is sufficient
2. Investigate actual by predicted plot for patterns
3. Investigate residual by predicted plot for patterns
4. Investigate variance and mean of model fit error
5. Investigate variance and mean of model representation error

$$R^2 = 1 - \frac{SS_{res}}{SS_{tot}} = 1 - \frac{\sum_i (y_i - f_i)^2}{\sum_i (y_i - \bar{y})^2} \quad (50)$$

$$SS_{res} = \sum_i (y_i - f_i)^2 \quad (51)$$

$$SS_{tot} = \sum_i (y_i - \bar{y})^2 \quad (52)$$

Stochastic data presents challenges for creating surrogate models. If a surrogate model was developed that perfectly captured the true underlying measure, e.g. mean, then the

Goodness of Fit tests would indicate some error. This may lead the analyst to believe that the surrogate model is a worst representation of the data than it truly is. This error arises as a result of the stochasticity of the output and the fact that every measure, e.g. sample mean, contains some error.

In addition to the possibility of obtaining misleading results from the Goodness of Fit test, a common mistake for the use of least squares regressions is to use Ordinary Least Squares (OLS) with a constant number of replications for each case. The problem with this approach is that it assumes that the output data is homoscedastic, i.e. constant variance for each case. As was shown in the previous sections this may not be the case. In fact, it would be expected that most stochastic simulation outputs with different inputs would be heteroscedastic; heteroscedasticity was observed in the output of the MCM model in Figures 73 and 74. The issue with using a constant number of replications for each case for OLS is that cases with high error are given equal weighting to cases with low error. This can result in poor model representation because the model will be regressed to the error from the high error cases. Heteroscedastic simulation outputs can be better regressed by maintaining a constant CI width or by using Weighted Least Squares (WLS).

Analysis will be performed to compare the performance of using OLS with constant sample sizes for each case ( $OLS_{CS}$ ), using OLS with constant CI widths ( $OLS_{CI}$ ), and using WLS. The weighting commonly used for WLS for regressing the sample mean is the inverse of the standard deviation [180]. This weighting will not work well for non-sample mean regressions, e.g. quantiles, therefore the 95% CI widths will be used instead. Since the CI width is a linear mapping of the standard deviation of the sample mean it is a natural alternative. Additionally, CI widths are available for all stochastic measures. For these reasons the CI width will be used as the weighting for the WLS regressions.

A canonical example will be used to help determine how the regressors should be regressed for a heteroscedastic data set. Once the manner in which stochastic data should be regressed is decided, e.g.  $OLS_{CS}$ ,  $OLS_{CI}$ ,  $WLS$ , a heuristic will be presented on the number of replications required to create a surrogate model of stochastic simulation data.

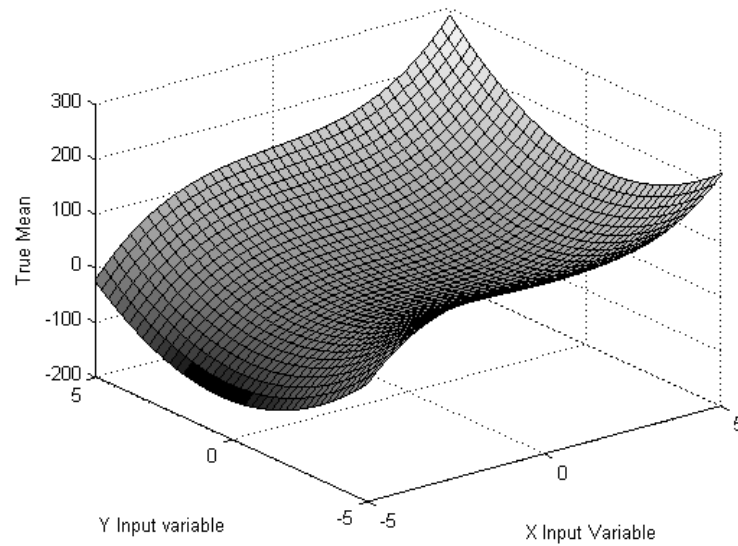


### 5.3.1 Experimental Set Up and Results of a Canonical Example

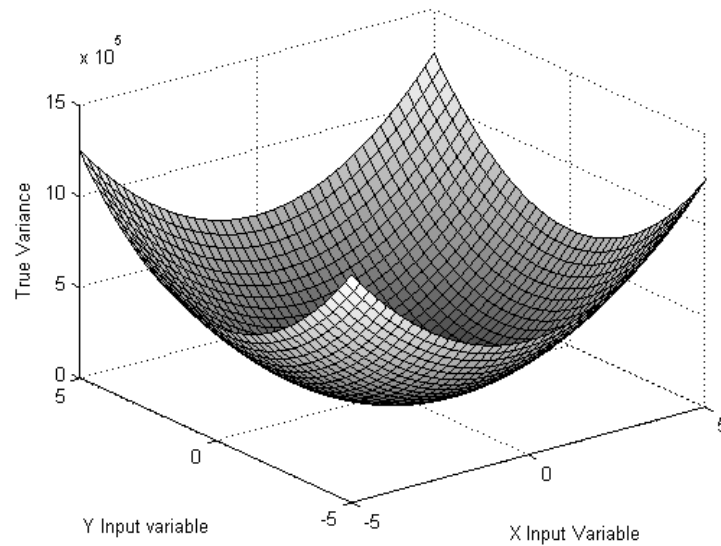
A canonical problem was developed that contained one output with two inputs: X and Y. The output follows a Gaussian Distribution. The mean of the output is defined by Equation 53 and is graphed in Figure 93. The variance of the output is defined by Equation 54 and is shown in Figure 94. The canonical problem was given a large variance to emphasize the impact of sample size on the performance of the regression methods. The problem is bounded on the x-axis and y-axis between -5 and 5. The input space is sampled with a four-level full factorial design giving a total of 16 cases. The mean, binomial proportions, and quantiles will be regressed using the three least squares approaches. The variance measure is excluded because it was found in the previous two sections that variance is a poor measure for stochastic output data. Surrogate models were created for the stochastic measures for sample sizes ranging from 25 to 500 at intervals of 25. The  $OLS_{CI}$  case redistributes the samples to maintain a constant width while keeping the total number of runs constant. The true variance was used to calculate the required sample size. The  $R^2$  between the sample values and the true values will be calculated. This measure will determine the amount of variability that the surrogate model captures of the true model. This process is repeated 1,000 times to determine the expected  $R^2$ .

$$M = X + Y + 5Y^2 + X^3 + XY \quad (53)$$

$$\sigma^2 = 25000(X^2 + Y^2) \quad (54)$$



**Figure 93:** True Mean of the Canonical Example

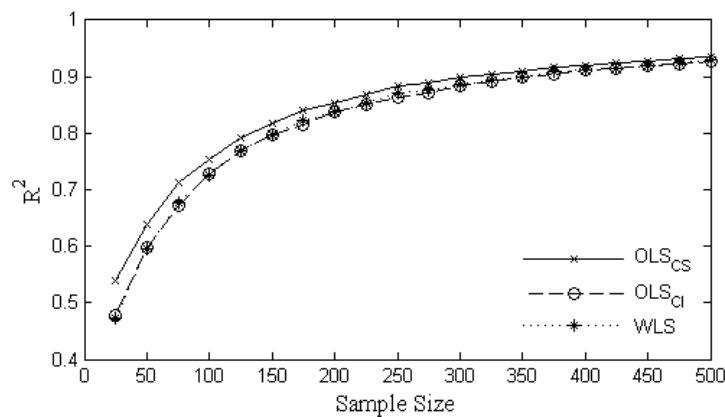


**Figure 94:** True Variance of the Canonical Example

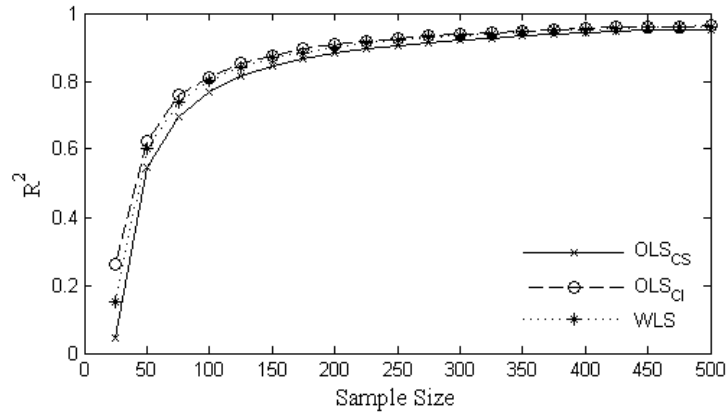
#### 5.3.1.1 Least Square Regressions of the Mean

All regression methods were regressed using the known model shown in Equations 53. An exact regression using any of the least squares regression methods would result in the following coefficients: 1,1,5,1,1. The results of the three regression methods are shown in

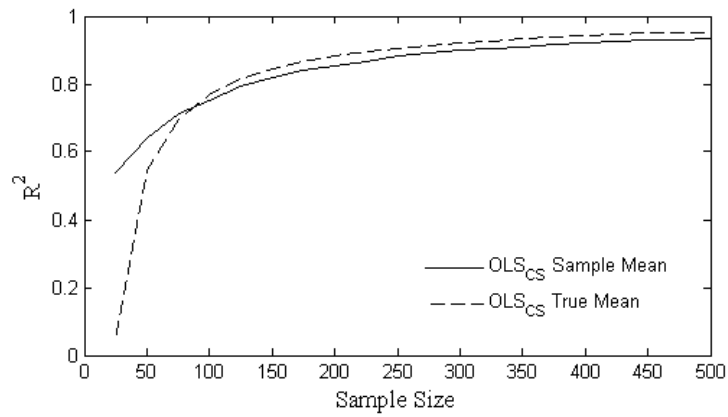
Figure 95 through Figure 98. Figure 95 compares the three methods using the Coefficient of Determination between the regressed model and the sample mean. It is observed that  $OLS_{CS}$  outperforms the other two methods,  $OLS_{CI}$  and  $WLS$ , whom perform at the same level. Additionally, it is observed that the  $R^2$  improves asymptotically as the sample size increases. This result is enlightening when compared to the result shown in Figure 96. Figure 96 shows the same three methods compared to the true mean as opposed to the sample mean. It is observed that  $OLS_{CS}$  performs the worst when compared to the true mean. This observation reasserts the claim made earlier that the use of  $OLS_{CS}$  for heteroscedastic data can result in misleading results. This is further emphasized in Figure 97 and Figure 98. Figure 97 compares the Coefficient of Determination of the  $OLS_{CS}$  method for both the sample and true mean. It is observed that performance with respect to the sample mean initially shows an inflated quality of performance. It is not until the sample size exceeds 100 that the regressed model performs better for the true mean than what would be reported for the sample mean. Note that the sample mean comparison would be the only measure available to test the quality of the fit. Figure 98 makes the same comparison for the  $OLS_{CI}$  and  $WLS$  methods. The same observation is made here as in Figure 97, though for a smaller sample size. The final observation made is that the  $OLS_{CI}$  method slightly outperforms the  $WLS$  method. This is best shown by Figure 96



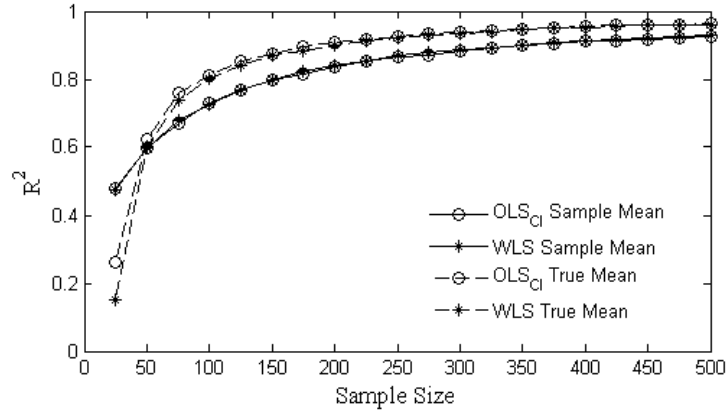
**Figure 95:** Variability Captured of the Sample Mean for Three Least Squares Methods



**Figure 96:** Variability Captured of the True Mean for Three Least Squares Methods



**Figure 97:** Variability Captured Comparison of the Sample and True Mean for  $OLS_{CS}$



**Figure 98:** Variability Captured Comparison of the Sample and True Mean for  $OLS_{CS}$  and  $WLS$

### 5.3.1.2 Least Square Regressions of the Binomial Proportions

The binomial proportion measurements will be made at the value that occurs for the true mean for an X and Y value of zero, which is zero. The binomial proportion will report the portion of observations that occurs greater than or equal to zero. The variance equation was reduced by two orders of magnitude for this study so that greater variability occurs for the binomial proportion values.

Resizing the sample sizes for each DOE case for the  $OLS_{CI}$  method presents some difficulty. To resize the sample sizes to maintain a constant CI width the following observations are made. The confidence interval width for the binomial proportions are based on the sample proportion value measured, e.g. 0.25, and the sample size. The interval width is at its minimum when the value is zero or one. For this case the Wilson Interval width is  $z_{1-\alpha/2}^2/(n + z_{1-\alpha/2}^2)$ . The interval width is at its maximum when the value is 0.5. For this case the Wilson Interval width is  $z_{1-\alpha/2}\sqrt{1 + z_{1-\alpha/2}/n^2}/(1 + z_{1-\alpha/2}/n)$ . Given the true mean, true variance, and the distribution the true binomial proportion for each of the 16 DOE cases is known. These values will then be taken to manually resize the samples such that the CI widths are constant and the total number of runs are maintained across the three least squares methods. The results for sample size 25 is shown in Table 44. This ratio of sample sizes are maintained as the sample sizes are increased.

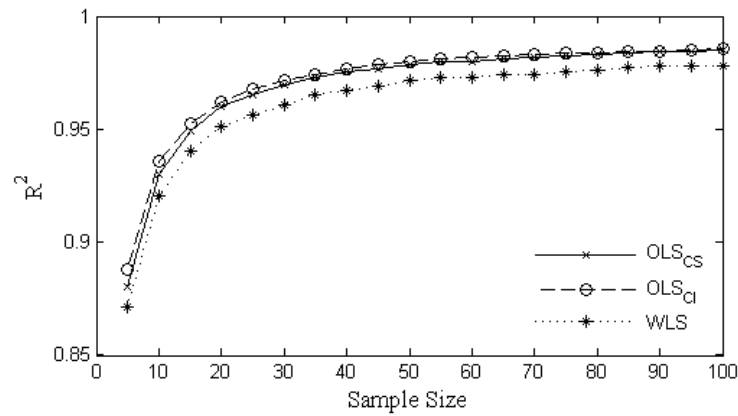
**Table 44:** Binomial Proportion Sample Sizes for 25 Repetition Case

$\hat{p}$	n	CI Width
0.55	45	0.28
0.93	16	0.28
0.92	17	0.28
0.98	11	0.29
0.09	19	0.28
0.59	45	0.28
0.66	41	0.28
0.95	14	0.28
0.07	16	0.28
0.57	45	0.28
0.75	35	0.28
0.97	12	0.29
0.41	44	0.28
0.92	17	0.28
0.96	13	0.28
0.99	10	0.29

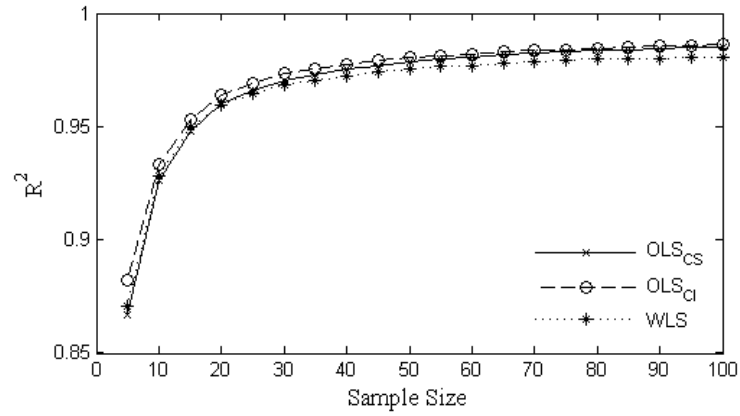
The model form that will be regressed was found using Stepwise Regression with the minimum BIC stopping rule in JMP. The data used for finding the model form is from the true values from the canonical problem. The best equation found that matched that data is as follows  $0.650 + 0.038X - 0.007X^2 + 0.002X^3 + 0.003Y + 0.011Y^2 + 0.002XY - 0.002XYY$ , where the coefficients were rounded to the third decimal place. The  $R^2$  of the model was found to be 0.99. This model form was then used for the three least squares regression methods.

The results can be seen in Figure 99 through Figure 102. The first observation made is that the fit was much better for the binomial proportions than for the sample means.

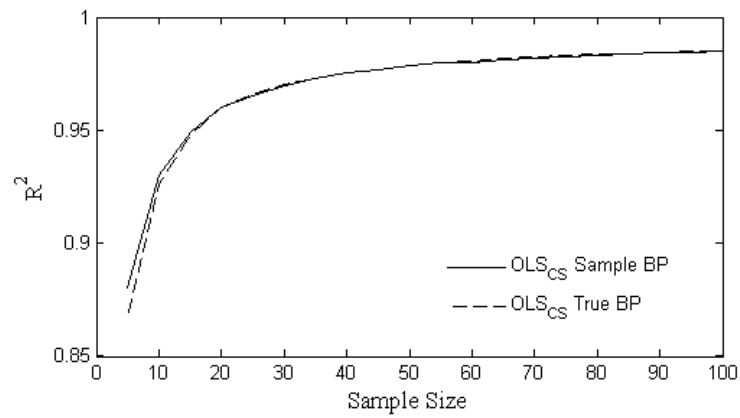
Note that the sample sizes have been scaled down by a factor of five. It is expected that the fit would degrade if there existed numerous values of one and zero. This would be due to the limitation of polynomials to fit bounded curves. The  $R^2$  measure with respect to the sample measure, shown in Figure 99, has a surprising result. The  $OLS_{CS}$  and  $OLS_{CI}$  perform relatively the same and the  $WLS$  performs the worst. This is not changed when the  $R^2$  is taken with respect to the true value, shown in Figure 100.  $OLS_{CI}$  is observed to perform the best. The  $R^2$  measure for the sample and true measure for  $OLS_{CS}$  is shown in Figure 101. Interestingly, the  $R^2$  measure is relatively the same for both measures. The same result is observed for the  $OLS_{CI}$  method in Figure 102; however, the true measure of the  $WLS$  method reported higher coefficients of determinations.



**Figure 99:** Variability Captured of the Binomial Proportion Estimates for Three Least Squares Methods

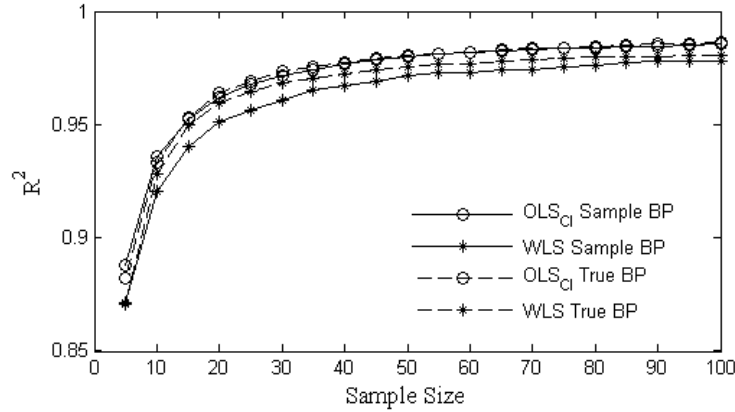


**Figure 100:** Variability Captured of the True Binomial Proportion for Three Least Squares Methods



**Figure 101:** Variability Captured Comparison of the Sample and True Binomial Proportion for  $OLS_{CS}$





**Figure 102:** Variability Captured Comparison of the Sample and True Binomial Proportion for  $OLS_{CS}$  and  $WLS$

### 5.3.1.3 Least Square Regressions of the Quantiles

The quantile measurements will estimates will estimate the 50% quantile. The quantile will report the value that occurs at the median for each of the 16 DOE cases. Since the distribution is symmetric the mean and median are the same; therefore, the model form that will be regressed is the same as was used for the mean. This model form was then used for the three least squares regression methods.

The confidence interval width for the quantiles are based on the predefined quantile value, e.g. 0.50, the sample size, and the values observed. Resizing the sample sizes for each DOE case for the  $OLS_{CI}$  method presents some difficulty. To resize the sample sizes to maintain a constant CI width the following observations are made. Every point in the DOE will use the same  $X_r$  and  $X_s$ . There is no method for approximating the number of samples each case would require to maintain a constant CI width. The sample sizes will have to be incremented based on the observations made. A pseudocode for determining how many samples each case should have is shown below. Note that every case will start with 10 repetitions based on the findings in the previous section on CI accuracy. Additionally, it should be noted that the CI widths were not found to be as similar for the two previous measures.

$$\text{Runs Remaining} = \text{Cases} * (\text{Samples} - 10)$$

```

while Runs Remaining > 0

Find case with largest CI width

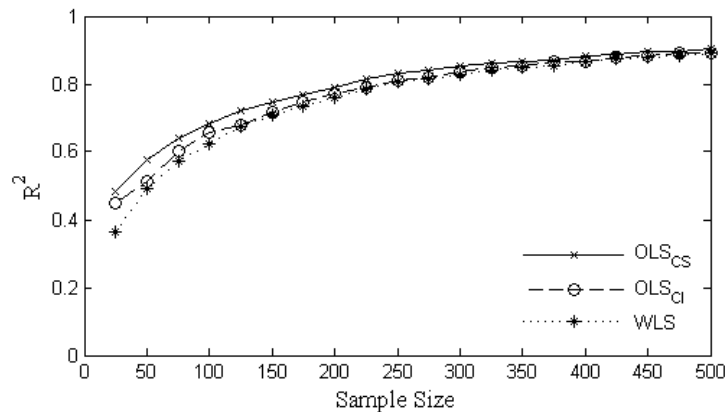
Add additional repetition to found case

Runs Remaining = Runs Remaining - 1

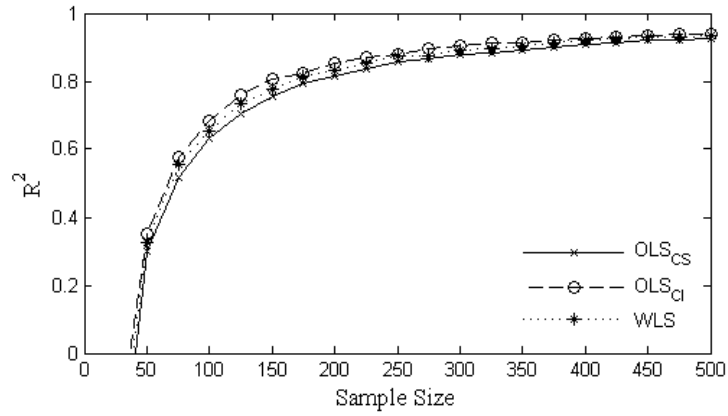
end

```

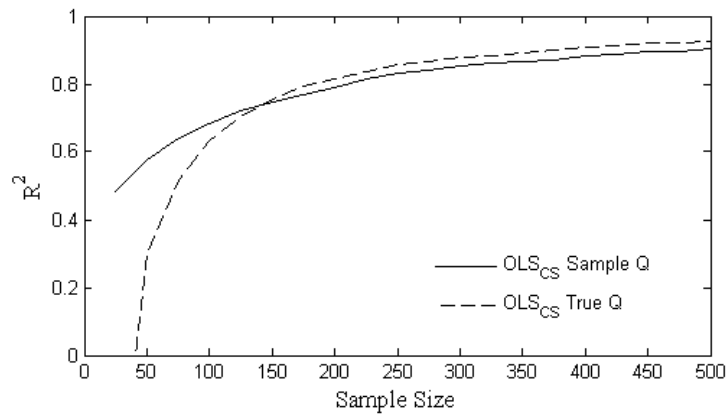
The results can be seen in Figure 103 through Figure 106. The first observation made is that the fit was worse for the quantile fit than for the sample mean or binomial proportion estimate. The  $R^2$  measure with respect to the sample measure, shown in Figure 103, has the expected result. The  $OLS_{CI}$  and  $WLS$  perform relatively the same and the  $OLS_{CS}$  performs the best. When  $R^2$  is compared to the true quantile the  $OLS_{CI}$  method performs the best,  $OLS_{CS}$  performs the worst, and the  $WLS$  method falls in between the two. The  $R^2$  measure for the sample and true measure for  $OLS_{CS}$  is shown in Figure 105. The same observation for this figure is made as was with the sample mean. Initially, the  $R^2$  measure over predicts the quality of the fit when compared to the sample value. With more samples the  $R^2$  under predicts when compared to the sample value. In Figure 102 both the  $OLS_{CI}$  and  $WLS$  methods show an improvement when compared to the true value. This figure shows the slight performance improvement of the  $OLS_{CI}$  method over the  $WLS$  method.



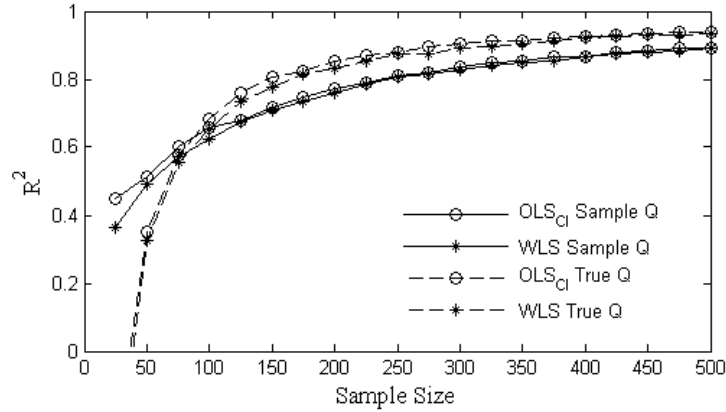
**Figure 103:** Variability Captured of the Quantile Estimates for Three Least Squares Methods



**Figure 104:** Variability Captured of the True Quantile for Three Least Squares Methods



**Figure 105:** Variability Captured Comparison of the Sample and True Quantile for  $OLS_{CS}$



**Figure 106:** Variability Captured Comparison of the Sample and True Quantile for  $OLS_{CS}$  and  $WLS$

### 5.3.2 Conclusions

The question that this section attempted to answer is “what method should be used when creating surrogate models of stochastic measures?” The three methods investigated were ordinary least squares with constant sample sizes, ordinary least squares with constant confidence interval widths, and weighted least squares. These methods were applied to the stochastic measures sample mean, binomial proportion estimate, and quantile estimate. In application, it was found that maintaining constant confidence interval widths created technical difficulties. If this method were selected the experimental model must be adapted to select replications in order to maintain a constant confidence interval width. Maintaining precise widths were not found to be possible for all cases; however, approximations were found to be sufficient.

The primary observation is that the coefficient of determination is a misleading measure of fit for stochastic measures. For small sample sizes the measure consistently over predicts the quality of fit, while for large sample sizes it consistently under predicts the quality of fit. Additionally, it was observed for all three stochastic measures that the ordinary least squares with constant confidence interval widths method performed the best. The ordinary least squares with constant sample sizes method performed the worst for the sample mean and quantile estimates. The weighted least squares estimate performed slightly below the

constant interval width method or the sample mean and quantile estimates. Interestingly, the weighted least squares performed the worst for the binomial proportion estimate, though all three methods performed very well for this estimate. From these observations it is concluded that the ordinary least squares with constant confidence interval widths method should be used; however, if it is not possible to use heterogeneous sample sizes for the cases then weighted least squares should be used for sample mean or quantile estimates and ordinary least squares for binomial proportion estimates.

**Research Question 7 Hypothesis:** The ordinary least squares with constant confidence interval widths method should be used for least squares regressions of stochastic measures in the creation of a surrogate model; however, if it is not possible to obtain constant confidence interval widths then the weighted least squares method should be used for sample mean or quantile estimates and ordinary least squares for binomial proportion estimates.

#### **5.4 *Investigation into the Required Replications for Creating Surrogate Models***

The last section addressed the method for regressing stochastic measures. This section will address the question of ‘How many replications are needed for surrogate modeling of a stochastic measure?’ This question is Research Question 8 reproduced below. The quality of the surrogate model fit is expected to be inversely proportional to the uncertainty of the stochastic measure, i.e. the confidence interval width of the stochastic measure. To investigate this question the quality of the surrogate model fit to the stochastic measures will be compared to the uncertainty of the stochastic measure. The stochastic measures investigated are mean, binomial proportions, and quantiles. The variance measure was not found to be a sufficient measure in the previous section; therefore, it will be excluded. Two methods will be investigated: *OLSC<sub>I</sub>* and *WLS* methods. The *OLSC<sub>S</sub>* method was found to be an insufficient method in the previous section. The canonical problem developed in the previous section and its experimental setup will be used in this analysis. Based on the observations from the canonical problem for the three measures and the two methods a

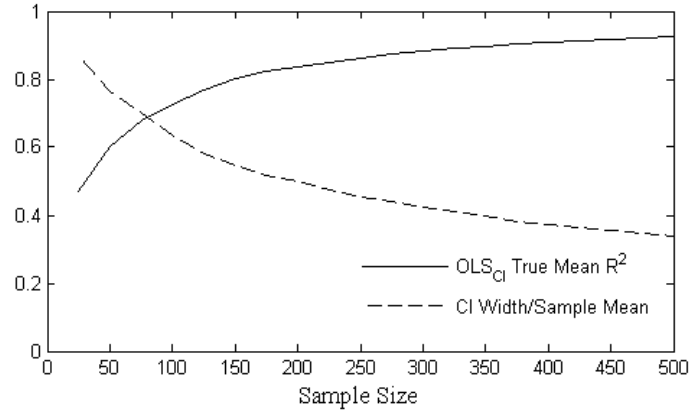
heuristic will be presented on the number of replications needed for a surrogate model of a stochastic measure.

**Research Question 8** How many replications are needed for creating an accurate surrogate model of a stochastic measure?

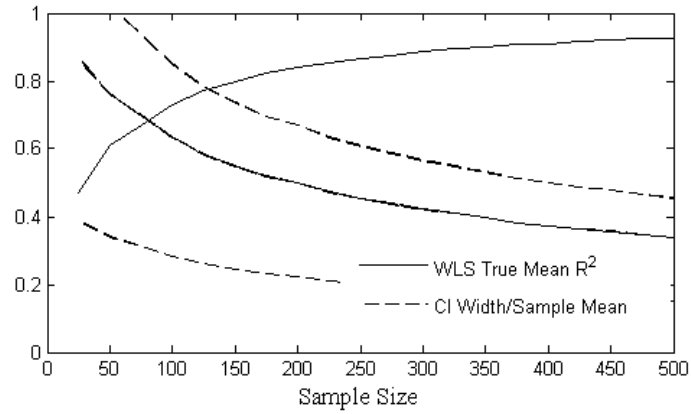
#### 5.4.1 Impact of Replications on a Surrogate Model of the Sample Mean

A comparison between the coefficient of determination with respect to the true mean and the ratio between CI width and sample mean range for the  $OLS_{CI}$  method is shown in Figure 107. The solid line is a plot of the coefficient of determination with respect to the true mean. The dashed line is the ratio between 95% CI width of the sample mean and the range of all sample means from the 16 DOE cases. Since the CI width was held constant, only one line is present. The first observation made is that the  $R^2$  improves as the uncertainty in the measure drops, i.e. the CI width decreases. Secondly, the CI width to estimate range ratio shows to be a good indicator for the quality of the surrogate model fit to the true measure. Finally, a heuristic that can be drawn from this figure is that a value of 0.5 for the CI width to estimate range ratio can provide a lower bound for the number of replications needed to fit a surrogate model to a stochastic measure.

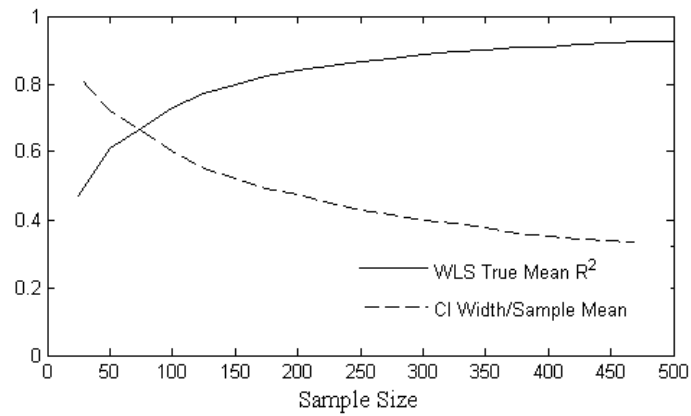
Figure 108 shows a similar graphic for the  $WLS$  method. The solid line is a plot of the coefficient of determination with respect to the true mean. The dashed line is the range of all sample means from the 16 DOE cases. Three lines appear due to the symmetric nature of the variance. The 16 DOE cases fall on one of the three lines. It is also observed in this figure that the CI width to estimate range ratio shows to be a good indicator for the quality of the surrogate model fit. Unfortunately, a point of diminishing returns for the CI width to estimate range ratio cannot be determined for this figure; however, the mean of the CI width to estimate range ratio for the 16 DOE cases can be plotted. This new value is shown in Figure 109. The heuristic of using 0.5 for the mean CI width to estimate range ratio appears to be sufficient for the  $WLS$  method. The mean CI width to estimate range for the  $WLS$  method will be used in the remaining analysis.



**Figure 107:** Comparison of the  $R^2$  to the CI Width to Sample Mean Range for  $OLS_{CI}$



**Figure 108:** Comparison of the  $R^2$  to the CI Width to Sample Mean Range for  $WLS$

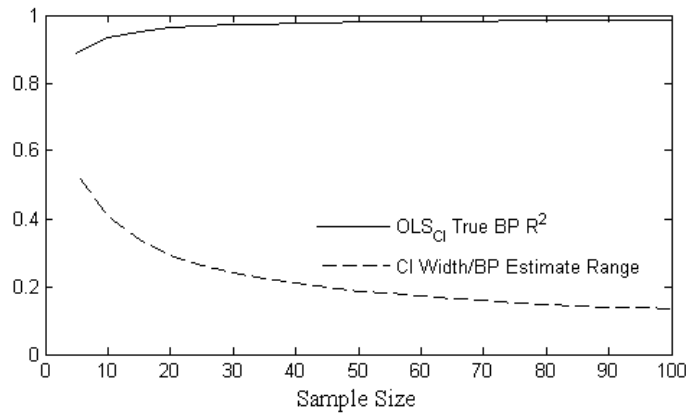


**Figure 109:** Comparison of the  $R^2$  to the Mean CI Width to Sample Mean Range for  $WLS$

### 5.4.2 Impact of Replications on a Surrogate Model of the Binomial Proportion Estimation

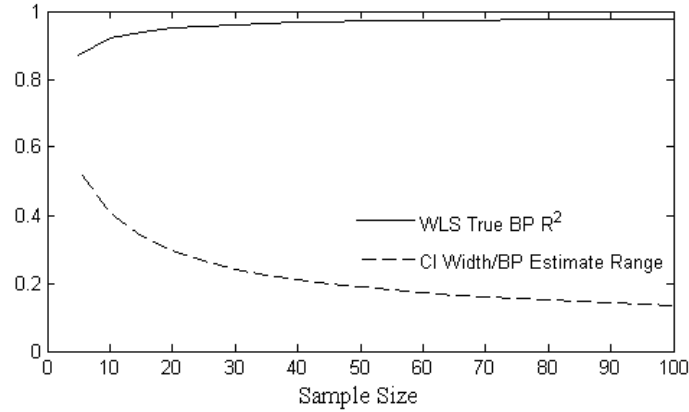
A comparison between the coefficient of determination with respect to the true binomial proportion and the ratio between CI width and binomial proportion range for the  $OLS_{CI}$  method is shown in Figure 110. This figure takes the same form as the one in the previous section. The first observation made is that the binomial proportion estimate fit is much better than the sample mean fit. It is observed that the CI width to estimate range ratio shows to be a good indicator for the quality of the surrogate model fit to the true measure. The low ratio value may be part of the cause for the improved fit over the sample mean fit.

Figure 111 shows the coefficient of determination with respect to the true binomial proportion and the ratio between CI width and binomial proportion range for the  $WLS$  method. Since there are multiple CI widths for the  $WLS$  method, the widths were averaged. This approach was used in the previous section to simplify the graph and draw a heuristic for the width to range ratio. Unfortunately, due to the nature of the binomial proportion estimate, the CI width to estimate range is relatively small. Therefore, a heuristic could not be found. For this measure it is suggested to default to the previous heuristic presented to estimate accurate CI bounds, e.g.  $n \geq 3/\min(p, 1 - p)$ .



**Figure 110:** Comparison of the  $R^2$  to the CI Width to Binomial Proportion Estimate Range for  $OLS_{CI}$





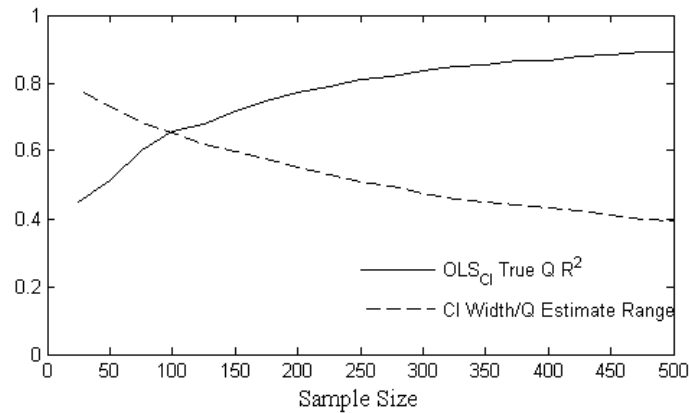
**Figure 111:** Comparison of the  $R^2$  to the Mean CI Width to Binomial Proportion Estimate Range for  $WLS$

### 5.4.3 Impact of Replications on a Surrogate Model of the Quantile Estimation

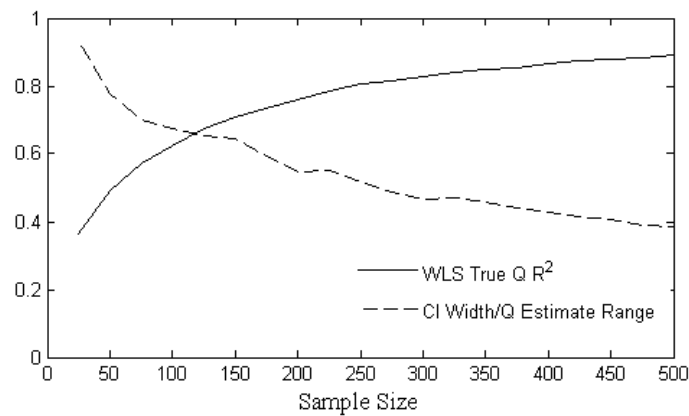
A comparison between the coefficient of determination with respect to the true quantile and the ratio between CI width and quantile estimate range for the  $OLSC_I$  method is shown in Figure 112. This figure takes the same form as the one in the previous two sections. The first observation made is that the quantile estimate fit is worse than the sample mean and binomial fit. It is observed that the CI width to estimate range ratio shows to be a good indicator for the quality of the surrogate model fit to the true measure. However, for both curves the shape is more shallow than for the sample mean measure, despite both measures having the same true values. Note the symmetry of the distribution results in the same mean and median. This indicates that the quantile measure requires more replications to reduce the CI widths than the mean measure. This may indicate as to why the  $R^2$  measure falls short of 0.9 with 500 samples. This is about 0.03 below that of the sample mean fit.

Figure 113 shows the coefficient of determination with respect to the true quantiles and the ratio between CI width and quantile estimate range for the  $WLS$  method. Since there are multiple CI widths for the  $WLS$  method, the widths were averaged. This approach was used in the previous two sections to simplify the graph and draw a heuristic for the width to range ratio. The observations made for the previous figure are made for this figure. Applying the heuristic from the sample mean analysis, i.e. 0.5 ratio value, to these two

figures would indicate that a little more than 250 samples would be required for each DOE case. For this canonical example the expected  $R^2$  to the true value would be slightly below 0.8. The expected  $R^2$  for the quantile surrogate model is surprisingly close to the expected  $R^2$  for the mean surrogate model.



**Figure 112:** Comparison of the  $R^2$  to the CI Width to Quantile Estimate Range for  $OLS_{CI}$



**Figure 113:** Comparison of the  $R^2$  to the Mean CI Width to Quantile Estimate Range for  $WLS$

#### 5.4.4 Conclusion

The question that this section attempted to answer is “how many replications are needed for surrogate modeling?” This question is highly dependent on the uncertainty of the stochastic measure. For example, as the variance increases both the sample mean and

quantile estimate confidence interval widths increase. This will result in requiring larger sample sizes for the same quality of fit. Because the dispersion of the output distributions vary, a heuristic based on the confidence interval width to the range of the estimate was created. This is similar to the inverse of a signal-to-noise ratio. It was found that achieving a value of 0.5 for the interval width to range of the estimate ratio resulted in a  $R^2$  measure with respect to the true stochastic measure of 0.8. A ratio value of about 0.4 resulted in a  $R^2$  measure with respect to the true stochastic measure of 0.9. Unfortunately, to reduce the interval width by a factor would require about an inverse squared of the factor increase in sample size, e.g. reducing the width by half would require four times the sample size.

The binomial proportion does not face the issue of large ratio values to the degree of the sample mean and quantile estimates. The largest value for the interval width to estimate range that can be achieved for ten samples is 0.53. The binomial proportion estimates naturally have a stronger signal to noise ratio. In the absence of a heuristic based on the interval width to estimate range ratio it is suggested that the heuristic found for Research Question 6 should be used, i.e.  $n \geq 3/\min(p, 1 - p)$ . The conclusion to Research Question 8 is shown below.

**Research Question 8 Hypothesis:** The ratio between the confidence interval width to the range of the estimates should be used as an indicator for the quality of fit of a surrogate model to a stochastic measure. Assuming the correct model form is used for a stochastic measure of mean or quantile, a ratio value of 0.5 will achieve a coefficient of determination of about 0.8 and a ratio value of 0.4 will achieve a coefficient of determination of about 0.9 with respect to the true stochastic measure. Replications required for a surrogate model of a binomial proportion should be the number of replications required to achieve accurate interval estimates, i.e.  $n \geq 3/\min(p, 1 - p)$ .

## CHAPTER VI

### METHODOLOGY PROOF OF CONCEPT

“In theory, there is no difference between theory and practice. But, in practice, there is.” (Jan L. A. van de Snepscheut)

Chapters two through five defined a methodology based on Balci’s 1986 model development procedure, shown in Figure 13 on page 42. This methodology was created to aid in the development of a model of a non-observable system. Now the methodology must be applied to a problem that requires modeling and simulation of a non-observable system.

Before the application of the methodology is detailed, a brief summary of the methodology is presented in the following section. This is followed by a selection of a test system for which the methodology will be applied. The following section, 6.3, will provide the problem definition. Once the problem is defined, Balci’s model development procedure will be followed while applying the methodology developed in this thesis. The first step addressed in the model development phase of Balci’s procedure is System and Objectives Definition, which is covered in Section 6.4. The next step is to develop the conceptual model, which is covered in Section 6.5. This step is followed by the creation of communicative models using UML, which is covered in Section B.5. Once the communicative model is developed the computerized model is developed within the step of Programmed Model. A section is not given on programming. Once the computerized model is developed an experimental model is created and simulation results are obtained. This is covered in Section 6.6. Finally, to close the loop on the methodology the simulation results are compared to the System and Objectives Definition and the Conceptual Model for model validation. This is covered in Section 6.7.

## ***6.1 Summary of Model Development Methodology***

Numerous research questions have been posed in order to develop a model development methodology for non-observable systems. These research questions are based on the model development phase of Balci's 1986 Simulation Life-Cycle, referred to here as a model development procedure. The conclusions drawn from the numerous research questions result in the developed methodology. This methodology is summarized in Figure 114.

The model development methodology begins after the problem formulation phase. The first step is to define the objectives of the simulation study. Once the objectives are defined, the system is to be defined using SysML. SysML contains several viewpoints that contribute to defining a system. The SysML system definition should contain as many viewpoints in quantity and variety such that those involved in the simulation study are satisfied to its completeness with respect to the simulation study objectives. The modeler should work closely with the customer and SMEs on these two steps to make sure the modeling effort is started on a good foundation. This concludes the System and Objectives Definition step in Balci's procedure.

The next step in Balci's procedure is the conceptual model. The methodology developed in this thesis begins this step by decomposing the variables of the system creating the impact matrix. It is suggested that one starts with the output measures of interest and then define the set of impact variables that directly affect them. These impact variables are then further decomposed into another level of impact variables. This will result in an impact matrix. The matrix at this state is empty. It only contains the structure.

Once the impact matrix is created impact weightings are applied by SMEs between each level of the impact matrix. These weightings are made through a series of pairwise comparisons of direct impacts. Next, the indirect impacts are calculated through matrix multiplication. At this point the impact matrix is fully defined.

The importance of the direct impacts are then calculated by removing the direct impact link and determining the amount of variability that is lost on the top level metrics. This will provide guidance for the selection of a model representation.

Each direct impact is then addressed on how it will be represented within the computerized model. This is accomplished utilizing a Morphological Matrix. Using a Morphological Matrix the possible model representations of the direct impacts are enumerated. Using the importance values of the impact relationships a selection is made. It is important to document the reasoning behind the selection. Once all decisions are made and documented the conceptual model is complete.

The next step is to develop the communicative model. UML is utilized to create the communicative model. Like SysML, UML contains several viewpoints that contribute to defining the communicative model. The communicative model should contain as many viewpoints in quantity and variety such that those involved in the simulation study are satisfied to its completeness with respect to the conceptual model defined.

Once the communicative model is complete the computerized model is programmed and verified. This methodology does not affect the programming of a computerized model. Standard programming practices are suggested. Once the computerized model is created an experimental model is created. Results are then gathered from the simulation of the experimental model. Brownian correlation values are then calculated from the simulation results for each of the impact relationships.

Next, the simulation results are compared to the original SME estimated impact matrix values. The modelers, analyst, and SMEs should be involved in comparing the subjective estimates to the observed simulation results. It is expected for there to be some differences between the initial subjective estimates and the simulation observations, for if the SME could produce a perfect impact matrix decomposition, then there is less need for a model.

Two possibilities exist for any disagreement; either the model is wrong or the SME is wrong. If it is determined that the model is incorrect then the development of the conceptual model must be readdressed. This would start with reviewing the system definition for any inaccuracies or omissions. The structure of the impact decomposition would then be addressed for accuracy. This active is followed by investigating the impact weightings. Finally, the model representation would be addressed. A higher level of fidelity may be required to accurately represent the area of disagreement. Any corrections made would

have to be propagated throughout the model development methodology, the current model would have to be modified, and new results would have to be gathered. If the disagreement continues either the development of the conceptual model can be addressed again or it can be concluded that the SME estimates are inaccurate.

SME subjective estimates can be inaccurate for numerous reasons; these include but are not limited to variable range, system/scenario evolution, or inaccuracies. The variable range plays a major role in the importance of variables in a system. Consider a specific process in a manufacturing line. The time required to complete the specific process can have a large impact on the efficiency of the manufacturing line; however, if the range of the time required is sufficiently below the arrival rate, then the impact of the time required becomes insignificant. The system or scenario evolution can affect the accuracy of the SME impact estimates to the simulation results. The SME may be developing their estimates from a historical experience based on a system that has changed, e.g. improved communications or fuel efficiency, or the scenario has changed. Scenario evolution can be very common among social systems. Finally, SMEs are used for their extensive knowledge and expertise; however they are still fallible. Great care should be taken to suggest that the initial estimate may have been wrong. SMEs like all other people do not like being wrong and may take offense to the suggestion; however, one should not default to iterating on the model development. Model development requires significant resources as do iterations. Both possibilities should be addressed within the group and a path forward should be suggested. Once there is sufficient agreement between the subjective models and the simulation results further analysis should be performed on the model that helps to answer the questions in the objectives definition.

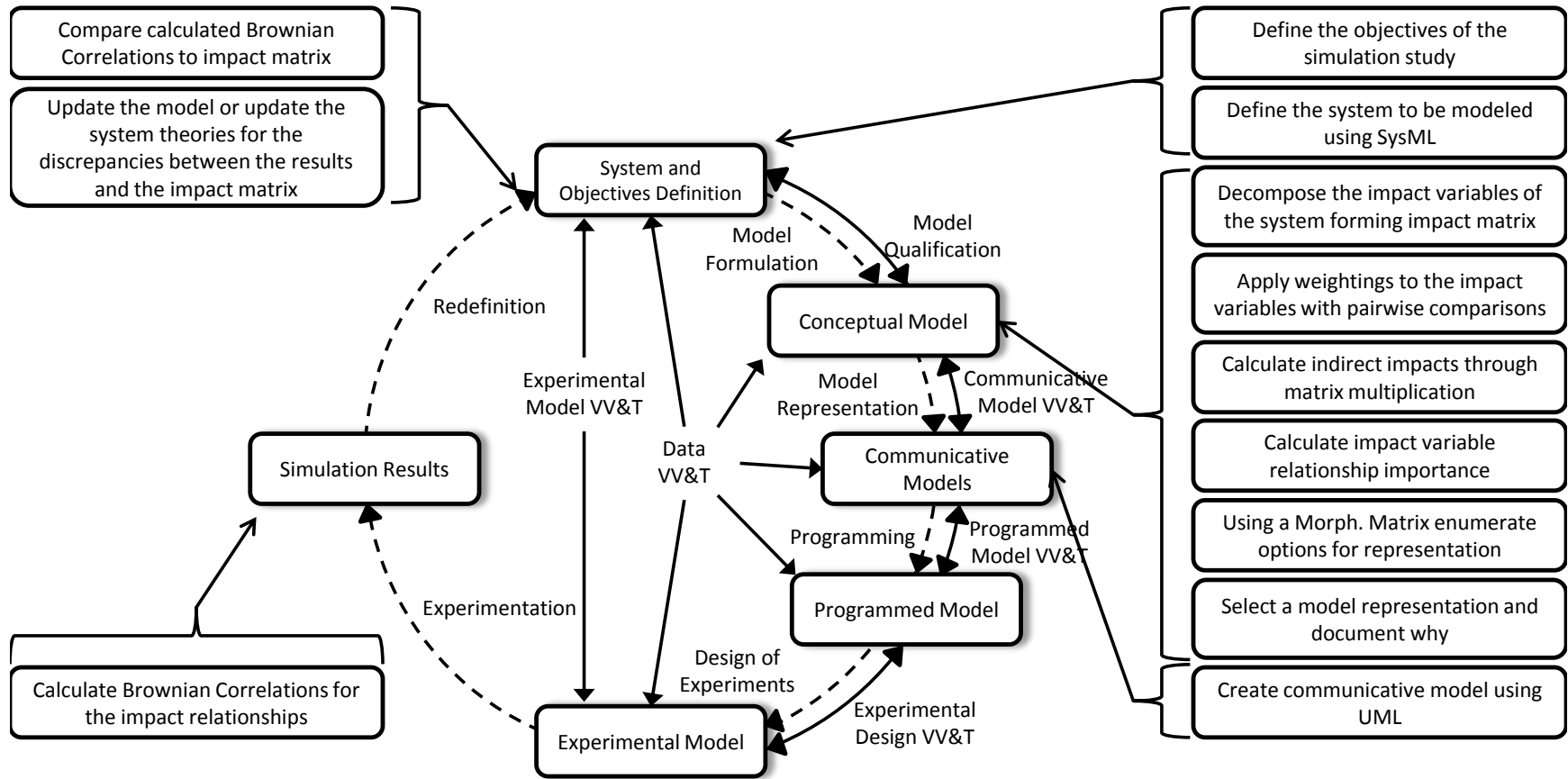
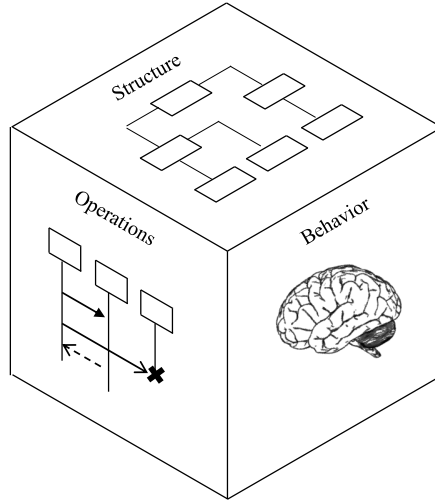


Figure 114: Example System Decomposition Graph



## *6.2 Selection of a Test System*

A scenario is required for the application of the methodology presented in this thesis. This scenario will be a system-of-systems scenario, which will include numerous interacting systems that coordinate to accomplish a task. For the purposes of this thesis, standard M&S terminology will be used; therefore, the system-of-systems will be referred to as a system. Three dimensions are identified as important for the selection of a scenario: structure, operations, and behavior. The structure of the scenario is a description of how the system decomposes into its subsequent parts and how the parts physically relate to each other. An example of a model that is built strictly for the structure is a sizing model, e.g. FLOPS [107]. The operations of the scenario is a descriptions of how the parts of the system interact and function. An example of a model that is built strictly for the operations is a discrete event industrial process model. Finally, the behavior of the scenario is a description of how the system or its parts make decisions and react to its environment or to other systems. An example of a model that is built strictly on the behavior of a system is an economic model. A scenario is defined by these three dimensions, Figure 115. Some scenarios require greater description in some dimensions than others. The selected scenario must encompass each of the three dimensions defined to be considered a reasonable test of the proposed methodology in its application to model development of non-observable systems.



**Figure 115:** Three Dimensions of Scenarios for Modeling and Simulation

A set of potential scenarios were identified from the literature. The potential scenarios are Economic Exclusion Zone (EEZ) enforcement, Mine Counter Measures (MCM), port security against maritime Improvised Explosive Devices (MIED), anti-piracy operations, and Humanitarian Aid/Disaster Relief (HA/DR).

### 6.2.1 Scenario Set One

The scenarios EEZ enforcement and anti-piracy operations will be considered as one class of scenario, because the two scenarios contain very similar elements. The EEZ scenario is based on the demonstrative example in Chapter 1. This scenario contains a potential set of underwater vehicles, surface vessels, and air vehicles who coordinate efforts to patrol an area and enforce EEZ rights. Few examples of models developed to simulate and analyze an aspect of EEZ enforcement were found in the literature. The first publication by Sutinen [187] investigates the allocation of patrol vessels to patrol tuna fishing areas off the coast of Costa Rica. To find an efficient allocation a linear program model was developed. The second source is based on two presentations given at the MORS Symposium in 2012 by Turner and Andriano [196, 10]. This work is the basis of the demonstrative example given in Chapter 1.

The anti-piracy operation scenario contains a potential set of underwater vehicles, surface vessels, and air vehicles who coordinate efforts to patrol an area to protect civilian vessels from piracy by identifying pirate vessels, capturing pirate vessels, and assisting civilian vessels under duress. Numerous examples of models developed to simulate and analyze an aspect of anti-piracy operations were found in the literature. Two of the publications found were found in the Winter Simulation Conference Proceedings by the Naval Postgraduate School (NPS). The first publication, Walton et al. [206], investigated the use of armed Sea Marshals on high value ships and the deployment of patrol craft to combat piracy from small boat attack in the Straits of Malacca. Walton et al. used an agent based model developed in Map Aware Non-Uniform Automata (MANA) to analyze the two alternatives. It was found that the use of Sea Marshals was the best alternative to counter piracy from small boat attack. The second publication, Esher et al. [64], used an agent based model to predict the potential location of pirates. This was conducted in an effort to estimate the risk to commercial fishing vessels.

Numerous other publications were found by the Agent Technology Center at the Czech Technical University where an advanced agent based model, called AgentC, was developed to improve maritime security with a focus on fighting maritime piracy [5]. A recent publication from the center by Vanek et al. discusses AgentC and provides the results of a what-if analysis resulting in the conclusion that a similar approach to the International Recommended Transit Corridor (IRTC) within the Gulf of Aden cannot be applied to the Indian Ocean [204]. In another recent paper by Vanek et al. the AgentC model was used to suggest a new transit scheme for the IRTC [205].

The characteristics of these two scenarios are very similar, particularly with respect to the three dimensions of a scenario. The structure of the two scenarios are composed of a set of diverse systems; therefore, the system decomposition, though not identical, will have similar structure. Each system of system will contain a set of systems. Each system will contain a set of components that enable their capabilities. In both scenarios the operations are critical and are often the central question asked of the modeling effort. Finally, in both scenarios behavior is a critical component to the simulation. For the EEZ patrol

the behavior of the fishing vessels must be modeled. The work performed by Turner and Andriano addressed behavior directly in model design and analysis. The work performed by Sutinen represented the behavior based on historical data of observations. Each of the anti-piracy operation publications included the behavior dimension into their model development and analysis. This class of scenario provides strong representation into the three dimensions of scenario.

### **6.2.2 Scenario Set Two**

The second class of scenarios includes MCM and port security against MIED, because the two scenarios contain very similar elements. The MCM scenario contains a potential set of underwater vehicles, surface vessels, and air vehicles who coordinate efforts to detect, identify, and neutralize mines within a given area. Numerous examples of models developed to simulate and analyze an aspect of MCM operations were found in the literature. These examples primarily use mathematical models. Reed et al. use multiple models to investigate the use of sidescan sonar in mine detection [148]. Their work includes the use of a Markov Random Field model, a cooperating statistical snake model, and a sonar simulator. Sariel et al. make use of the US Navy's Autonomous Littoral Warfare Systems Evaluator - Monte Carlo (ALWSE-MC) simulator to test the performance of their multi robot cooperation framework [173].

The port security against MIED scenario contains a potential set of underwater vehicles, surface vessels, and air vehicles who coordinate efforts to detect, identify, and neutralize mines within a port. A publication by Paulo details a modeling and analysis effort to identify a system that can mitigate a MIED threat using the Joint Conflict and Tactical Simulation (JCATS) program [141]. One of the major conclusions of the study was that the use of mobile and easily deployable systems is important to enhanced port security.

The characteristics of these two scenarios are very similar, particularly with respect to the three dimensions of a scenario. The structure of the two scenarios are composed of a set of diverse systems; therefore, the system decomposition, though not identical, will have similar structure. The operations of both scenarios are critical and often the focus of the

analysis. The behavior dimension of the scenario is very small. The mines or MIEDs do not have a behavior to the degree of a human opponent, and the vehicles typically follow a defined operation.

### 6.2.3 Scenario Set Three

The final scenario considered is a HA/DR operation. The HA/DR scenario contains a potential set of ground units, surface vessels, and air vehicles who coordinate efforts to provide security and deliver resources to a disaster affected area. Numerous examples of models developed to simulate and analyze an aspect of HA/DR operations were found in the literature. Balci and Beamon use a mathematical model, which is similar to a maximal covering location model, to predict the optimum number of supply stocks and the amount of supplies to be stored at each location [26]. Lee et al. presents a model that simulated the supply chain and distribution operations of a HA/DR scenario [101]. The model, i-DRuM, was used to analyze the distribution of resources in response to a hurricane disaster. Turner et al. uses an agent based model to simulate a HA/DR mission to determine the affect of the size, number, and placement of resource distribution centers on crime rates and population resource need [198]. A publication by Cohen et al. [50], which is based on a master's thesis by Alexander et al. [7], models a HA/DR mission by integrating a behavioral system dynamics model and a throughput discrete event simulation to assess the satisfaction of the population based on the humanitarian aid effort.

The structure of the scenario is composed of a set of diverse systems, which include surface vessels, air vehicles, and people. The operation dimension of the scenario is often the focus of HA/DR modeling and analysis, e.g. Balcik and Lee et al. Every found example of HA/DR M&S activities included an operational dimension. The behavior component is also a very important dimension to the problem. Several works found in the literature included behavioral modeling alongside operational modeling, e.g. Alexander et al. and Turner et al. All three dimensions of the scenario are found to be important for HA/DR; however, the operational aspect and the behavioral aspect of the scenario has been found to be capable of being modeled separately, e.g. Alexander et al. and Cohen et al.

#### 6.2.4 Scenario Selection

The second scenario set, which included MCM and port security against MIED, showed to be lacking in the dimension of behavioral modeling. The models found in this category were found to primarily focus on operational modeling with some structural modeling. Since behavioral modeling is not well represented the second scenario set will be removed from consideration. The first scenario set, which included the EEZ and anti-piracy operation, and the third scenario set, which included HA/DR operations, maintained a strong need to model of all three scenario dimensions. One primary difference between the two scenario sets is that the HA/DR operations enabled the separation of the structural and operational modeling from the behavioral modeling. This provides a great benefit for its use as a test scenario. This will enable an easier analysis of the application of the methodology to a non-observable system. This separation will allow observations into how the methodology performs for different kinds of models. For this reason the HA/DR scenario is selected as the test scenario.

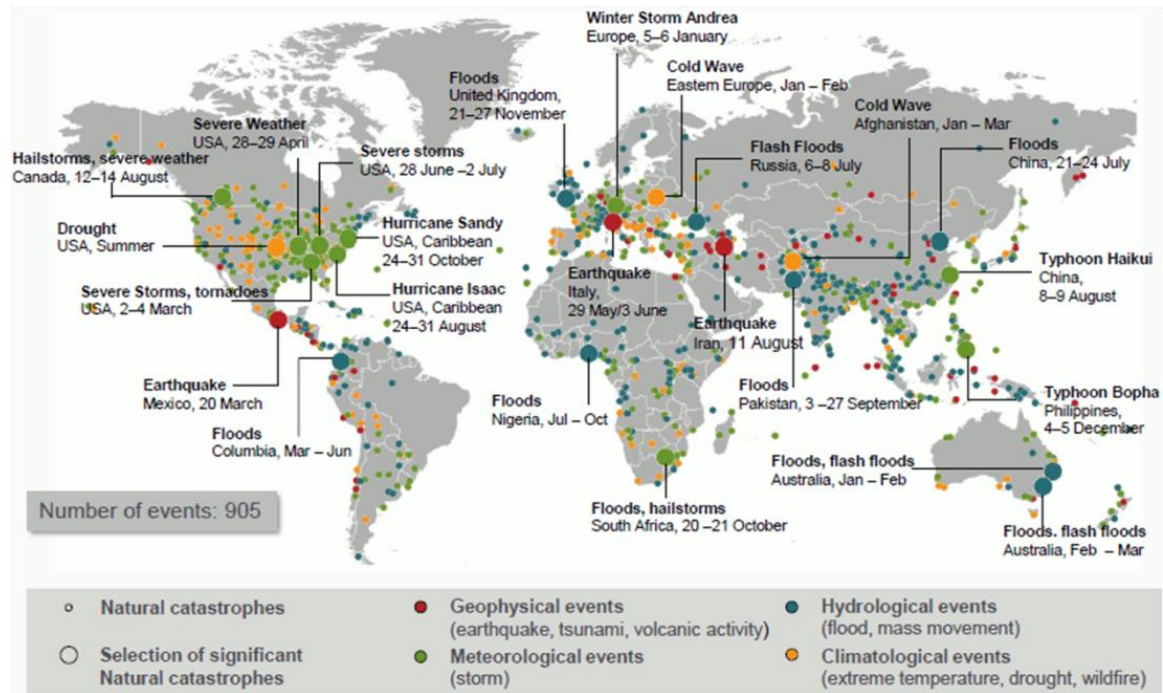
### 6.3 Problem Formulation

Balci defines the problem formulation as “the process by which the initially communicated problem is translated into a formulated problem sufficiently well defined to enable specific research action” [20]. This will be accomplished here by detailing the scenario in question and defining the questions asked of the scenario. First a brief overview of HA/DR will be given in the following section. Next the specific scenario will be defined. The scenario developed will be based on the works of Alexander [7] and Cohen [50].

#### 6.3.1 Humanitarian Aid/Disaster Relief Overview

HA/DR operations continue to be a critical need around the globe. This is evident by numerous disasters that occur every year. These disasters include but are not limited to earthquakes, hurricanes, flooding, droughts, war, insurgencies, and riots. The type and severity of the disaster varies greatly from disaster to disaster. For example, a 7.0 magnitude earthquake struck outside of Port-au-Prince, Haiti on January 12th, 2010 resulting in

100,000 [201] to 159,000 [89] deaths. Later that year on the 3rd of September a 7.1 magnitude earthquake struck near Christchurch, New Zealand, which resulted in two serious injuries [186]. The Pacific Regional Disaster Preparedness Center provides a good visual of the frequency, diversity, and severity of disasters shown in Figure 116.



**Figure 116:** World Wide Disaster Map for 2012

HA/DR is required throughout the globe and various Non-Governmental Organizations (NGOs), e.g. The Red Cross, international organizations, e.g. United Nations, NATO, and nation states participate in relief efforts. In general, the United States address relief efforts in the following ways. Relief operations can be divided into two categories: natural disasters and complex emergencies. A natural disaster is not directly caused by human behavior and includes earthquakes, hurricanes, flooding, droughts, etc. Complex emergencies develop as a result of human conflict, e.g. war, insurgency, riots [200]. In both of these types of operations the U.S. Disaster Assistance Response Team (DART) and the U.S. military coordinate efforts. For relief operations in response to natural disasters the DART will have greater control in operation planning. For relief operations in response to complex emergencies

the military will have more control on the operational planning and the DART will focus on humanitarian issues [200]. The USAID Field Operations Guide assumes that food, water, medical, quality of life supplies are transported by the U.S. military and provided by NGOs, international organizations, and individual nation states [200]. The U.S. military has an inherent lift capability that can transport massive amounts of material and be rapidly deployed world wide in a relatively short time span [50]. At the current date the U.S. military is the only organization in the world with this ability. The selected scenario focuses on the operational efforts of the U.S. military and its impact on the satisfaction of the populace.

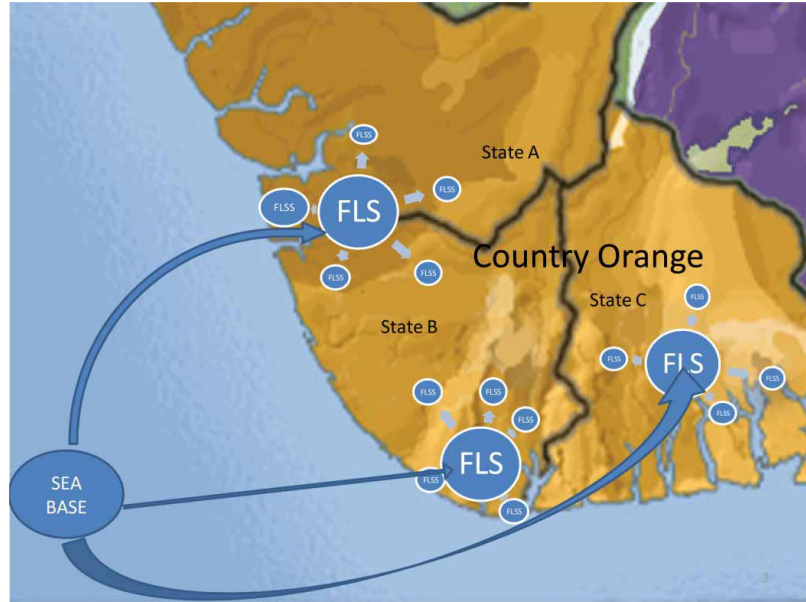
### **6.3.2 Selected Humanitarian Aid / Disaster Relief Scenario**

The scenario developed is based on the work conducted at the NPS. Two sources are used as a basis to develop the scenario: Alexander et al. and Cohen et al. [7, 50]. The specific scenario concerns the 60 day humanitarian relief effort in response to severe flooding in a populated region of West Africa. The timeframe set for this missions is 2020. The affected area is encompassed within the three southern states of country Orange, which includes more than 179 million people. The northern most state is referred to as State A. The southwestern most state is referred to as State B. The eastern most state is referred to as State C. The three states in country Orange are home to a diverse set of ethnic groups. There are about 70 million Muslims and 70 million Christians in the North and South of the region, respectively. The south is plagued by terrorist activities and thievery by insurgent groups and criminal gangs. This civil unrest is a result of a non-optimal distribution of resources from the revenue of the oil and gas industry, which is a primary source of revenue in the region. In the north external Muslim radicals have presented issues in the relations between the north and south.

It is assumed that the massive flooding has damaged the seaports and the airports; therefore, the relief effort will be delivered from the sea. The relief effort will take place in three stages. The first is the transportation of material and goods from a Seabase to the Forward Logistic Sites (FLS). The second is the distribution of material and goods from



the FLS to the Forward Logistic Satellite Sites (FLSS). The final stage is to distribute the material and goods from the FLSS to the people. A visual of this operation can be seen in Figure 117.



**Figure 117:** West Africa HA/DR Operational View One

The broad questions being asked of the scenario are how well is aid being distributed to the population, what is the effect of HA/DR operations on the population, and how should a Command and Control (C&C) system be designed to enable effective collaboration?

Four Courses of Action (COA) are presented to provide aid, i.e. material and goods and security. These COAs are summarized in Table 45, which is reproduced from [50]. The first column indicates the COA identification. The second column indicates how aid will be delivered from the seabase to the FLS and if security will be provided by the military. The third column indicates how aid will be delivered to the FLSS from the FLS and if security will be provided. COA 1 uses the military assets to deliver the aid from the seabase to the FLS using surface ships and helicopters. The military also provides security at the three FLS locations. The NGOs will then deliver the aid to the FLSS and distribute the aid to the people. COA 2 will transfer aid using surface connectors from the seabase to the FLS locations. Air connectors will transfer aid from the seabase directly to the FLSS. Security

will be provided at both logistic types. COA 3 and 4 distribute aid using surface connectors between the seabase and the FLS and using air connectors between the FLS and the FLSS. The difference between the two is that COA 3 will not have security present at the FLSS, and COA 4 will have security present at the FLSS.

**Table 45:** Courses of Action

COA	Stage 1 Connectors	Stage 2 Connectors
1	Air + Surface + Security	NGOs
2	Air + Surface + Security	
3	Surface + Security	Air
4	Surface + Security	Air + Security

More in depth questions are posed by the Alexander et al. masters thesis that expand upon the two general questions of the simulation. Four questions are presented that inquire into the transferring of aid from the seabase to the FLS. These questions are listed directly below. This example problem will not be as in depth as the one conducted at the NPS; therefore, a subset of the questions will be addressed. From the set below the third question will be addressed in this example problem. The conclusions to the other questions within the NPS thesis will be used as inputs.

1. What configurations of ships are required?
2. What functions can/cannot be supported via seabase?
3. What is the throughput of aid via the seabase?
4. What limitations does the seabase impose?

Four questions are presented that inquire into the transferring of aid from the FLS to the FLSS. These questions are listed below. Questions one through three will be address in this example problem. The NPS thesis conclusion to question four will be used as in input.

1. What is the throughput of aid via a distributed system?

2. How much of the population can we reached?
3. How much security is required to protect all assets?
4. What is the best way to develop a Command, Control, and Communication architecture under the current scenario?

Finally, three questions are presented that address the reaction of the population to the humanitarian assistance. These questions are listed below. Of this question set only question one will be addressed in this example. The conclusions reached in the NPS thesis will be used as inputs where applicable.

1. Using qualitative analysis, what are the resulting effects of aid on the population?
2. What social response will the population have to foreign military forces?
3. How effectively will the military and NGOs interact?

#### ***6.4 System and Objectives Definition***

This section will address the process of System Investigation. System Investigation involves the development of an objective definition and a system definition. An objective definition is a statement of what is to be achieved in the simulation study, which is presented in Section 6.4.1. Additionally, the objective definition provides a statement on the resources available to achieve the stated objectives. A system definition is the process of decomposing the complex system into manageable parts such that a model can be formulated. The system definition of the defined scenario is shown in Section 6.4.2.

##### **6.4.1 Objective Definition**

The objective definition is a statement of what is to be achieved in the simulation study with respect to questions being answered and the resources available to develop the model and analyze the simulations. The overall objective is to develop a model using the methodology presented in this thesis to show how it is applied, identify any shortcomings, and make additional observations about the modeling process. As stated above, two broad questions

being asked of the scenario. This first is how well is aid being distributed in terms of population reached and amount of material distributed? The second broad question is what is the effect of aid on the population? In addition to the more focused questions presented in Problem Formulation, Alexander et al. address these questions with an effects based decomposition [7] shown in Table 46. These sets of questions and measures constitute the statement on what the simulation study should answer.

**Table 46:** Effects Based Decomposition

<i>MOE<sub>1</sub></i>	Effectiveness of HA over 60 day time frame
<i>MOP<sub>1.1</sub></i>	Elapsed time until all HA at Objectives
<i>MOP<sub>1.2</sub></i>	Percent of HA at the Objective in 60 days
<i>MOP<sub>1.3</sub></i>	Percent of water at Objective in 60 days
<i>MOP<sub>1.4</sub></i>	Percent of food at Objective in 60 days
<i>MOP<sub>1.5</sub></i>	Percent of shelter at the Objective in 60 days
<i>MOP<sub>1.6</sub></i>	Percent of misc. supplies at objective in 60 days
<i>MOE<sub>2</sub></i>	Effectiveness of HA over 15 day time frame
<i>MOP<sub>2.1</sub></i>	Percent of HA at the Objective in 15 days
<i>MOP<sub>2.2</sub></i>	Percent of water at Objective in 15 days
<i>MOP<sub>2.3</sub></i>	Percent of food at Objective in 15 days
<i>MOP<sub>2.4</sub></i>	Percent of shelter at the Objective in 15 days
<i>MOP<sub>2.5</sub></i>	Percent of misc. supplies at objective in 15 days
<i>MOE<sub>3</sub></i>	Effectiveness of Marine Support
<i>MOE<sub>3.1</sub></i>	Percentage of Supplies to support MEU delivered
<i>MOE<sub>3.2</sub></i>	Percentage of Time Force Protection/Security Measures are above minimum Threshold of 80%
<i>MOE<sub>4</sub></i>	Effectiveness of Essential Functions
<i>MOP<sub>4.1</sub></i>	Level of Command and Control provided
<i>MOP<sub>4.2</sub></i>	Logistical functions
<i>MOE<sub>5</sub></i>	Stage 3 Effects of Humanitarian Aid on the population
<i>MOP<sub>5.1</sub></i>	Percentage of population influenced COA 1
<i>MOP<sub>5.2</sub></i>	Percentage of population influenced COA 2
<i>MOP<sub>5.3</sub></i>	Percentage of population influenced COA 3
<i>MOP<sub>5.4</sub></i>	Percentage of population influenced COA 4

#### 6.4.1.1 Resource Availability

The resources available to the simulation study are one desktop computer with a 2.66 GHz Intel quad core processor and 2 GB of RAM. The person-hours available is about two full work weeks by an individual. This results in about 80 person-hours. Numerous software programs are available for developing the model and analyzing the simulation outputs. These software programs include but are not limited to MatLab, JMP, Microsoft Office, NetLogo, Python (SimPy), and MagicDraw. This set of available resources constitute resource availability portion of the objective definition.

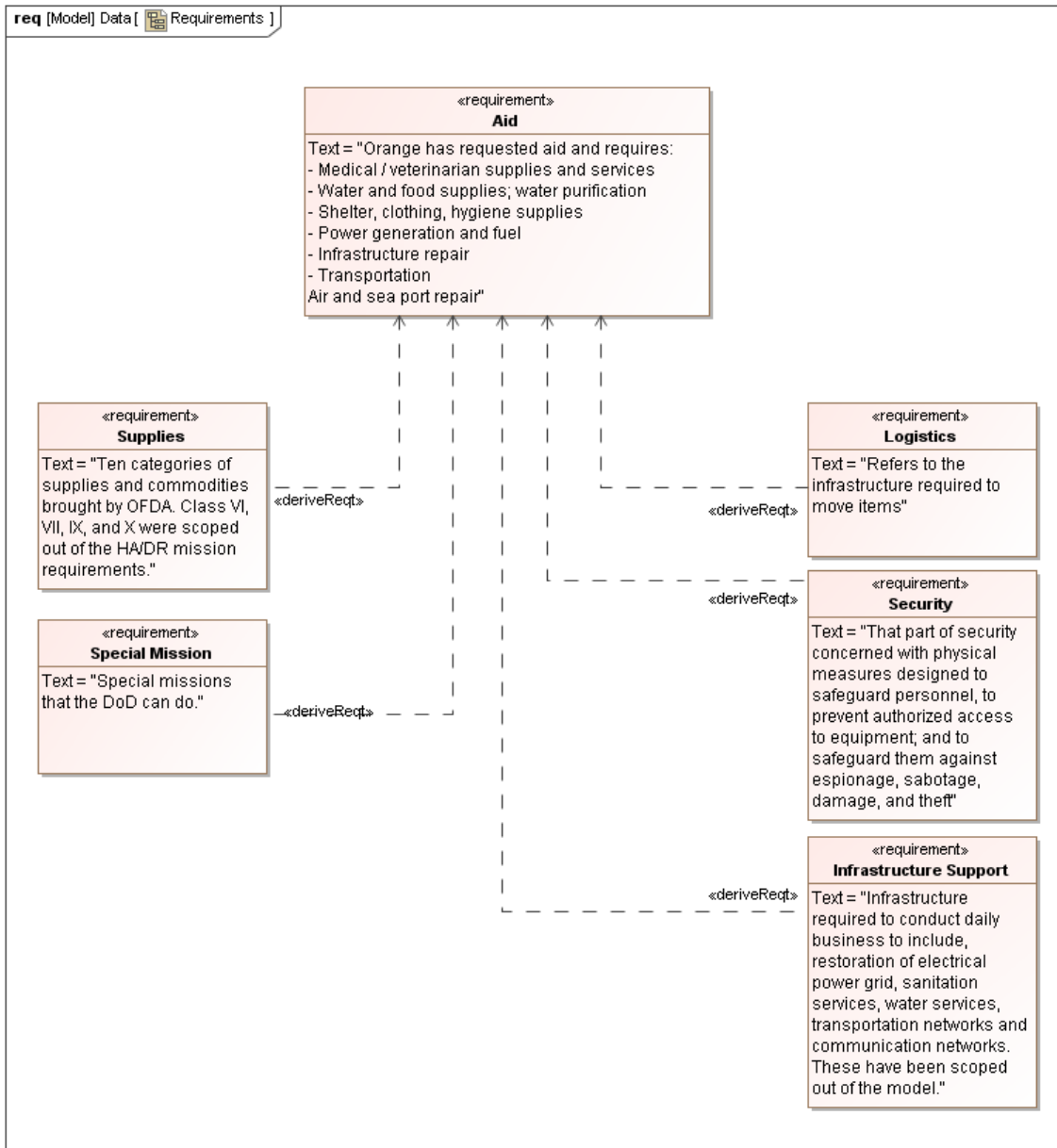
#### 6.4.2 System Definition

The modeling of multiple COA would require the methodology to be executed for each COA. Many of the parts would be reusable, e.g. SysML diagrams, system decomposition, UML diagram, computer code, but this would be a time consuming process. Since the overall objective of the simulation study is to apply the methodology and two weeks are given for the study only COA 2 will be modeled using the full methodology. COA 2 was selected because it was shown to perform well with respect to the number of days to deliver aid and the percent coverage of aid [50]. COA 4 will be used sparingly to demonstrate how the application of the methodology changes for different system definitions. The selection of one COA will reduce on the model time requirements in the proceeding stages of model development and will still accomplish the stated goals of the study.

The system definition process involves the decomposition of the complex system into manageable parts such that a model can be formulated. Additionally, it serves as a process to familiarize the analyst with the entire system that is to be modeled. This will help to ensure a more complete model development process. It was decided in Chapter 3 that The Systems Modeling Language (SysML) would be used for this task. Each COA defined in the formulated problem would require a separate SysML model. Creating four SysML models to account for each COA is a time consuming process. This is why the problem was scoped to COA 2.

The SysML process begins with a requirements definition of the HA/DR operation. The

top level requirements are modeled with a block definition diagram and are shown in Figure 118. This figure along with the proceeding SysML figures were reproduced from the text or diagrams located within the Alexander et al. masters thesis [7]. The requirements definition is further decomposed in Figures 147 through 152 located in Appendix B.1.

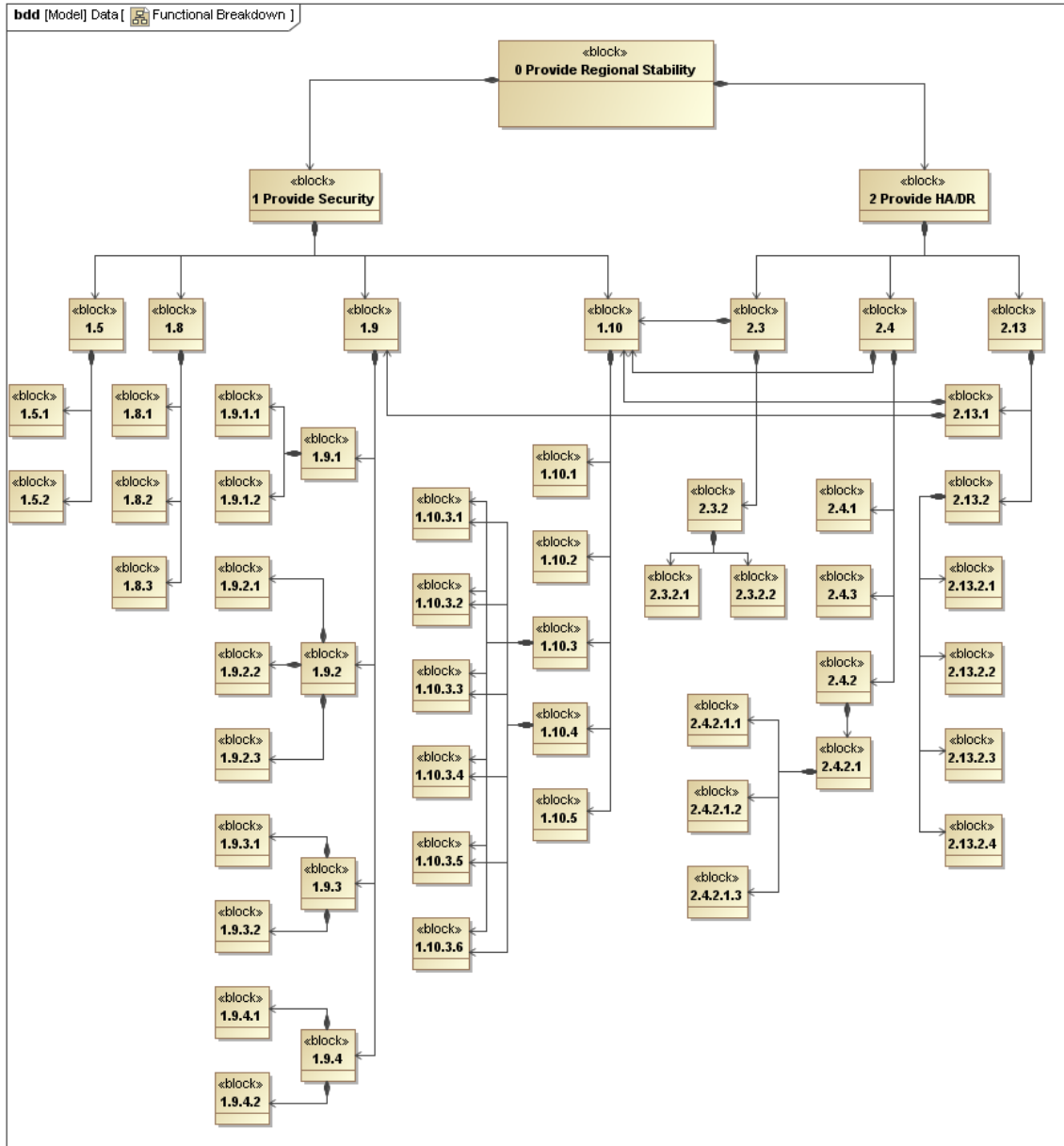


**Figure 118:** Requirements using Block Definition Diagram

The functional hierarchy of the HA/DR operation is modeled using a block definition diagram shown in Figure 119. In order to conserve space the function names were omitted

from the SysML diagram. The list of the functions and their decomposition are presented in full within Appendix B.2. It can be seen that the two general goals of the HA/DR operation represent the major branch in the functional hierarchy. The first function is defined as providing regional stability. This is accomplished by providing security and providing aid and other services to the affected people. The providing security function strongly aligns with the question on the effect of aid on the population? The providing HA/DR function strongly aligns with the question on how well is aid being distributed in terms of population reached and amount of material distributed? After this functional bifurcation little interaction occurs. The exception is the reliance of some 'Provide HA/DR' sub-functions on the functions of 'Provide Command and Control' and 'Operate Sea Base Connector', functions 1.9 and 1.10, respectively.





**Figure 119:** Functional Hierarchy using Block Definition Diagram

Next the system definition is modeled using a block definition diagram shown in Figure 120. Numerous system are defined. The first system defined is Command and Control located in the upper left. Tracking along the left side of the figure, the Logistic Site is defined as a general system type for which the forward logistic site (FLS) and the forward logistic satellite site (FLSS) are generalizations. The Amphibious Ships is defined as a general system type for which the LHD, LPD, and LSD are generalizations. These systems

make up the seabase. The Surface Connectors is defined as a general system type for which the LCAC and LCU are generalizations. The Air Connectors is defined as a general system type for which the MH-53E, S-60B, and MV-22 are generalizations. Two other systems defined are the Civilian and the Security Personnel located in the upper right of the figure. Finally, along the bottom of the figure are objects that are used within the flows between the defined systems.

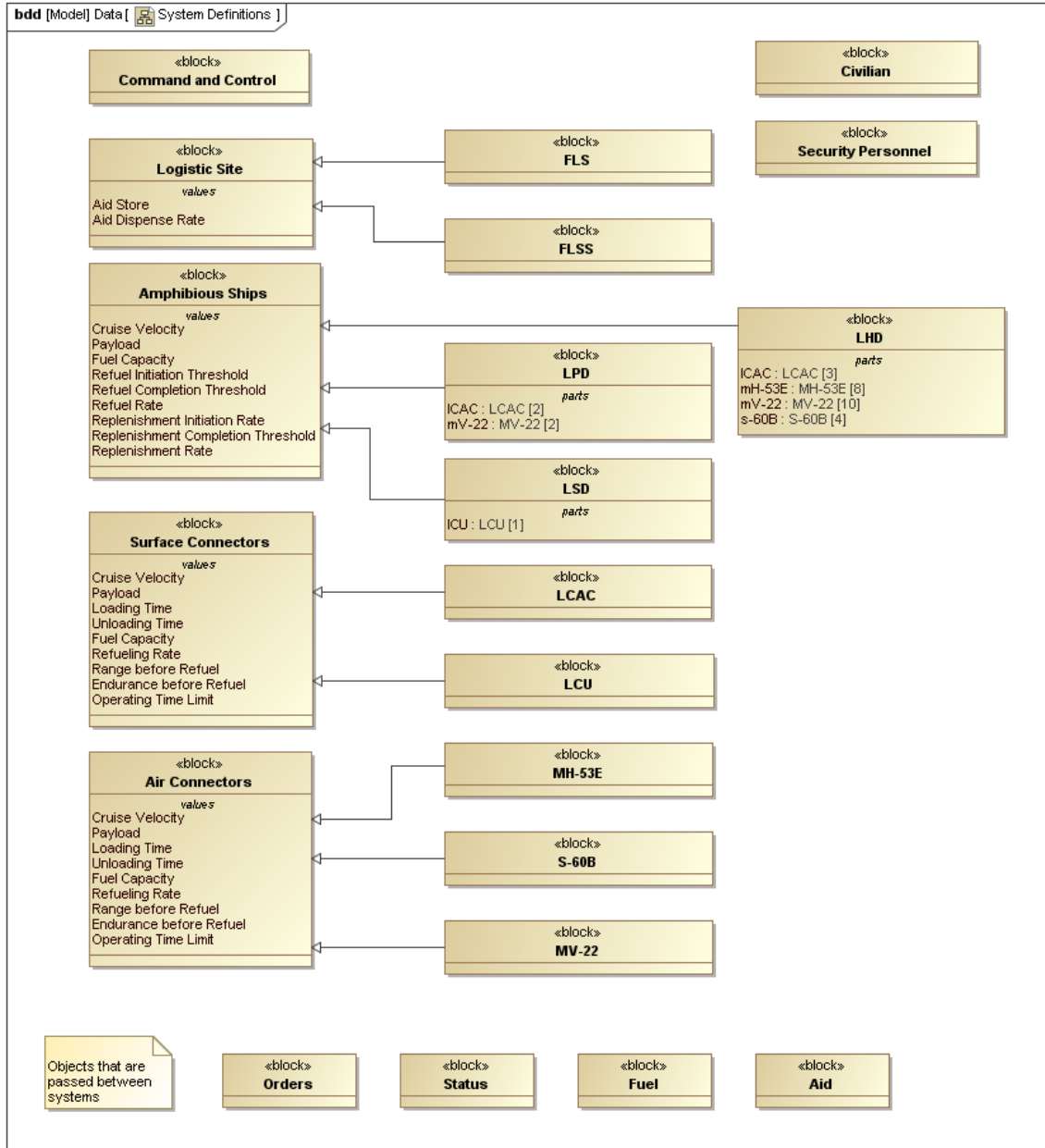


Figure 120: System Definitions using Block Definition Diagram

Next the system decomposition is modeled using a block definition diagram shown in Figure 121. A new block is defined to describe the entire HA/DR system. The HA/DR system contains one Command and Control, LHD, LPD, and LSD. It is noted that the Command and Control is physically located within the LHD. The HA/DR system also contains three FLS and 41 FLSS. Also contained within the HA/DR system are Civilians, who the system is designed for, and Security Personnel, who help maintain peaceful operations. As can be seen the LHD is equipped with three LCACs, eight MH-53Es, ten MV-22s, and four S-60Bs. The LPD is equipped with two LCACs and two MV-22s. Finally, the LSD is equipped with one LCU and no air connectors.

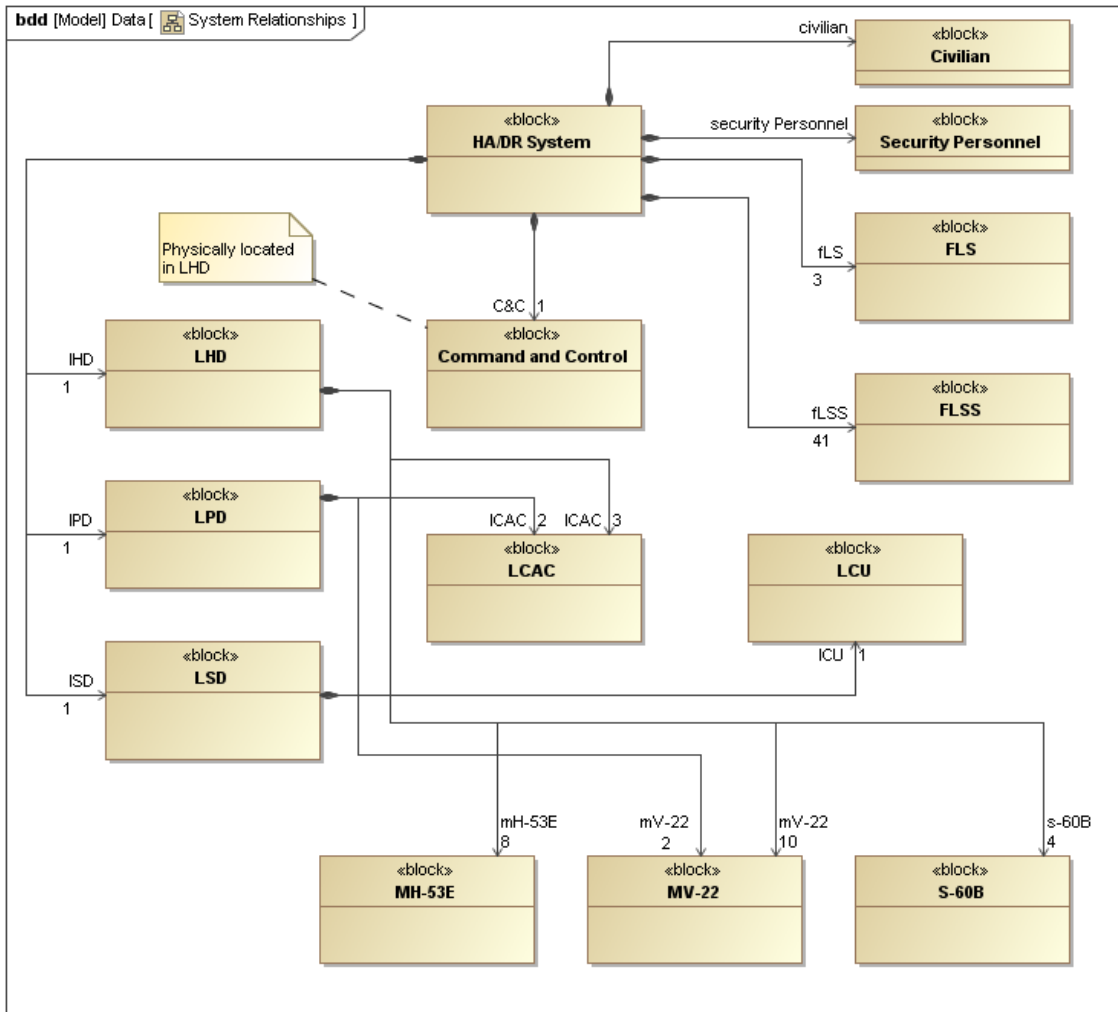
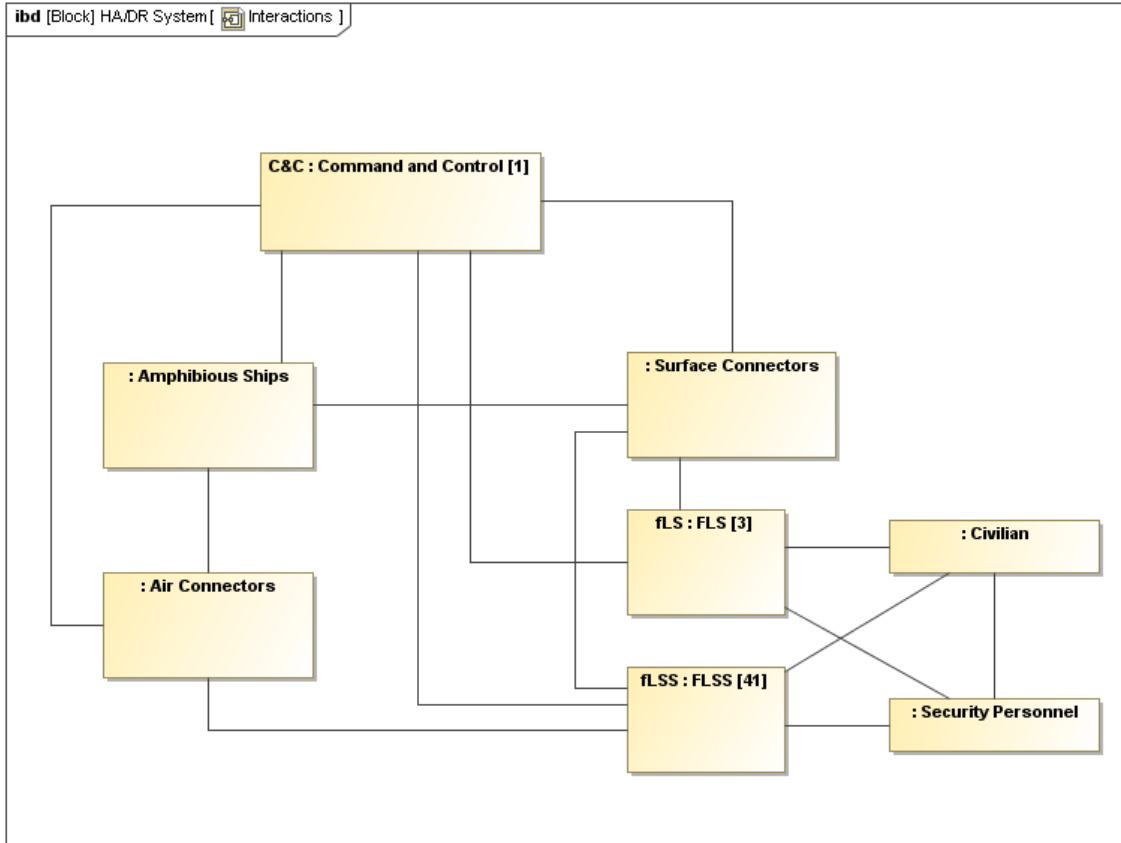


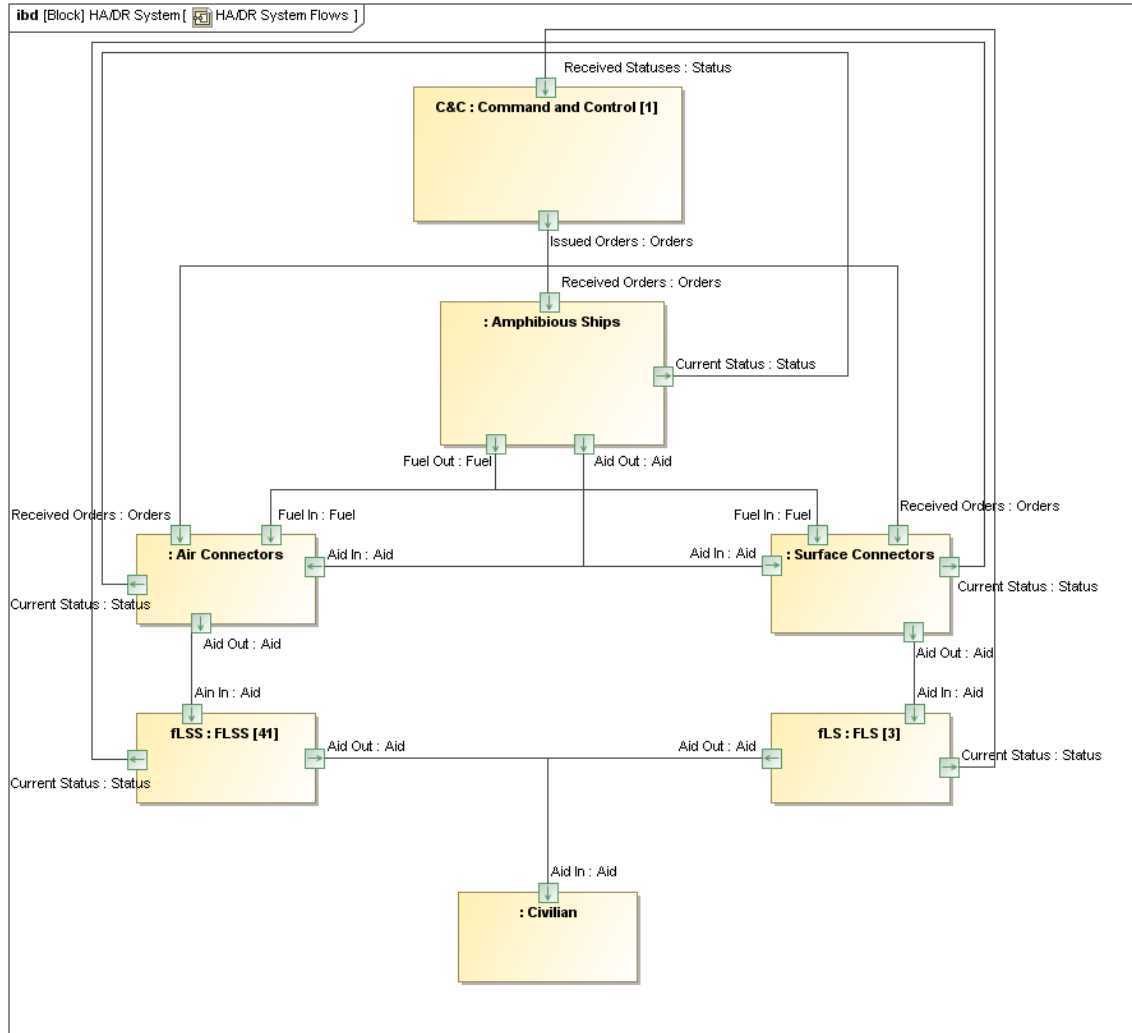
Figure 121: System Decomposition using Block Definition Diagram

Next the system interactions are modeled using an internal block diagram shown in Figure 122. Each connector indicates that some interaction takes place. The Command and Control system receives status updates from the Amphibious Ships, Air Connectors, Surface Connectors, FLS, and FLSS. The Command and Control system issues orders to the Amphibious Ships, Air Connectors, and Surface Connectors. The Amphibious Ships provide resources, i.e. aid material, to the Surface Connectors and the Air Connectors. The Surface Connectors provide aid to the FLS. The Air Connectors provide aid to the FLSS. Civilians receive aid from the FLS and the FLSS. Security Personnel is present at the FLS and FLSS. Finally, the Civilian and Security Personnel have a complex interaction. Civilians are less likely to commit criminal activities against other Civilians if Security Personnel is present. Civilians are also bother by the presence of the foreign Security Personnel which reduces their satisfaction. Civilians may commit criminal activities against the Security Personnel.



**Figure 122:** System Interactions using Internal Block Diagram for COA 2

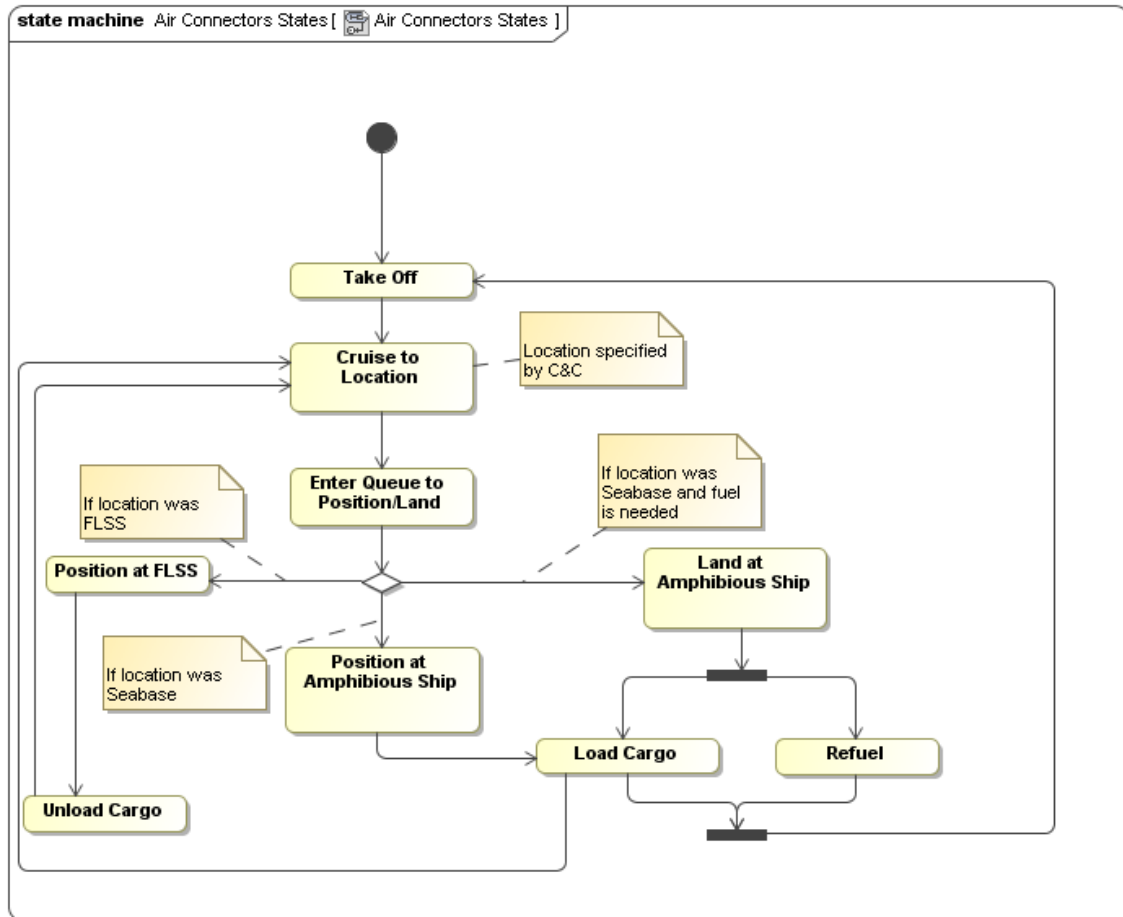
Next, the flow of materials between the systems is modeled with an internal block diagram shown in Figure 123. This is a more detailed description than was shown in Figure 122; however, the interaction between the Civilian and Security Personnel is not present because no material is transferred between the two systems. As can be seen the Command and Control system receives status updates from the Amphibious Ships, Air Connectors, Surface Connectors, FLS, and FLSS. The Command and Control system issues orders to the Amphibious Ships, Air Connectors, and Surface Connectors. The Amphibious Ships provide Fuel and Aid to the Surface Connectors and the Air Connectors, who in turn deliver Aid to the FLS and FLSS, respectively. The Aid from the logistic sites is transferred to the Civilians.



**Figure 123:** System Flows using Internal Block Diagram for COA 2

The Air Connector operations are described with a state machine diagram shown in Figure 124. The operations begin with the Air Connectors sitting on the Amphibious Ships; therefore, the first state that the Air Connector would enter would be Take Off. From the Take Off state the Air Connector would enter the Cruise to Location state. The location would be defined by the Command and Control system. Once the connector arrives to the location it must enter a queue to land or position itself to drop or pick up cargo, for there may be other Air Connectors in the way. This state is the Enter Queue to Position/Land state. At this point, depending on the location, the connector will either enter the state to Position at FLSS, Position at Amphibious Ship, or Land at Amphibious Ship. If the

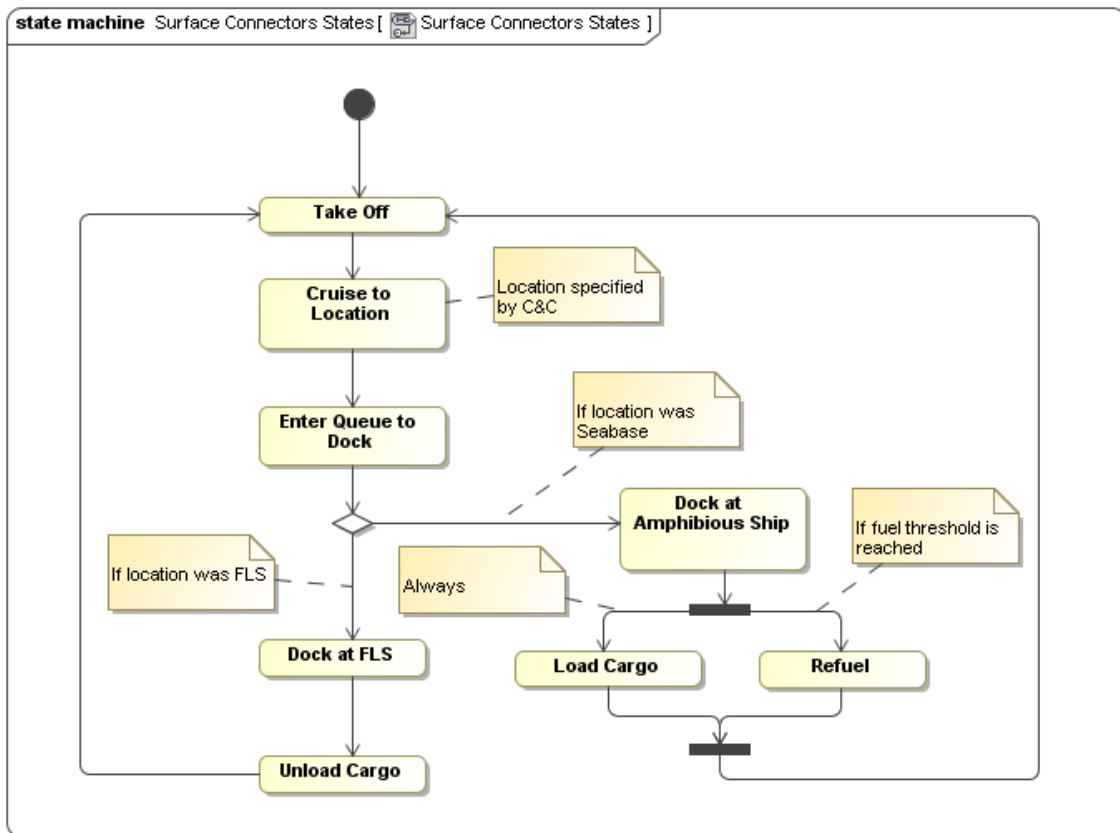
connector positions at the FLSS it will then enter the state Unload Cargo. This is followed by the state Cruise to Location. If the connector positions at the Amphibious Ship it will then enter the state Load Cargo followed by the state Cruise to Location. This location is most likely a FLSS. Finally, if the connector lands at an amphibious ship then it will enter both states of Load Cargo and Refuel. Once both states are completed then it will enter the state Take Off.



**Figure 124:** Air Connector States using State Machine Diagram for COA 2

The Surface Connector operations are described with a state machine diagram shown in Figure 125. The state machine diagram for the Air Connectors and the Surface Connectors are very similar given that their operations are very similar on this level of abstraction. The operations begin with the Surface Connectors docked in the Amphibious Ships; therefore, the first state that the Surface Connector would enter would be Take Off. From the Take

Off state the Surface Connector would enter the Cruise to Location state. The location would be defined by the Command and Control system. Once the connector arrives to the location it must enter a queue to dock, for there may be no room to doc. This state is the Enter Queue to Dock state. At this point, depending on the location, the connector will either enter the state to Dock at FLS or Dock at Amphibious Ship. Following the state Land at FLSS, the connector would enter the state Unload Cargo. Following the state Land at Amphibious Ship, the connector would enter the state Load Cargo and potentially Refuel, if the fuel threshold is reached. Once the connector exits these states it returns to the Take Off state.

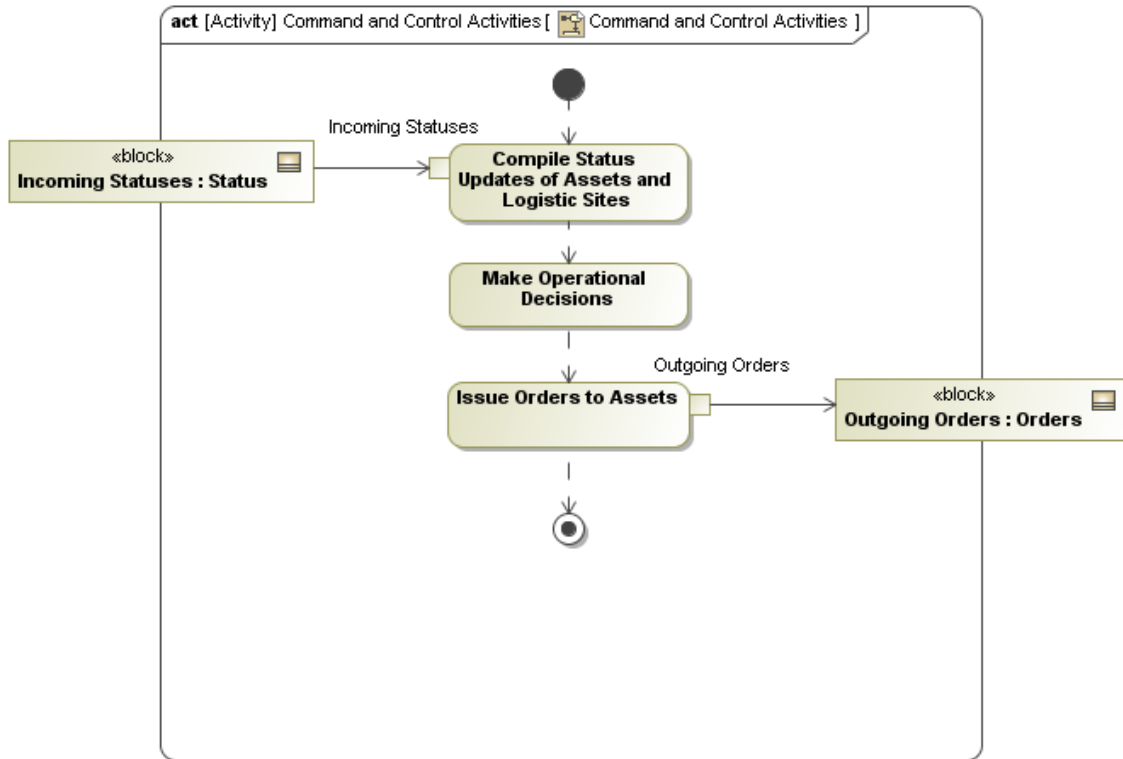


**Figure 125:** Surface Connector States using State Machine Diagram for COA 2

The activities of the Command and Control system are model with an activity diagram shown in Figure 126. The first activity of the Command and Control system is to compile the statuses of the military assets and the logistic sites. This information is provided by a

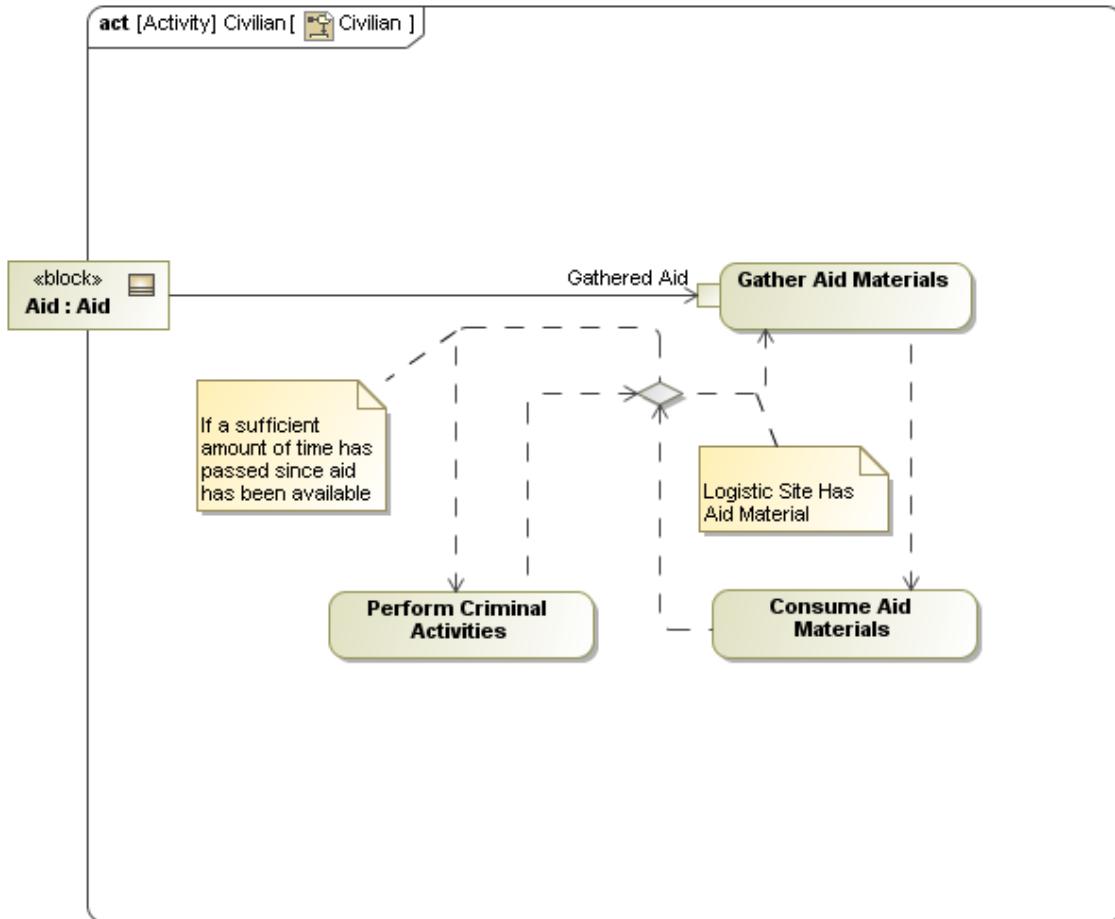


flow of incoming statuses. Once the statuses are compiled an operational decision is made. This is primarily a decision on the direction of the connectors, e.g. sending MH-53E(1) to FLSS(1). Once the decision is made the orders are issued. The information is sent out by a flow Orders.



**Figure 126:** Command and Control Activities using an Activity Diagram

Finally, the activities of the Civilian is modeled using an activity diagram shown in Figure 127. There is no natural beginning for the civilian activities; therefore, no start and end is given. The Civilian will perform the activity of Gather Aid Material. The flow of resources needed for this activity is Aid. The following activity is Consume Aid Material. The following activity can result in either gathering more aid materials or resulting to criminal activities. This is dependant upon the time that the civilian as gone without aid. Once the civilian performs criminal activities they may either return to gathering aid from the logistic sites or return to criminal activities. The determination of this is based on the presence of aid at the logistic sites and the presence of Security Personnel.



**Figure 127:** Civilian Activities using an Activity Diagram

The parameters for the military assets are shown in the tables below and are reproduced from Alexander et al. [7]. The parameter values for the amphibious ships that make up the seabase are shown in Table 47. The parameter values for the connectors are shown in Table 48.

**Table 47:** Craft Specifications - Seabase

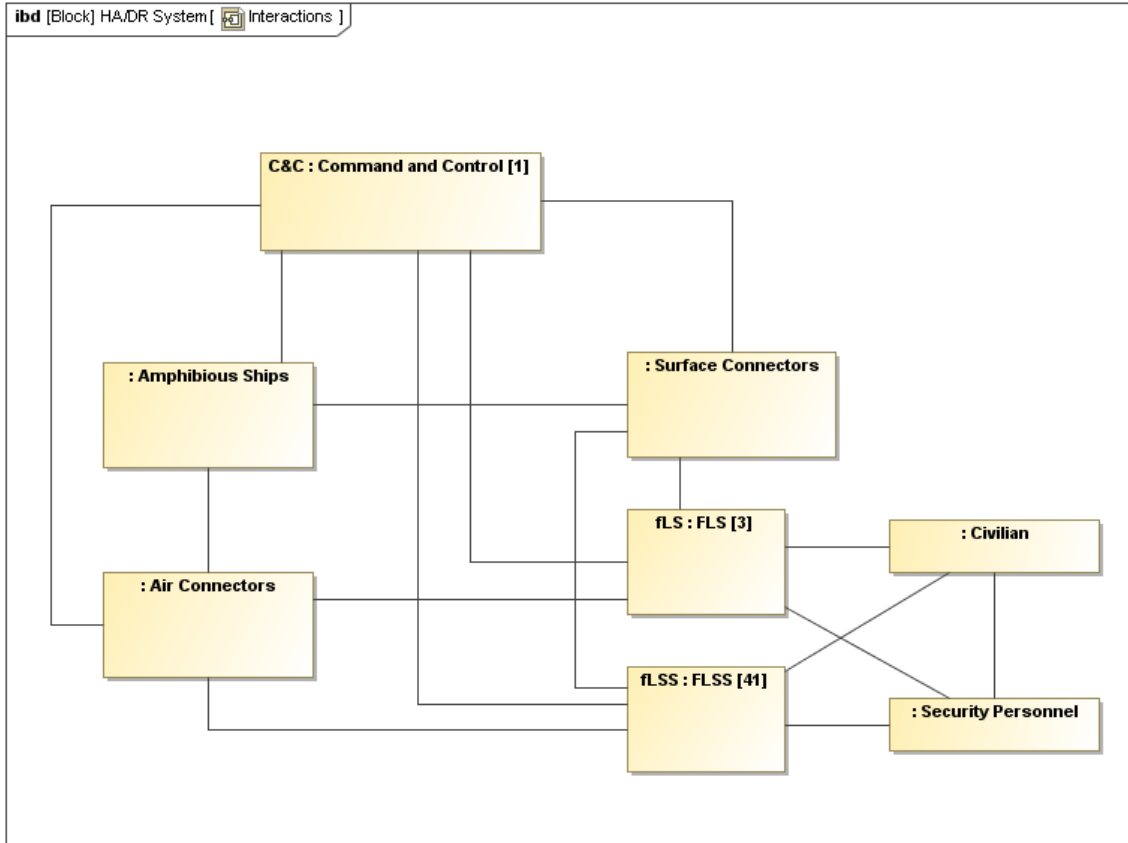
Parameter Value	LHD	LPD	LSD
Cruise Velocity (kts)	17	17	17
Payload (U.S. tons)	2711.69	2535.32	1234.59
Fuel Capacity (Gal)	585000	314160	50000
Refuel Initiation Threshold (%)	30	30	30
Refuel Completion Threshold (%)	95	95	95
Refuel Rate (Gal/Hr)	252000	252000	252000
Replenishment Initiation Threshold (%)	0	0	0
Replenishment Completion Threshold (%)	100	100	100
Replenishment Rate (tons/Hr)	210	210	210

**Table 48: Craft Specifications - Connectors**

Parameter Value	LCAC	LCU	MH-53E	MV-22	S-60B
Cruise Velocity (kts)	40	8	150	215	160
Payload (U.S. tons)	75	170	18	17.5	3
Fuel Capacity (Gal)	5000	N/A	2277	1448	590
Refuel Initiation Threshold (%)	30	30	30	30	30
Refuel Completion Threshold (%)	100	100	100	100	100
Refuel Rate (Gal/Hr)	60000	N/A	60000	60000	60000
Replenishment Initiation Threshold (%)	0	0	0	0	0
Replenishment Completion Threshold (%)	100	100	100	100	100
Load Time (Hr)	2	4.5	1/60	2/60	1/60
Unload Time (Hr)	2	4.5	1/60	2/60	1/60
Range before Refuel (Nmi)	200	1200	700	950	450
Endurance before Refuel (Hr)	5	150	4.67	4.4	2.8
Operating Time Limit (Hr/Day)	16	16	8	8	8

#### 6.4.2.1 SysML COA 4 Comparison

COA 4 will be used to demonstrate how the SysML model changes for different system definitions. Only three of the SysML diagrams produced changes. The first is the system interaction internal block diagram shown in Figure 128. The only change is that the Air Connectors now also interact with the FLS. The second is the system flow internal block diagram shown in Figure 129. The change is that the flow of Aid into the Air Connectors comes from the FLS instead of the Amphibious Ships. Finally, the Air Connector state machine diagram changes which is shown in Figure 130. Several changes are made in this state diagram. First, the Air Connector can travel to the FLS, so the state Position at FLS was added. This state is followed by the Load Cargo state. Another change is that the states following the Land at Amphibious Ship only involves the Refuel state.



**Figure 128:** System Interactions using Internal Block Diagram for COA 4

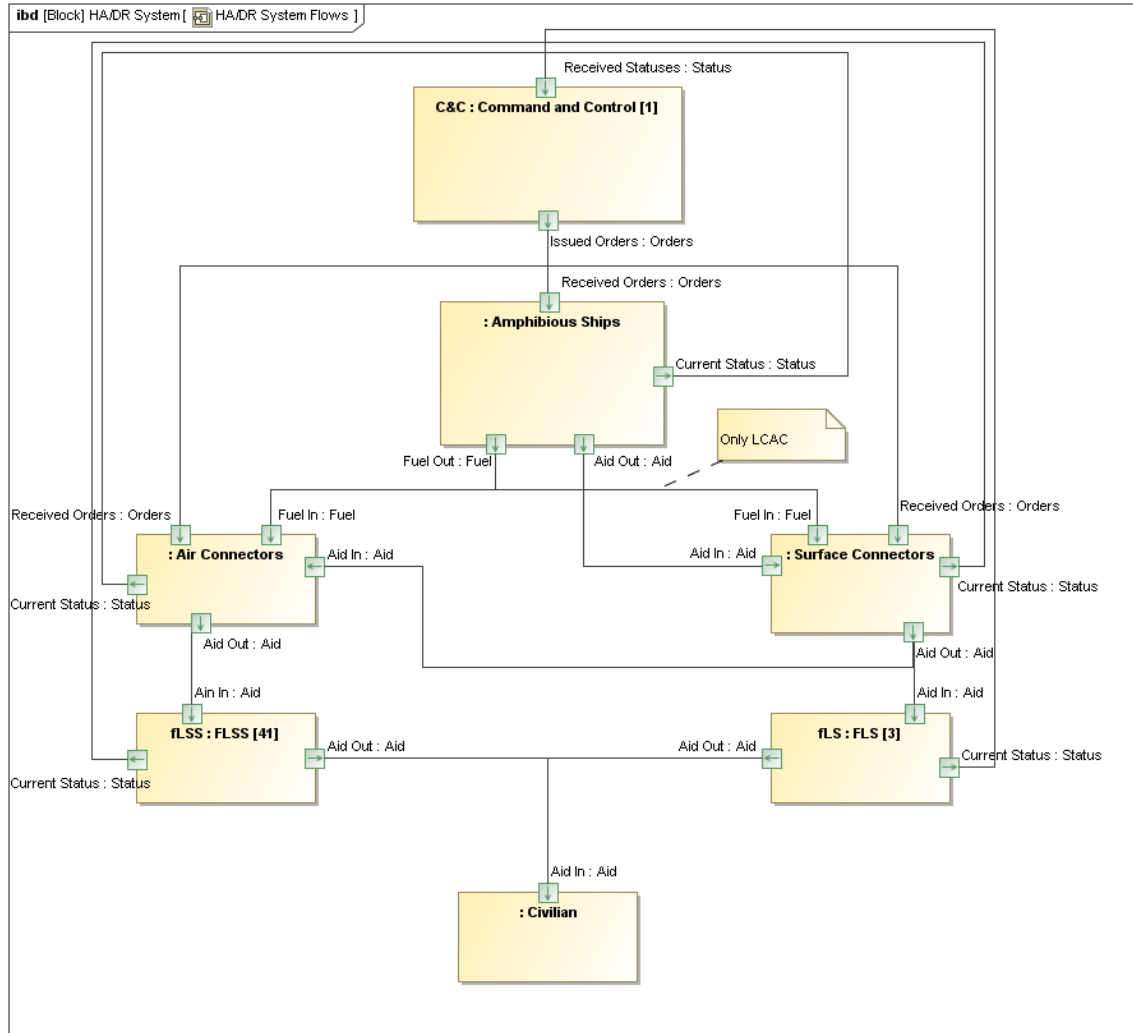


Figure 129: System Flows using Internal Block Diagram for COA 4

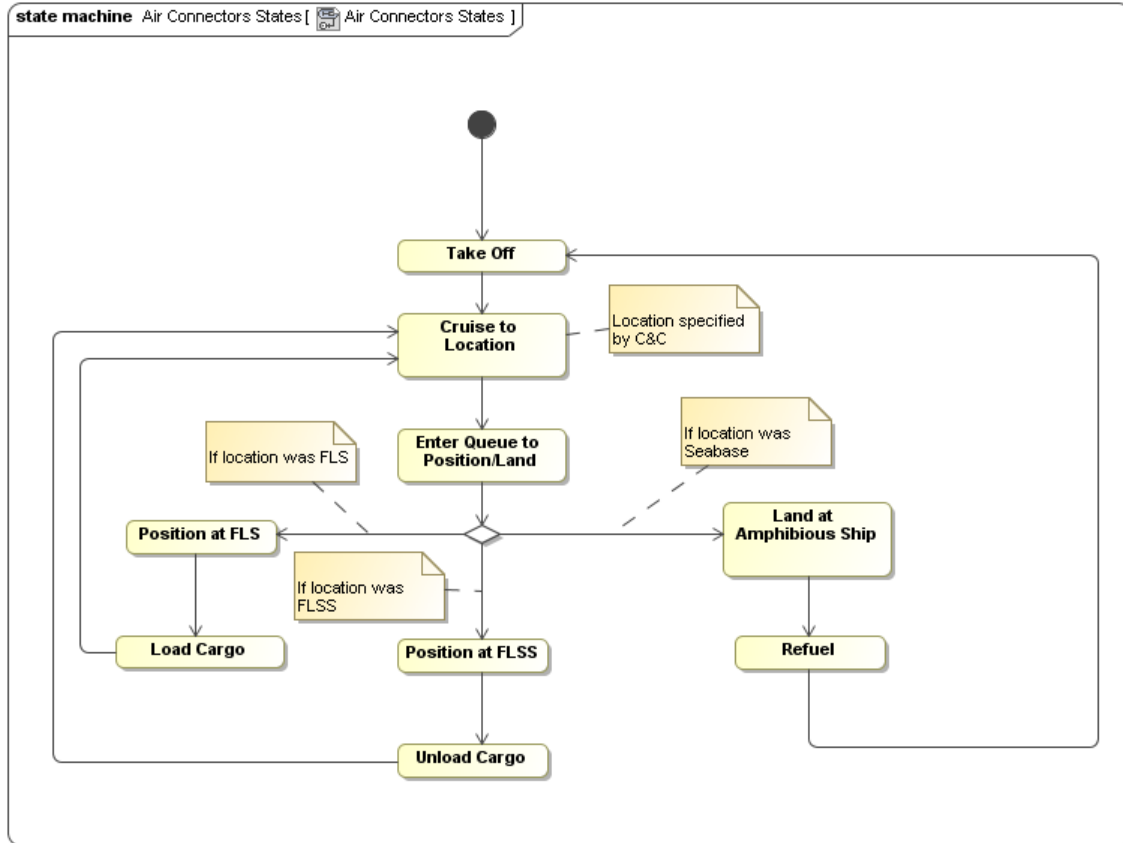


Figure 130: Air Connector States using State Machine Diagram for COA 4

### 6.5 Conceptual Model Development

The development of the conceptual model will follow the process developed in Section 4.1. The first step is to define the decomposition of the impacts. This decomposition is developed from the questions presented in the formulated model, the effects decomposition defined in the objective definition, and the various views shown in the system definition section. The following section will decompose the impacts forming the impact matrix. Once the impact matrix is developed, weightings are applied to the impacts using a relative weightings scheme. This is accomplished in Section 6.5.2. Based on the impact weightings, the importance of the impact relationships is calculated. These results are shown in Appendix B.3. Finally, given the importance ratings of the impact relationships, a method for representing the relationship within the model is selected for each impact relationship using a morphological matrix. This is accomplished in Section 6.5.3.

### 6.5.1 Decomposition of Impacts

The measures of effects that were developed by Alexander et al. [7] and are shown in Table 46 are used to define a new hierarchy of impact measures. This new hierarchy of measures is defined within two categories based on the two objectives. The first objective, distribute aid to the population, is defined by two effect level impact measures. These measures are the aid flow rate to the population and the population that is reached with the aid. If compared to Table 46 both sets of measures capture the time to deliver the aid and the coverage of the aid. Given the average flow rate and the total amount of aid, the time can be calculated. The second objective, provide security for peaceful HA/DR operations, is defined by the effect level impact measure ‘Criminal Events’. This hierarchy of measures is summarized in Table 49.

The performance level of impact measures contains every performance variable of every asset within the defined system of systems. There are a total of 12 assets excluding command and control, see Figure 120. There are a total of 90 variables listed with in Tables 47 and 48, which does not include variables form logistic sites, civilians, nor security personnel. The mapping of this large number of performance level impacts onto the effect level impacts is a daunting task and would most likely be rife with errors. For this reason, the decomposition of impacts must be broken into segments.

**Table 49:** Decomposition of Impact Measures

$Obj_1$	Distribute aid to the population
Impact $_1^E$	Average throughput rate of aid from Seabase to Civilians (ton/day)
Impact $_2^E$	Population reached with aid (millions)
$Obj_2$	Provide Security for peaceful HA/DR operations
Impact $_3^E$	Criminal events (hundreds)



#### 6.5.1.1 $Impact_1^E$ : Average Throughput Rate of Aid from Seabase to Civilians

The decomposition of the impacts for  $Impact_1^E$  is shown in Figures 131 through 134. The decomposition of the impacts began with the analysis of the system flows shown in Figure 123. The flow rate of aid to the civilians is dependent upon the flow rate of aid out of the FLS and the FLSS and to the civilians. This is shown in each of the figures. Figure 131 shows the decomposition of the FLSS impact, which based on Figure 123 is impacted by the flow rates from the air connectors. Additionally, the FLSS flow rate is impacted by the FLSS operations, represented as a category of variables. The snip single rectangle represents a category of variables and does not contain a measurable value, as opposed to the rounded triangles that represent a measurable value. For example, the flow rate of aid from the FLSS to the Civilians is measurable and has the units tons per hour. In the following section the relative impact will be estimated between the FLSS Operations as a whole and the flow rate of aid to the FLSS from the air connectors. This is done to simplify the problem.

The FLSS Operations is decomposed into the storage capacity of each FLSS and the wait time that a civilian would experience at an FLSS. The larger the storage capacity of the FLSS the fewer trips each air connector may have to make to each FLSS and thus the less likely the FLSS would run out of aid. The longer the wait time for the civilian to acquire aid the lower the flow rate of aid to the civilian would be. This measure impacted by the number FLSS facilities, the process rate of the FLSS, and the operating time. The increase of each of these impacts would decrease the wait time at the FLSS.

The flow rate of aid from the air connectors to the FLSS locations is based on the flow rate of aid from the three air connectors to the FLSS locations in addition to the number of drop-off locations for the aid at each FLSS location. An important decision was made on the location of the 'FLSS facilities' impact within the decomposition. The impact 'FLSS facilities' is an obvious impact on the FLSS flow of aid to civilians. The more FLSS facilities the greater the operational throughput of aid. If the number of locations is doubled then the potential operational throughput is doubled assuming all FLSS operation variables are constant. However, the impact 'FLSS facilities' also has an impact on the flow rate of aid

from the aid connectors to the FLSS locations. It is possible that queues develop at the FLSS to drop of aid or if air connector throughput outpaces the FLSS throughput and the FLSS storage capability then the air connector throughput would be limited. Increasing the number of FLSS locations would reduce this possibility. The inclusion of this impact on multiple levels of the decomposition would create issues in the calculation of its impact in the following section. For this reason, 'FLSS facilities' remains at the higher level and its impact must be considered there.

The flow rates of aid for the MH-53E, S-60B, and MV-22 air connectors are decomposed in Figures 131, 132, and 133, respectively. The air connectors are impacted by three categories of variables: FLSS Connection, Seabase Connection, and Lift Parameters. The FLSS Connection addresses the variables that impact the air connector state of unloading the cargo at the FLSS. The only variable that is included in this category is the time to unload the cargo. The number of drop-off locations per FLSS would also impact this state, but since each air connector contains this variable in the same form, it was moved to the level above. The Seabase Connection addresses the variables that impact the air connector state of either Position at Amphibious Ship or Land at Amphibious Ship. The variables that affect these states are the time to load cargo and the refuel rate if the connector had to land to refuel. Additionally, the class of variables, Seabase Operations, impact the Seabase Connection. Seabase Operations contain variables of the seabase that may affect either the Position at Amphibious Ship or Land at Amphibious Ship air connector states. The final category of variables that impact the connector flow rate is the Lift Parameters. The Lift Parameters include the remaining variables of the air connector that impact the air connector flow rate. The variable travel time if decomposed into its impacting variables: travel distance, cruise velocity, and the category of weather variables.

The flow rate of aid from the FLS impacts were decomposed in a similar fashion as the FLSS impact decomposition and is shown in Figure 134. The major differences are the connectors and a few variables. Otherwise, the decompositions are similar.

Each of these performance level impact variables, i.e. non-flow rate measures, can be further decomposed into technical performance parameter level impact variables. In fact,

the decomposition can progress continuously to the individual parts of the assets and their material properties. This however is not a realistic approach. This level of decomposition will provide a solid framework for the following tasks of applying impact weightings and determining impact relationship importance. If it is found that a particular performance level impact metric has a high level of importance then it can then be decomposed further.

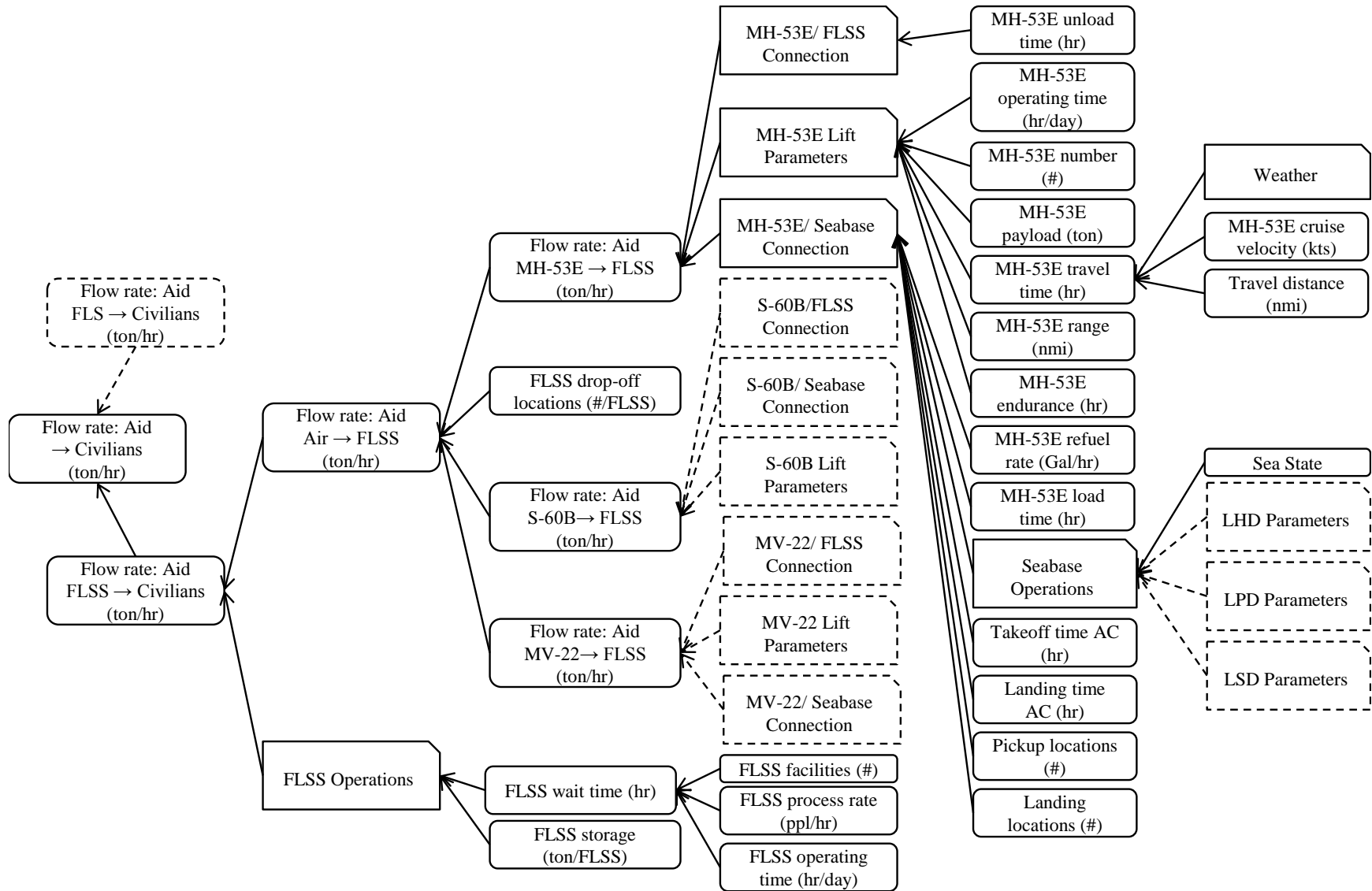


Figure 131: Decomposition of Impacts on Flow Rate of Aid to Civilians (MH-53E Connectors)

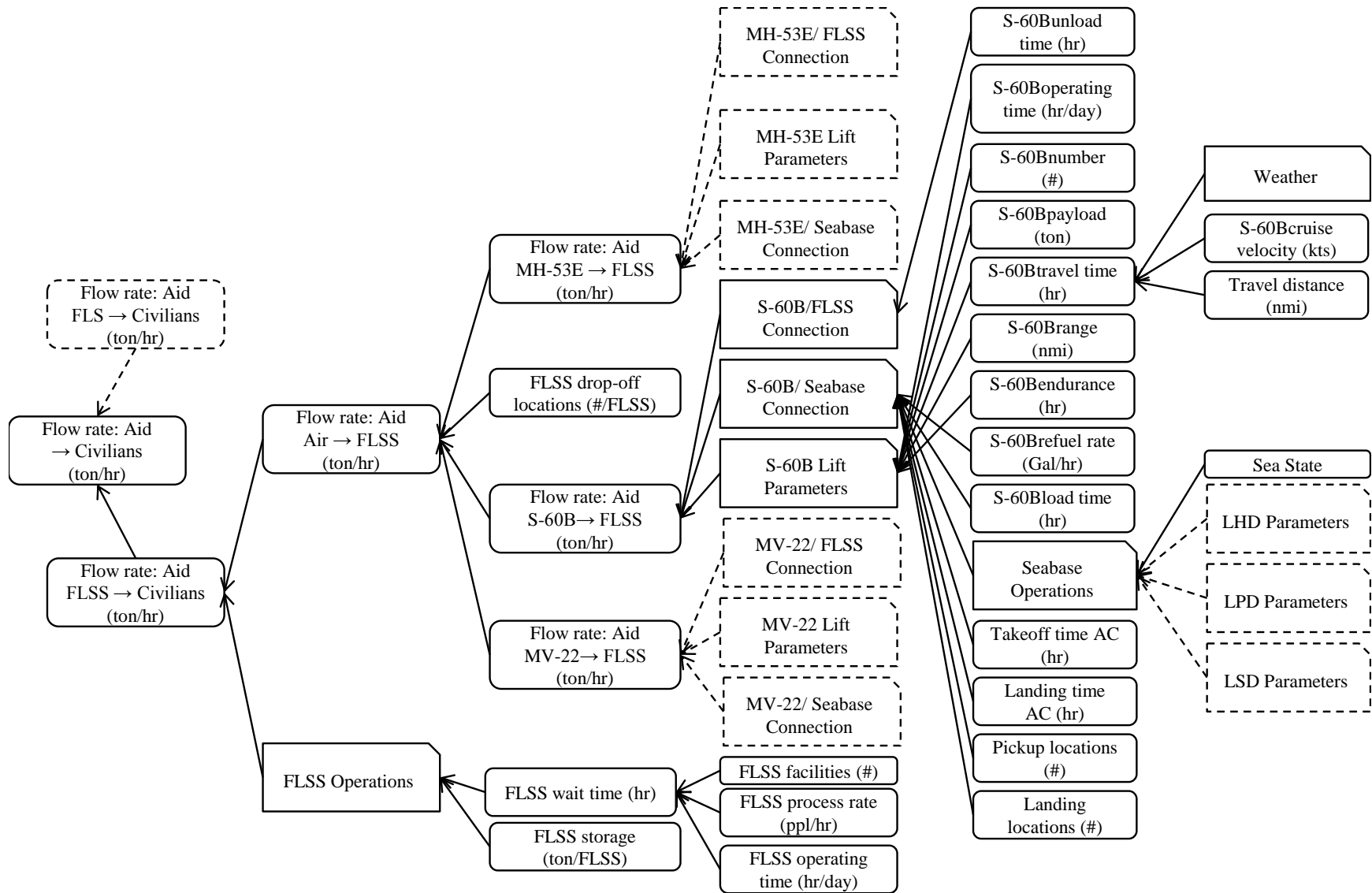


Figure 132: Decomposition of Impacts on Flow Rate of Aid to Civilians (S-60B Connectors)

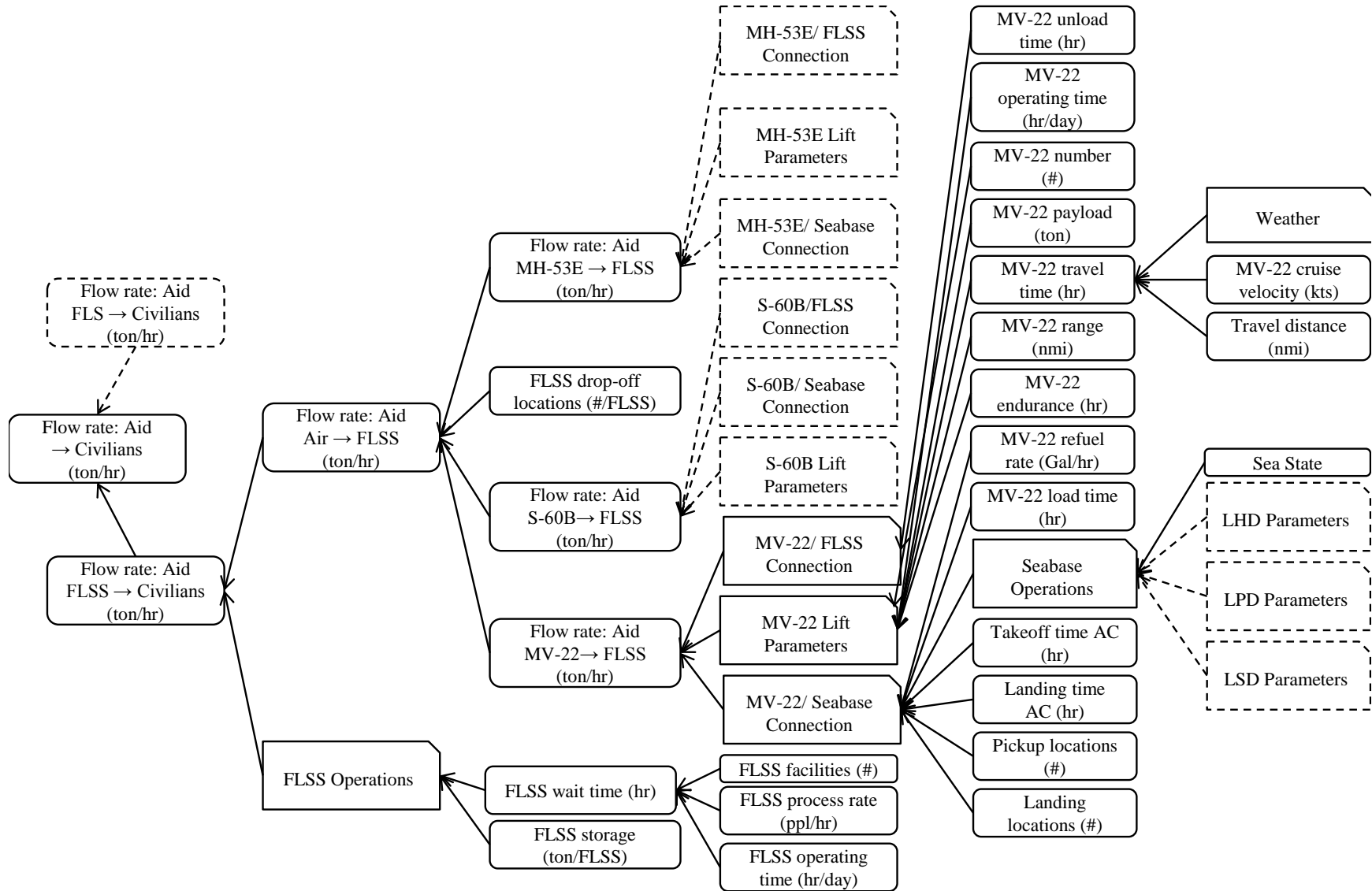


Figure 133: Decomposition of Impacts on Flow Rate of Aid to Civilians (MV-22 Connectors)

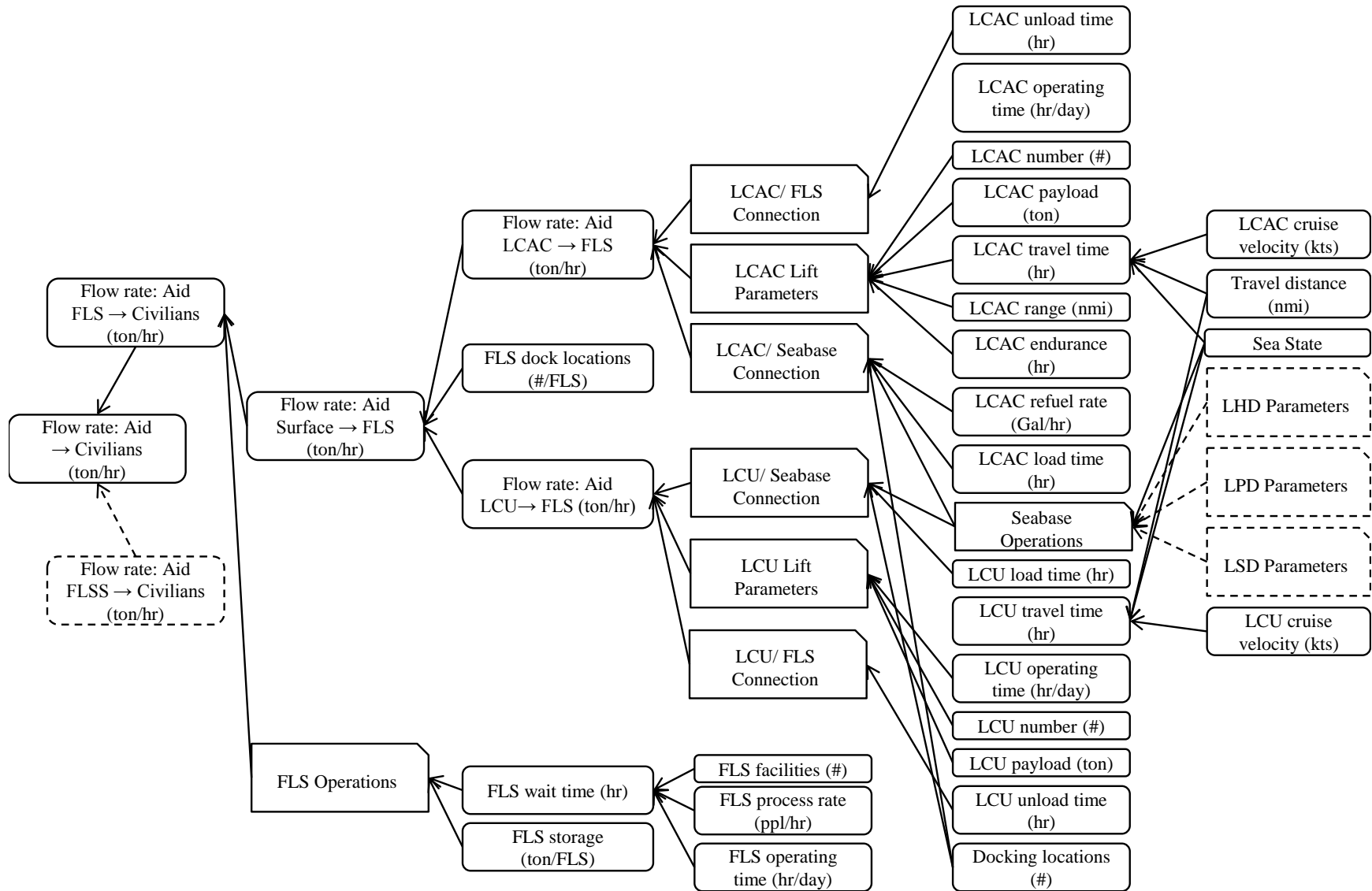


Figure 134: Decomposition of Impacts on Flow Rate of Aid to Civilians (Surface Connectors)

#### 6.5.1.2 $Impact_2^E$ : Population Reached with Aid

The decomposition of the impacts for  $Impact_2^E$  is shown in Figures 135 through 139. The decomposition of the impacts began with the decomposition of the measure ‘Population Reached’ into the population reached from the FLS and the population reached from the FLSS. The population reached by each of these measures is a function of the access of the logistic site to the civilian population and the total amount of aid delivered from the logistic site to the civilian population.

‘Access to (FLS/FLSS)’ is a category of variables that accounts for how easily the civilians can travel to the logistic site and acquire aid supplies. The impact variables are the frequency of visits to the logistic sites and time required to acquire aid. The frequency of visits to the logistic site from the civilian is a function of two impacts: the aid that is given to each visitor and the amount of aid that the civilian can transport home with them. The time required for a civilian to acquire aid is a function of the average time to travel to the logistic site and the wait time at the logistic site. The travel time is a function of the distance to a logistic facility, the number of facilities, and the civilian travel velocity. Since the average distance to a logistic site and the travel distance are so closely related, only the number of facilities is used as an impact on average travel time. Finally, the average wait time at a facility is a function of the impacts: facility process rate and facility operating hours.

The total amount of aid delivered from the logistic site to the civilian population is a function of the logistic site operations and the aid that is delivered to the logistic site. The logistic site operations is a category of variables that includes only the average wait time for a civilian to acquire aid at the site. This category serves the purpose of keeping the average wait time on the same level within the impact decomposition. If this were not done, then issues would arise when one calculates the impact relationship importance values.

The total amount of aid delivered to the logistic sites follows the same decomposition as was shown for  $Impact_1^E$  decomposition, where the sea connectors contribute to the FLS and the air connectors contribute to the FLSS. The primary difference is the effect measures in this decomposition are in the units of tons whereas the effect measures in the previous



decomposition are in units of tons/hour. This change does not affect the decomposition, but it does affect the impact weightings that will be applied in the next section. Notice that the 'Travel Time' and 'Seabase Operations' impact variables are the same as the ones used in the previous decomposition. These variables are decomposed further; however, the decomposition was not shown in the figures below due to a limitation of space.

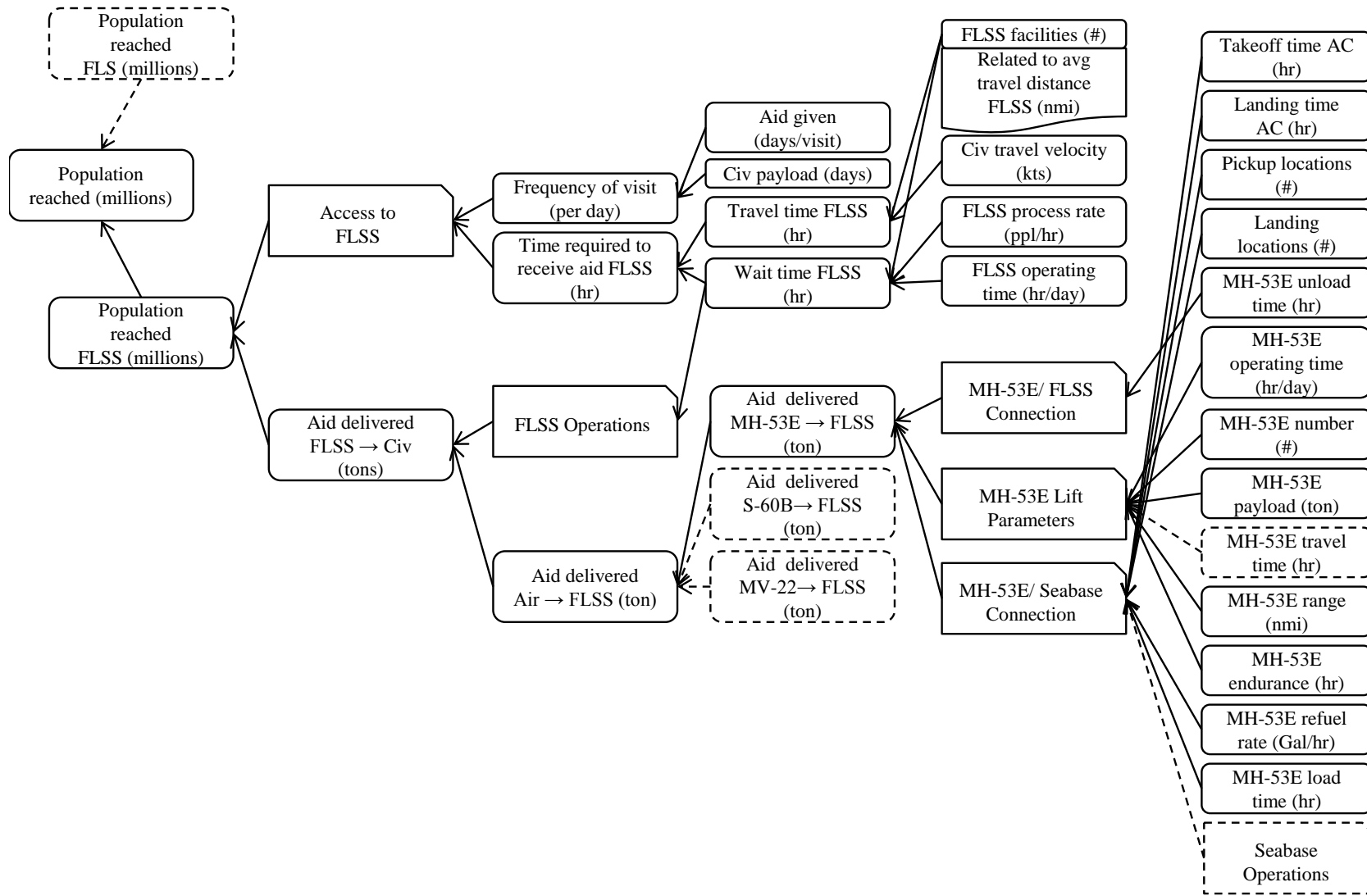


Figure 135: Decomposition of Impacts on Population Reached (MH-53E Connectors)

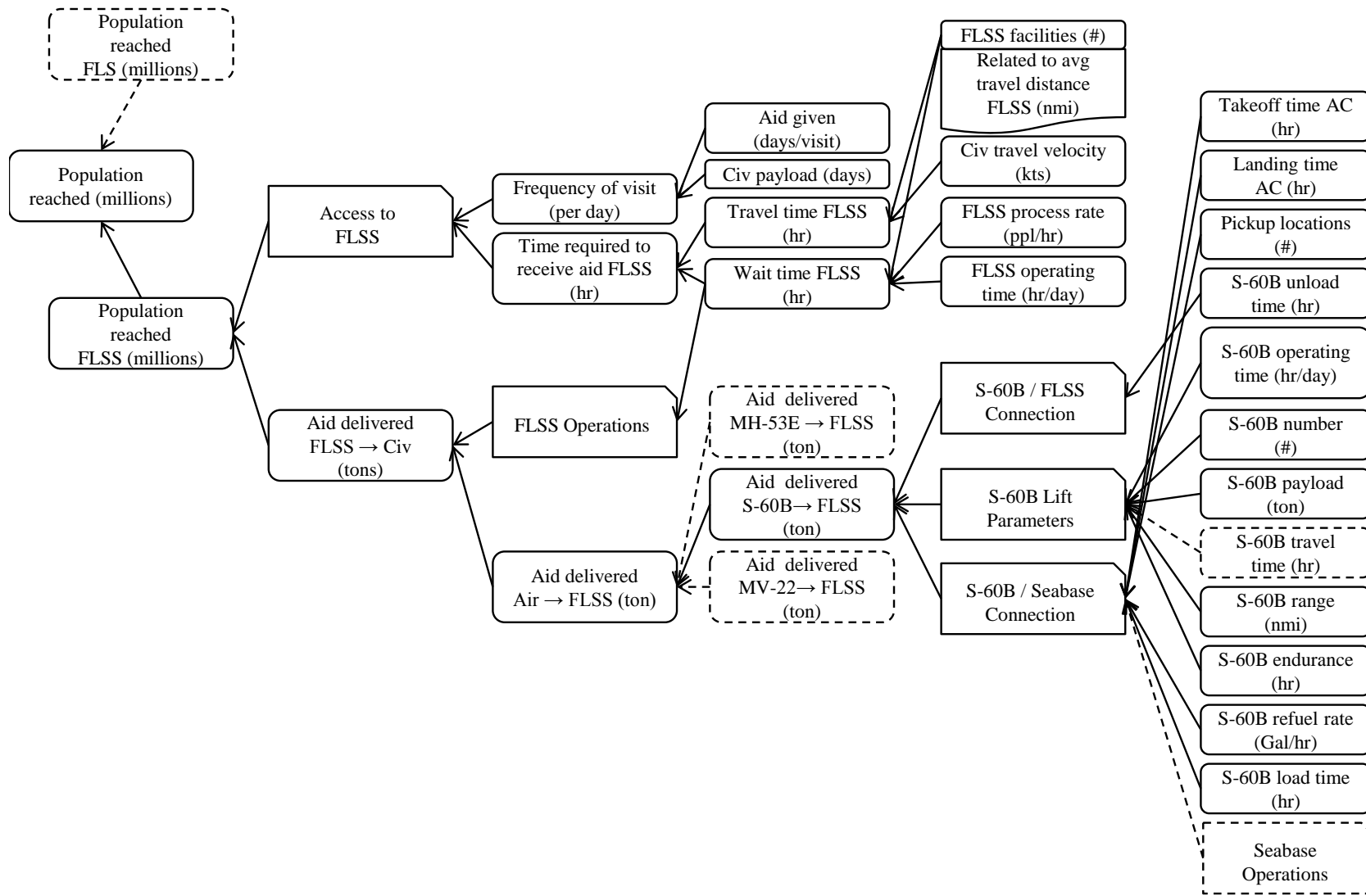


Figure 136: Decomposition of Impacts on Population Reached (S-60B Connectors)

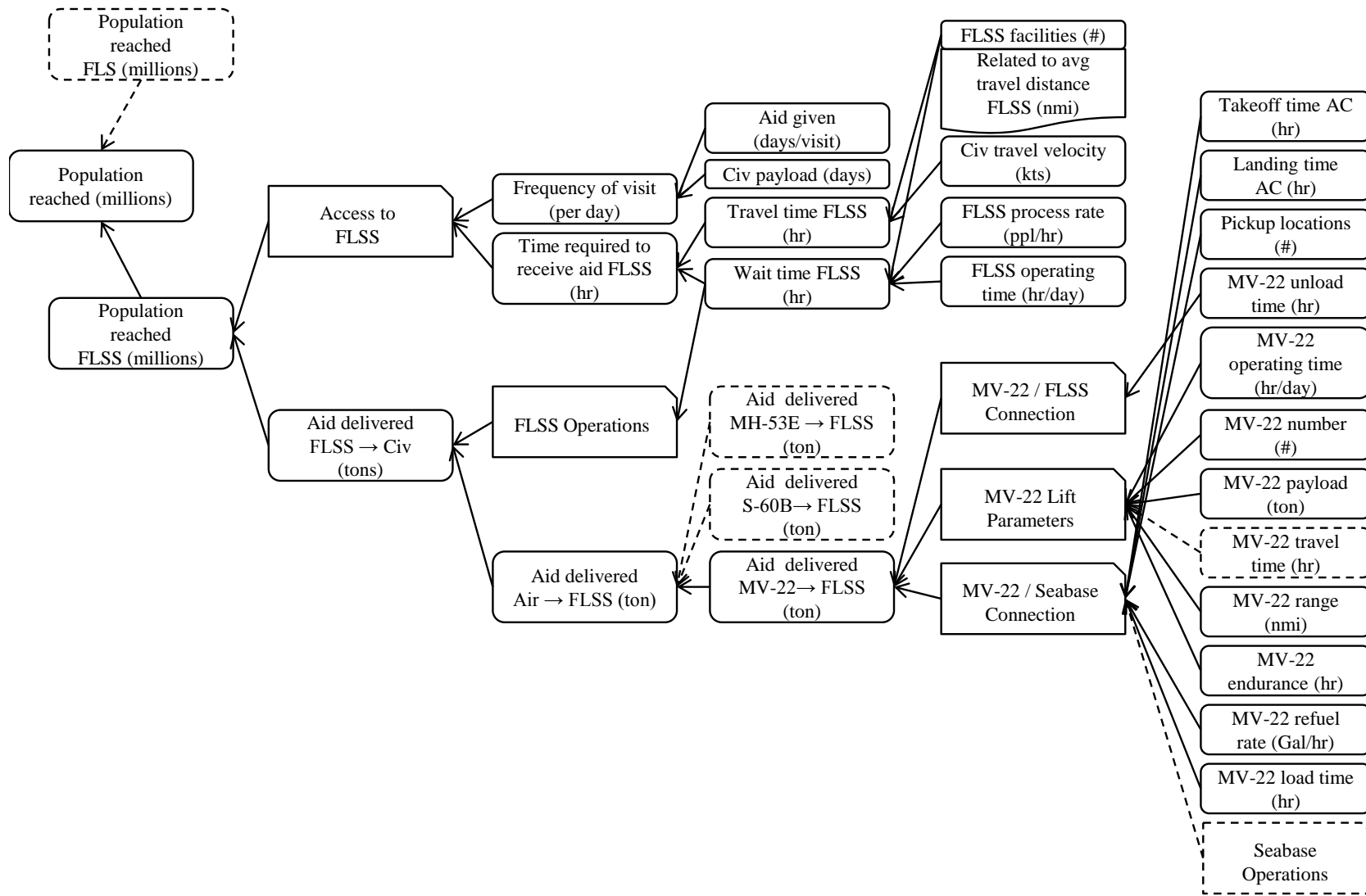


Figure 137: Decomposition of Impacts on Population Reached (MV-22 Connectors)

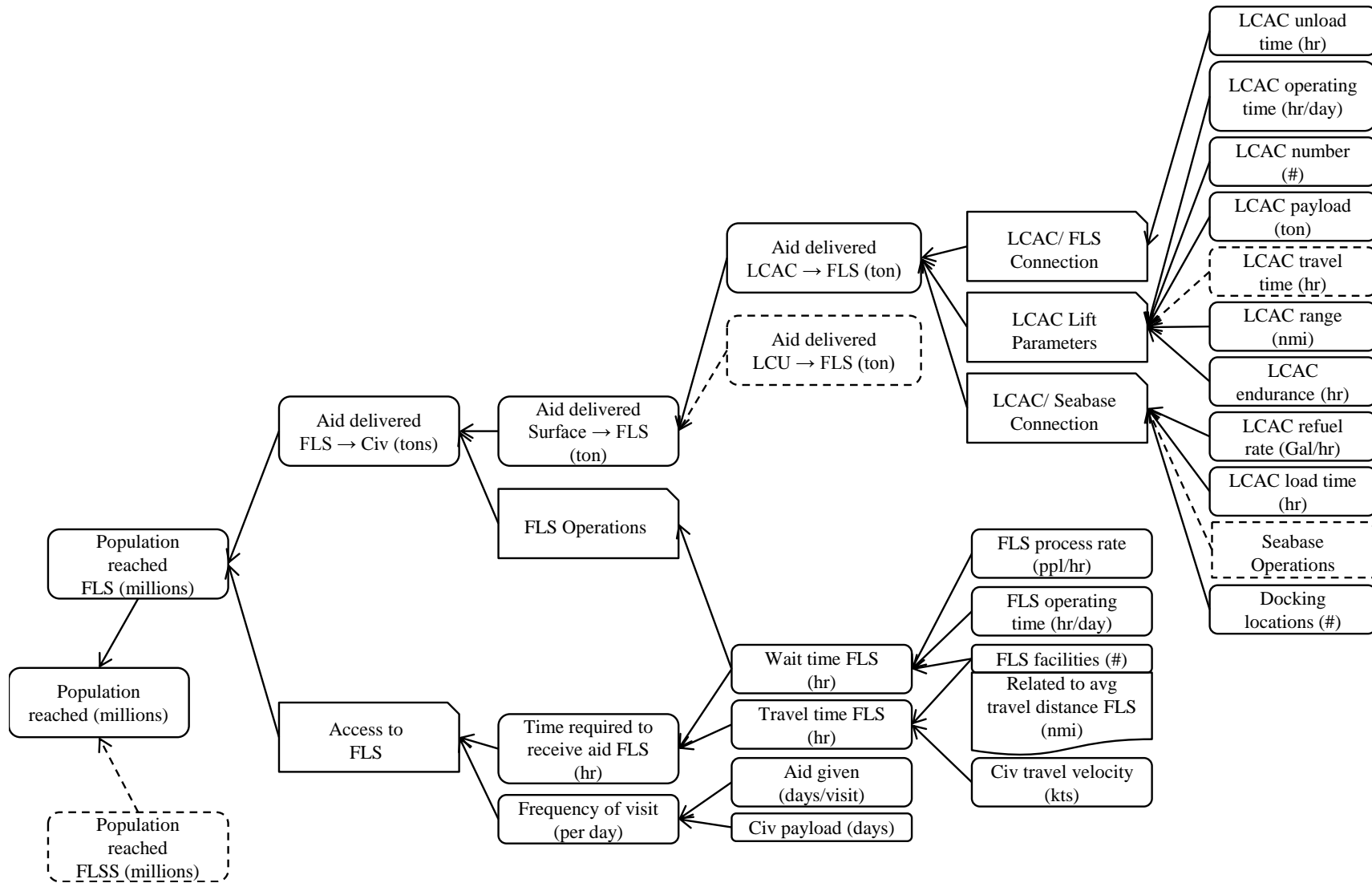


Figure 138: Decomposition of Impacts on Population Reached (LCAC Connectors)

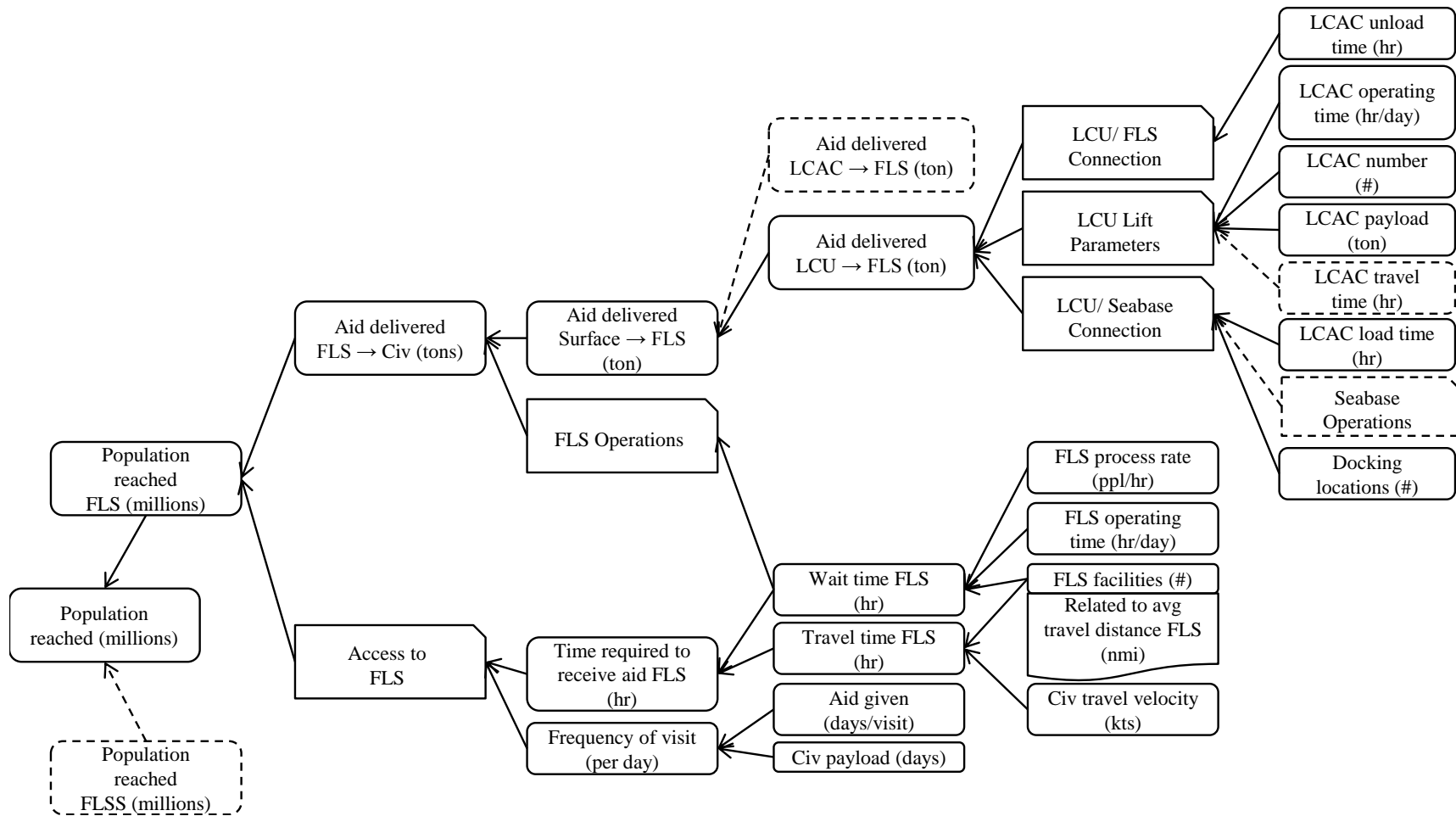


Figure 139: Decomposition of Impacts on Population Reached (LCU Connectors)

### 6.5.1.3 $Impact_3^E$ : Criminal Events

The decomposition of the impacts for  $Impact_3^E$  is shown in Figure 140. The decomposition of the impacts began with the decomposition of the measure ‘Criminal Events’. Three impacts were identified that affect the number of criminal events: Security, Population Reached, and Aid Consumption. The Security category of variables is decomposed into the number of security personnel at the FLS and the FLSS sites.

The Population Reached impact is  $Impact_2^E$  which was presented in the previous section. The Aid Consumption by the population is only impacted by  $Impact_1^E$ , which was presented in the section before. The combination of the three effect level impacts is shown in Figure 141.

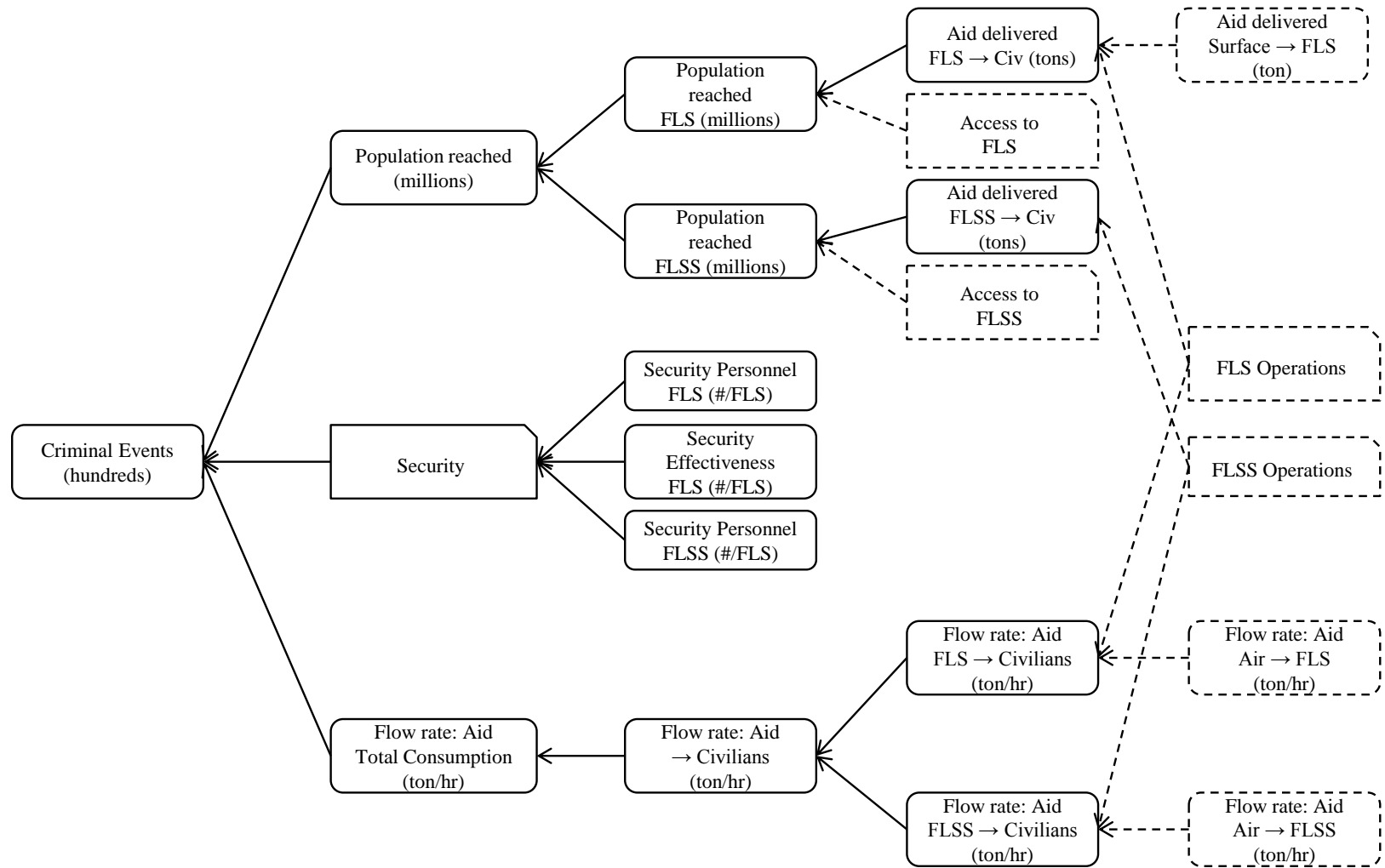


Figure 140: Decomposition of Impacts on Criminal Events



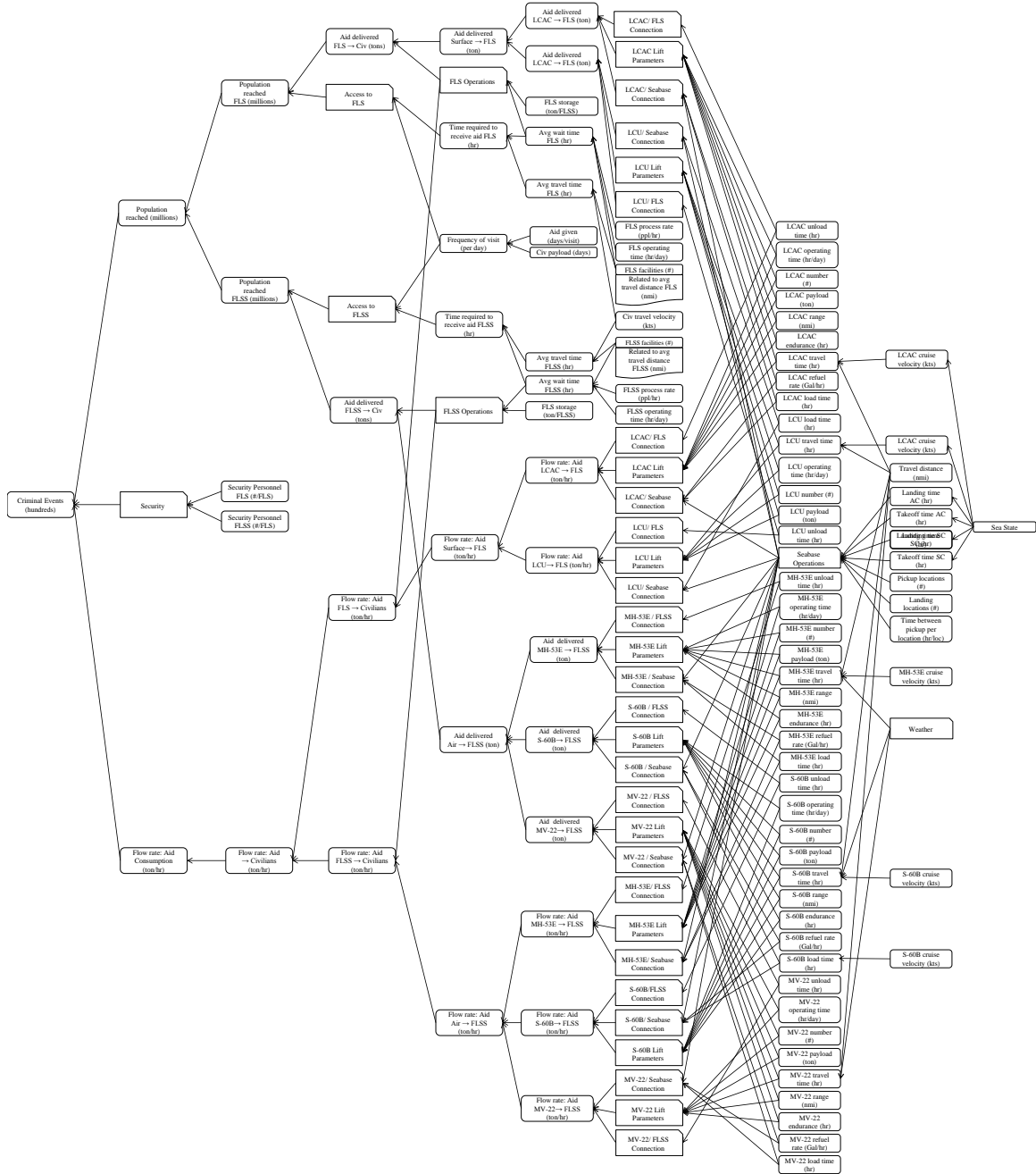


Figure 141: Full Decomposition of Impacts

### 6.5.2 Assigning Impact Values

Given an impact decomposition, the impact weightings can be assigned. It was decided in Section 4.1.6 that the impact weightings would be assigned using pairwise comparisons. This process is begun with the  $Impact_1^E$  decomposition and is detailed in Section 6.5.2.1.  $Impact_2^E$

decomposition, Population Reached, is detailed in Section 6.5.2.2. Impact<sup>E</sup><sub>3</sub> decomposition, Criminal Events, is detailed in Section 6.5.2.3.

*6.5.2.1 Flow Rate of Aid Impact Weightings*

The first pairwise comparison of Impact<sup>E</sup><sub>1</sub> is between the flow rates of the FLS and FLSS with respect to their impacts on the total flow rate of aid material to civilians. It is known from the system decomposition that there are 3 FLS and 41 FLSS. No values were given in the NPS thesis on the processing rates of the FLS and FLSS; therefore, it is assumed that the FLS has five times the processing rate. Given this information it is estimated that the FLSS flow rate is 2.7 times more important to the flow rate of aid to the population than the FLS flow rate. This results in the FLS and FLSS flow rates contributing 27% and 73% to the flow rate to the population, respectively.

On the next level, level 2, the flow rate of the connectors to the logistic site and the logistic site operations are impacts on the logistic site flow rates to the population. The failure of one of the impacts would result in the failure of the whole system, e.g. if the FLS/FLSS operations were poor and could not dispense aid adequately, then the flow rate from the surface/air connectors would not improve the flow rate of aid from the FLS/FLSS to the population and vice versa. For this reason the two impacts are considered equivalent. This estimate should minimize error with respect to the true value. The direct impacts of level 1 and level 2 are shown in Table 50. Additionally, the indirect impacts are shown.

**Table 50:** Flow Rate: Level 1 and 2 Impacts

	Flow Rate from FLS	Flow Rate from FLSS	Flow Rate from SC	FLS Operations	Flow Rate from AC	FLSS Operations
Flow Rate to Civilians	0.27	0.73	0.1350	0.1350	0.3650	0.3650
Flow Rate from FLS	1	-	0.5	0.5		
Flow Rate from FLSS		1			0.5	0.5

The next level of decompositions, level 3, will be discussed separately for each branch of the decomposition due to space limitations. The two branches being FLS flow rate and FLSS flow rate. Table 51 shows the level 3 impacts for the FLS branch and Table 52 shows the level 3 impacts for the FLSS branch.

For the FLS branch the 'FLS Dock Locations' impact was found to be insignificant and was assigned the impact value of 0.01. The reason for this is that there exist six sea connectors: five LCACs and one LCU. There are three FLSs with at least one docking location. The load and unload time are equivalent for each sea connector: 2 hours for LCAC and 4.5 hours for LCU. From this it is assumed that a natural rhythm would develop such that no more than one sea connector would be attempting to dock at an FLS at one time. Therefore the 'FLS Dock Locations' impact was found to be insignificant.

The impacts of the flow rate of aid from the LCAC and LCU into the FLS are compared against each other. There are five LCACs and one LCU that transport 75 and 170 tons of aid per trip at 40 and 8 knots, respectively. For each trip the LCACs would deliver 375 tons of aid and the LCU would deliver 170 tons of aid. With the LCAC traveling at five times the speed it is estimated that the LCAC will make two trips in a day and the LCU would make about one trip in a day. This results in the LCAC flow rate being about 4.4 times as important as the LCU flow rate.

The impacts on the FLS Operations are FLS Wait Time and FLS Storage capacity. It is not expected that the FLS will meet its storage capacity often, if it ever does. For this reason the FLS Wait time is considered to be ten times as important as the FLS Storage capacity.

For the FLSS branch the 'FLSS Dropoff Locations' impact was found to be insignificant and was assigned the impact value of 0.01. The reason for this is that the unload time for the air connectors is between one and two minutes. If every air connector arrived at the same FLSS at the same time, which would not happen, it would take 36 minutes to process them all. At which point the connectors would be staggered and would not face the same situation again. Therefore, the 'FLSS Dropoff Locations' impact was found to be insignificant.

**Table 51:** Flow Rate: Level 3 Impacts (FLS)

	Flow Rate from LCAC	Flow Rate from LCU	FLS Dock Locations	FLS Wait Time	FLS Storage
Flow Rate from SC FLS Operations	0.8066	0.1833	0.01	10/11	1/11

The impacts of the flow rate of aid from the MH-53E, S-60B, and MV-22 into the FLSS are compared against each other. There are 8 MH-53Es, 4 S-60Bs, and 12 MV-22s that carry 18, 3, and 17.5 tons of aid at 150, 215, and 160 knots, respectively. The payload delivered per trip for the MH-533, S-60B, and MV-22 are 144, 210, and 12 tons respectively. The MH-53Es and 4 S-60Bs travel at about the same rate but the MV-22 travels about 39% faster. Quick calculations result in the flow rate of aid from the MV-22 being 2 times more important than the MH-53E and 22 times more important than the S-60B. Thus flow rate of aid from the MH-53E is 10 times as important than the S-60B.

Finally, the impacts on the FLSS Operations are FLSS Wait Time and FLSS Storage capacity. The impact weightings were found to be the same as the FLS Operations impacts.

**Table 52:** Flow Rate: Level 3 Impacts (FLSS)

	Flow Rate from MH-53E	Flow Rate from S-60B	Flow Rate from MV-22	FLSS Dropoff Locations	FLSS Wait Time	FLSS Storage
Flow Rate from AC FLSS Operations	0.3	0.03	0.67	0.01	10/11	1/11

The weightings for the level 4 impacts of the FLS branch, shown in Table 53, starts with a weighting of the impacts on the flow rate from LCAC and LCU. The FLS Connection categories contain the variable for unload time. The Lift Parameters include the variables that define the sea connector and the Seabase Connection includes the load time and additional seabase variables. It is easily concluded that the Seabase Connection impact for both LCAC and LCU are more important than the FLS connection, since the load time and unload time are equivalent. Now it must be determined whether the Lift Parameters or the Seabase Connection is more important. The Lift Parameters would not change much, but they are very important to the flow rate of aid. The Seabase Connection is composed of the Seabase Operations which controls the aid from the amphibious ships; another very important set of variables. The difference between the two sets is difficult to assess; therefore, they will be considered equivalent. The Lift Parameters and the Seabase Connection will be considered three times as important as the FLS connection for each sea connector. This was decided because the load and unload times are equivalent. The remaining variables of the Seabase Connection are considered twice as important as the unload time.

The FLS Wait time is impacted by the FLS Process Rate, Operating Time, and Facilities. These three impacts are dependant on the other two impacts, i.e. if the process rate falls then the overall wait time increases no matter how high the operating time or number of facilities. Because of this dependence these impact variables are weighted as equivalent.

**Table 53:** Flow Rate: Level 4 Impacts (FLS)

	LCAC/FLS Connection	LCAC Lift Parameters	LCAC/Seabase Connection	LCU/FLS Connection	LCU Lift Parameters	LCU/Seabase Connection	FLS Process Rate	FLS Operating Time	FLS Facilities
Flow Rate from LCAC	1/7	3/7	3/7						
Flow Rate from LCU				1/7	3/7	3/7			
FLS Wait Time							1/3	1/3	1/3

The weightings for the level 4 impacts, shown in Table 54, of the FLSS branch starts with a weighting of the impacts on the flow rate from MH-53E, S-60B, and MV-22. The FLSS Connection categories contain the variable for unload time. The unload time is on the order of one minute therefore it is considered insignificant and is given an impact weighting of 0.01.

The Lift Parameters include the variables that define the air connector and the Seabase Connection includes the load time and additional seabase variables. The load time is insignificant. It is estimated that the dependence of the air connectors on the seabase variables is less than that of the surface connectors. This is because the air connectors are greater in number and have higher velocities resulting in them transporting more cargo per day. The air connector's List Parameters are then estimated to be three times as important as the Seabase Connection.

The FLSS Wait time is impacted by the FLSS Process Rate, Operating Time, and Facilities. These three impacts are dependant on the other two impacts, i.e. is the process rate falls then the overall wait time increases no matter how high the operating time or number of facilities. Because of this dependence these impact variables are weighted as

**Table 54:** Flow Rate: Level 4 Impacts (FLSS Connectors)

	MH-53E/FLSS Connection	MH-53E Lift Parameters	MH-53E/Seabase Connection	S-60B/FLSS Connection	S-60B Lift Parameters	S-60B/Seabase Connection	MV-22/FLSS Connection	MV-22 Lift Parameters	MV-22/Seabase Connection
Flow Rate from MH-53E	0.01	0.66	0.33						
Flow Rate from S-60B				0.01	0.66	0.33			
Flow Rate from MV-22							0.01	0.66	0.33

equivalent. This is shown in Table 55.

**Table 55:** Flow Rate: Level 4 Impacts (FLSS Operations)

	FLSS Process Rate	FLSS Operating Time	FLSS Facilities
FLSS Wait Time	1/3	1/3	1/3

The weightings for the level 5 impacts are further broken into branches. Under the branch FLS two branches are defined: LCAC and LCU. The Seabase Connection is addressed later due to space limitations. The first branch are the level 5 impacts under the LCAC shown in Table 56. The LCAC/FLS Connection is only impacted by the LCAC Unload Time; therefore, it is given a weighting of one. Of the variables that impact the Lift Parameters the range and endurance is addressed first. Based on the cruise velocity it is found that the range and endurance are equivalent and will be equally weighted. The remaining variables are dependent on the others for the delivery of aid. A 20% increase

in any of the variables would roughly translate to a 20% increase in the rate of aid being delivered; therefore, the variable impacts are weighted equally. Based on the refuel rate and the fuel capacity, a refuel would require about 5 minutes. Based on this the range and endurance are considered insignificant and are given a weighting of 0.01.

**Table 56:** Flow Rate: Level 5 Impacts (LCAC)

	LCAC Unload Time	LCAC Operating Time	LCAC Number	LCAC Payload	LCAC Travel Time	LCAC Range	LCAC Endurance
LCAC/FLS Connection LCAC Lift Parameters	1	0.245	0.245	0.245	0.245	0.01	0.01

The second branch are the level 5 impacts under the LCU shown in Table 57. The LCU/FLS Connection is only impacted by the LCU Unload Time; therefore, it is given a weighting of one. Based on the discussion under the LACA branch the remaining variables are equally weighted.

**Table 57:** Flow Rate: Level 5 Impacts (LCU)

	LCU Unload Time	LCU Operating Time	LCU Number	LCU Payload	LCU Travel Time
LCU/FLS Conn. FCU Lift Par.	1	1/4	1/4	1/4	1/4

The Seabase Connection impacts on the sea connectors are shown in Table 58. It was already found that the Refuel Rate and the Dock Locations are insignificant; therefore,



they are weighted at 0.01. The Seabase Operations are considered the most important of the remaining variables. It is estimated that the Seabase Operations are three times as important as the Load Time. This results in the impact weightings shown in Table 58.

**Table 58:** Flow Rate: Level 5 Impacts (SC Seabase Connection)

	LCAC Refuel Rate	LCAC Load Time	LCU Load Time	Seabase Operations	Docking Locations
LCAC/Seabase Connection	0.01	0.245		0.735	0.01
LCU/Seabase Connection			0.2475	0.7425	0.01

Under the branch FLSS three branches are defined: MH-53E, S-60B, and MV-22. The Seabase Connection is addressed later due to space limitations. The first branch are the level 5 impacts under the MH-53E shown in Table 59. The MH-53E/FLSS Connection is only impacted by the MH-53E Unload Time; therefore, it is given a weighting of one. Of the variables that impact the Lift Parameters the range and endurance is addressed first. Based on the cruise velocity it is found that the range and endurance are equivalent and will be equally weighted. The time required to refuel the vehicle is on the order of two minutes; however, the range and endurance is not considered insignificant. The vehicle would be required to land to refuel. This whole process is estimated to take about 20 minutes. These variables are considered to have the least impact. The remaining variables are considered to have equal impact, similar to the sea connectors. It is then estimated that the Operating Time is five times as important as the endurance. The same arguments are used for the S-60B and MV-22 impacts. This impact weighting results are shown in Tables 59, 60, and 61.

**Table 59:** Flow Rate: Level 5 Impacts (MH-53E)

	MH-53E Unload Time	MH-53E Operating Time	MH-53E Number	MH-53E Payload	MH-53E Travel Time	MH-53E Range	MH-53E Endurance
MH-53E/FLSS Connection	1						
MH-53E Lift Parameters		5/22	5/22	5/22	5/22	1/22	1/22

**Table 60:** Flow Rate: Level 5 Impacts (S-60B)

	S-60B Unload Time	S-60B Operating Time	S-60B Number	S-60B Payload	S-60B Travel Time	S-60B Range	S-60B Endurance
S-60B/FLSS Connection	1						
S-60B Lift Parameters		5/22	5/22	5/22	5/22	1/22	1/22

**Table 61:** Flow Rate: Level 5 Impacts (MV-22)

	MV-22 Unload Time	MV-22 Operating Time	MV-22 Number	MV-22 Payload	MV-22 Travel Time	MV-22 Range	MV-22 Endurance
MV-22/FLSS Connection	1						
MV-22 Lift Parameters	5/22	5/22	5/22	5/22	1/22	1/22	

The Seabase Connection impacts on the air connectors are shown in Table 62. It was already found that the Refuel Rate and the Pickup Locations are insignificant; therefore, they are weighted at 0.01. The Seabase Operations are considered significantly more important than the remaining variables. It is estimated that the Seabase Operations are 10 times as important as the remaining variable for each vehicle platform. This results in the impact weightings shown in Table 62.

**Table 62:** Flow Rate: Level 5 Impacts (AC Seabase Connections)

	Seabase Operations	Landing Time	Takeoff Time	Pickup Locations	Landing Locations	MH-53E Refuel Rate	MH-53E Load Time	S-60B Refuel Rate	S-60B Load Time	MV-22 Refuel Rate	MV-22 Load Time
MH-53E/SB Conn.	0.7	0.07	0.07	0.07	0.07	0.01	0.01				
S-60B/SB Conn.	0.7	0.07	0.07	0.07	0.07			0.01	0.01		
MV-22/SB Conn.	0.7	0.07	0.07	0.07	0.07					0.01	0.01

The level 6 impacts are broken into the impacts into Travel Times and the impacts into the Seabase Operations, shown in Tables 63 and 64, respectively. For the Travel Time

impacts the Travel Distance and the velocity of the various platforms are considered to be equivalent. The Sea State and Weather are considered to be twice as important as the distance. This results in the impact weightings shown in Table 63.

**Table 63:** Flow Rate: Level 6 Impacts (Travel Times)

	Travel Distance	Sea State	LCAC Cruise Velocity	LCU Cruise Velocity	MH-53E Cruise Velocity	S-60B Cruise Velocity	MV-22 Cruise Velocity	Weather
LCAC Travel Time	0.25	0.5	0.25					
LCU Travel Time	0.25	0.5		0.25				
MH-53E Travel Time	0.25				0.25			0.5
S-60B Travel Time	0.25					0.25		0.5
MV-22 Travel Time	0.25						0.25	0.5

The Seabase Operations are impacted by the Sea State and the LHD, LPD, and LSD Parameters. The LHD Parameters are estimated to be three times as important as the LPD Parameters. The LPD Parameters are estimated to be two times as important as the LSD Parameters. Finally, it is estimated that the Sea State is equivalent in importance as the LHD Parameters. This results in the impact weightings shown in Table 64.

**Table 64:** Flow Rate: Level 6 Impacts (Seabase Operations)

	Sea State	LHD Parameters	LPD Parameters	LSD Parameters
Seabase Operations	0.4	0.4	0.1333	0.0667

### 6.5.2.2 Population Reached Impact Weightings

The first pairwise comparison of  $\text{Impact}_2^E$  is between the population that is reached by FLS and FLSS. It is known from the system decomposition that there are 3 FLS and 41 FLSS. Since it was assumed that the processing rate of the FLS is five times that of the FLSS it is not assumed that the FLSS is 13.67 times as important as the FLS to the total population reached. Instead made estimate made is that the FLSS is ten times as important as the FLS to the total population reached.

On the next level, level 2, the logistic sites are impacted by the total aid delivered to the site and the accessibility of the logistic site to the population. These two impacts are linked. If one of these impacts performs poorly then the other is limited as well. For this reason the impacts are equally weighted. The direct impacts of level 1 and level 2 are shown in Table 65. Additionally, the indirect impacts are shown.

**Table 65:** Population Reached: Levels 1 and 2 Impacts

	Pop Reached from FLS	Pop Reached from FLSS	FLS Aid Delivered	Access to FLS	FLSS Aid Delivered	Access to FLSS
Population Reached	1/11	10/11	0.0455	0.0455	0.4545	0.4545
Pop Reached from FLS	1		0.5	0.5		
Pop Reached from FLSS		1			0.5	0.5

The weightings for the level 3 impacts are shown in Table 66. The aid delivered to the logistic site is dependent on the aid delivered by the corresponding connector and the operations of the logistic site. It is estimated that the aid delivered is twice as important as the operations. This is because without a sufficient amount of aid then the aid will not reach a large number of people and will mostly go to the people closest to the site, and thus can

get in line first.

The access to the logistic site is impacted by the total time to receive aid from the site and the frequency of visits to said site. It is estimated that the frequency of visits to the site is a bigger impact on the access than the time to get to the site. The travel to the site and back is a very laborious activity. One may be willing to commit sufficient time for this activity but if the frequency increases then the civilian may resort to other means. It is estimated that the frequency is twice as important as the travel time.

**Table 66:** Population Reached: Level 3 Impacts

	Aid Delivered SC	FLS Operations	FLS Time to Receive Aid	Frequency of Visit	Aid Delivered AC	FLSS Operations	FLSS Time to Receive Aid	Frequency of Visit
FLS Aid Delivered	2/3	1/3						
Access to FLS			1/3	2/3				
FLSS Aid Delivered					2/3	1/3		
Access to FLSS							1/3	2/3

The level 4 impacts were broken into two branches, FLS shown in Table 67 and FLSS shown in Table 68. The aid delivered by the surface connectors are impacted by the aid delivered by the LCAC and LCU. As discussed in the previous section the LCAC has a much higher flow rate of aid, but how does this relate to total aid delivered? Since the operation is set for 15 days the two measures are equivalent; therefore, the impacts are based on the previous section.

The FLS Operations is impacted only by the FLS Wait Time therefore it is weighted at one. The total time to receive aid is impacted by the travel time to the logistic site and

the time waiting at the logistic site. The comparative weighting is largely based on the absolute values of these two variables. The larger value would have the bigger impact. It is estimated that the travel time is the biggest impact and is twice as important at the wait time.

The Frequency of Visit is impacted by the total aid given and the limit to how much the civilian can carry. Since the Civilian Payload is a limitation it is estimated that the Aid Given is twice as important as the payload.

**Table 67:** Population Reached: Level 4 Impacts (FLS)

	Aid Delivered LCAC	Aid Delivered LCU	FLS Wait Time	FLS Travel Time	Aid Given	Civilian Payload
Aid Delivered SC	0.8147	0.1851				
FLS Operations			1			
FLS Time to Receive Aid			1/3	2/3		
Frequency of Visit					2/3	1/3

The FLSS Operations and Frequency of visit are the same as above. The FLSS Wait Time impact weightings varies in that there are many more FLSS, and therefore the travel time would be less. This results in the FLSS Wait Time being twice as important as the FLS Travel Time. The impact of the specific air connectors on the total aid brought by air connector is based on the discussion in the section before. These impact weightings can be seen in Table 68.

**Table 68:** Population Reached: Level 4 Impacts (FLSS)

	Aid Delivered MH-53E	Aid Delivered S-60B	Aid Delivered MV-22	FLSS Wait Time	FLSS Travel Time	Aid Given	Civilian Payload
Aid Delivered AC	0.30	0.03	0.67				
FLSS Operations				1			
FLSS Time to Receive Aid				2/3	1/3		
Frequency of Visit						2/3	1/3

The level 5 impacts were divided up into 3 branches, FLS shown in Table 69, Air Connectors shown in Table 70, and FLS shown in Table 71. The impact weightings of the variable categories on the connectors is based on the weights found in the previous section. These weightings can be seen in Tables 69 and 70.

The FLS Wait Time is impacted by the FLS Process Rate, FLS Operating Time, and FLS Facilities. Since there are only three facilities, a change in the number would be drastic. The more facilities present the shorter the lines to get aid and the shorter the wait time. The number of facilities is the most important impact. This is followed by the process rate and the operating time. It is estimated that the number of facilities is twice as important as the process rate. The process rate is estimated to be twice as important as the operating time.

The FLS Travel Time is impacted by the FLS Facilities and Civilian Velocity. Since an increase in the number of facilities would drastically reduce the travel distance, the number of facilities is the most important impact. It is estimated that the number of facilities is three times as important as the Civilian Velocity.



**Table 69:** Population Reached: Level 5 Impacts (FLS)

	LCAC/FLS Connection	LCAC Lift Parameters	LCAC/Seabase Connection	LCU/FLS Connection	LCU Lift Parameters	LCU/Seabase Connection	FLS Process Rate	FLS Operating Time	FLS Facilities	Civilian Velocity
Aid Delivered LCAC	1/7	3/7	3/7							
Aid Delivered LCU				1/7	3/7	3/7				
FLS Wait Time							2/7	1/7	4/7	
FLS Travel Time									0.75	0.25

**Table 70:** Population Reached: Level 5 Impacts (AC)

	MH-53E/FLS Connection	MH-53E Lift Parameters	MH-53E/Seabase Connection	S-60B/FLS Connection	S-60B Lift Parameters	S-60B/Seabase Connection	MV-22/FLS Connection	MV-22 Lift Parameters	MV-22/Seabase Connection
Aid Delivered MH-53E	0.01	0.66	0.33						
Aid Delivered S-60B				0.01	0.66	0.33			
Aid Delivered MV-22							0.01	0.66	0.33

The FLSS Wait Time is impacted by the FLSS Process Rate, FLSS Operating Time, and FLSS Facilities. Since there are 41 facilities, a change in the number would not be drastic. The number of facilities is the least important impact. Similar to the FLS impact

weightings, it is estimated that the process rate is twice as important as the operating time. The operating time is estimated to be equivalent to the operating time.

The FLSS Travel Time is impacted by the FLSS Facilities and Civilian Velocity. Since an increase in the number of facilities would not drastically reduce the travel distance, the number of facilities is the least important impact. It is estimated that the Civilian Velocity is twice as important as the number of FLSS facilities.

**Table 71:** Population Reached: Level 5 Impacts (FLSS)

	FLSS Process Rate	FLSS Operating Time	FLSS Facilities	Civilian Velocity
FLSS Wait Time	0.5	0.25	0.25	
FLSS Travel Time			1/3	2/3

The remaining impact weightings are equivalent to the impact weightings found for the Flow Rate impact decomposition. The impact weightings of the FLS Connection and Lift Parameters on the LCAC and LCU are shown in Tables 72 and 72, respectively. The impact weightings of the Seabase Connection on the LCAC and LCU are shown in Table 74. The impact weightings of the FLSS Connection and Lift Parameters on the MH-53E, S-60B, and MV-22 are shown in Tables 75, 76, and 77, respectively. The impact weightings of the Seabase Connection on the MH-53E, S-60B, and MV-22 are shown in Table 78. Finally, the impact weightings for the Travel Time decomposition and the Seabase Operations are shown in Tables 79 and 80, respectively.

**Table 72:** Population Reached: Level 6 Impacts (LCAC)

	LCAC Unload Time	LCAC Operating Time	LCAC Number	LCAC Payload	LCAC Travel Time	LCAC Range	LCAC Endurance
LCAC/FLS Connection	1						
LCAC Lift Parameters	0.245	0.245	0.245	0.245	0.245	0.01	0.01

**Table 73:** Population Reached: Level 6 Impacts (LCU)

	LCU Unload Time	LCU Operating Time	LCU Number	LCU Payload	LCU Travel Time
LCU/FLS Conn. FCU Lift Par.	1	1/4	1/4	1/4	1/4

**Table 74:** Population Reached: Level 6 Impacts (SC Seabase Connection)

	LCAC Refuel Rate	LCAC Load Time	LCU Load Time	Seabase Operations	Docking Locations
LCAC/Seabase Connection	0.01	0.245	0	0.735	0.01
LCU/Seabase Connection	0.01	0	0	0.98	0.01

**Table 75:** Population Reached: Level 6 Impacts (MH-53E)

	MH-53E Unload Time	MH-53E Operating Time	MH-53E Number	MH-53E Payload	MH-53E Travel Time	MH-53E Range	MH-53E Endurance
MH-53E/FLSS Connection	1						
MH-53E Lift Parameters		5/22	5/22	5/22	5/22	1/22	1/22

**Table 76:** Population Reached: Level 6 Impacts (S-60B)

	S-60B Unload Time	S-60B Operating Time	S-60B Number	S-60B Payload	S-60B Travel Time	S-60B Range	S-60B Endurance
S-60B/FLSS Connection	1						
S-60B Lift Parameters		5/22	5/22	5/22	5/22	1/22	1/22

**Table 77:** Population Reached: Level 6 Impacts (MV-22)

	MV-22 Unload Time	MV-22 Operating Time	MV-22 Number	MV-22 Payload	MV-22 Travel Time	MV-22 Range	MV-22 Endurance
MV-22/FLSS Connection	1						
MV-22 Lift Parameters		5/22	5/22	5/22	5/22	1/22	1/22

**Table 78:** Population Reached: Level 6 Impacts (AC Seabase Connections)

	Seabase Operations	Landing Time	Takeoff Time	Pickup Locations	Landing Locations	MH-53E Refuel Rate	MH-53E Load Time	S-60B Refuel Rate	S-60B Load Time	MV-22 Refuel Rate	MV-22 Load Time
MH-53E/SB Conn.	0.7	0.07	0.07	0.07	0.07	0.01	0.01				
S-60B/SB Conn.	0.7	0.07	0.07	0.07	0.07			0.01	0.01		
MV-22/SB Conn.	0.7	0.07	0.07	0.07	0.07					0.01	0.01

**Table 79:** Population Reached: Level 7 Impacts (Travel Times)

	Travel Distance	Sea State	LCAC Cruise Velocity	LCU Cruise Velocity	MH-53E Cruise Velocity	S-60B Cruise Velocity	MV-22 Cruise Velocity	Weather
LCAC Travel Time	0.25	0.5	0.25					
LCU Travel Time	0.25	0.5		0.25				
MH-53E Travel Time	0.25				0.25			0.5
S-60B Travel Time	0.25					0.25		0.5
MV-22 Travel Time	0.25						0.25	0.5

**Table 80:** Population Reached: Level 6 Impacts (Seabase Operations)

	Sea State	LHD Parameters	LPD Parameters	LSD Parameters
Seabase Operations	0.4	0.4	0.1333	0.0667

### 6.5.2.3 Criminal Events Impact Weightings

The first pairwise comparison of Impact<sub>3</sub><sup>E</sup> is between the population reached, the flow rate of aid to the population, and Security. It is estimated that the order of importance to the number of criminal events is Security, Population Reached, and Total Consumption Flow Rate. It is estimated that the Security is twice as importance as the Population Reached and the Population Reached is twice as important as the Total Consumption Flow Rate.

The impact weightings for the Population Reached was defined in the previous section.

The total consumption is only impacted by the rate of aid flowing to the civilians; therefore, it is weighted at a value of one. The purpose of the total consumption impact is to place the lower level impacts on the same level of the decomposition. The Security is impacted by the FLS Security Personnel, the FLSS Security Personnel, and the Security Effectiveness. Based on the number of FLSS and the impact weightings defined for the other impact decompositions, it is estimated that the FLSS Security Personnel is 10 times as important as the FLS Security Personnel. Additionally, it is estimated that the Security Effectiveness is as important as the FLSS and FLS Security Personnel. The results of these impact weightings can be seen in Table 81. The remaining impacts are the same as were defined as above and will not be reproduced.

**Table 81: Criminal Events: Level 1-3 Impacts**

	Population Reached			Total Consumption Rate			Security		
	Population Reached	Total Consumption Rate	Security	FLS Population Reached	FLSS Population Reached	Flow Rate to Civilians	FLS Security Personnel	FLSS Security Personnel	Security Effectiveness
Criminal Events	2/7	1/7	4/7	0.0260	0.2597	0.1429	0.0260	0.2597	0.2857
Pop. Reached				1/11	10/11				
Tot. Con. Rate						1			
Security							1/22	5/11	1/2

### 6.5.3 Model Representation

The importance of the impact variables are calculated through matrix multiplication of the direct impacts. The calculated importance values with respect to the top level impacts can be found in Appendix B.3. The indirect impact matrices and the calculated same level correlation values are too large to be displayed in this document. Their calculated values will be stated as they are needed. Based on these calculation, decisions can be made on

how to represent the system within the programmed model.

In Section 4.1.5 it was concluded that a morphological matrix would be used to help enumerate the possible model representations of the identified impact relationships. Once the possible representations were enumerated, a selection would be made and supported by the importance of the impact relationships to the overall system.

The process of defining model representation will be started by studying the importance of the individual systems and the corresponding variables. Decisions will then be made on how to represent these systems within the programmed model. This will be followed by studying the interaction between these systems. Decisions will then be made on how to represent these interactions within the programmed model. Finally, the environmental factors will be studied, and decisions will be made on how to represent the environment within the programmed model.

#### *6.5.3.1 Model Representation of Systems*

The systems contained within the HA/DR scenario are the FLS, FLSS, Civilian, Security Personnel, LCAC, LCU, MH-53E, S-60B, MV-22, LHD, LPD, and LSD. These systems will be addressed within groups: logistic sites, actors, surface connectors, air connectors, and amphibious ships.

The FLSS was found to contribute 2.3 to 10 times more than the FLS to the overall system for the three top level impacts discussed above. Of the variables that belong to the logistic sites the wait time for each site was found to be the most important impact. Referring back to the impact decomposition, the wait time is impacted by the process rate and the number of facilities. The representation of the wait time can take many forms. These possible representations are listed in Table 82. The options are listed from left to right in decreasing model fidelity.

The highest fidelity option is to represent the wait time as a function of lower level variables. This would include keeping track of a queue of civilians and modeling the process rate. Another option would be to estimate the wait time with a distribution. This option would remove the need to keep track of a queue or model the process rate. Even simpler, the



wait time can be represented with a simple deterministic value. Finally, the wait time could not be modeled. This would result in the civilian arriving at the logistic site and instantly returning with aid. The highest fidelity option is selected for the wait time representation in the model, because the importance of the impact relationships from the three impact decompositions states that the wait time is an important variable.

**Table 82:** Model Representation of Wait Time

Impact	Options			
Wait Time	Function of lower level variables	Distribution	Simple value	Not modeled

The variables that impact the logistic site wait time are the number of facilities, process rate, and operating time. The importance of these impacts to the overall system are equal for Impact<sub>1</sub><sup>E</sup>. For Impact<sub>2</sub><sup>E</sup> and Impact<sub>2</sub><sup>E</sup> the order of importance of these impacts is number of facilities, process rate, and operating time for the FLS and process rate, number of facilities, and operating time for FLSS.

The number of facilities, *logistic sites*, for the FLS and FLSS not necessarily modeled but instead is an input to the simulation. An important aspect of the impact of the number of facilities is how it affects the placement of the logistic sites. This question of placement will be address in Section 6.5.3.3.

Similar to the wait time, the process rate can be represented as a function of lower level variables, a distribution, a simple value, or not modeled. Based on the importance value of the FLS Process Rate, this measure is very important to the system. Typically, this would result in the decision of modeling the process rate as a function of lower level variables. If this selection were made, then the system definition and impact decomposition would need to be readdressed, for this is the lowest level variable down this branch. The readdressing of the system definition and impact decomposition would investigate the operations that are responsible for processing people. This application of the proposed methodology is a proof

of concept; therefore, the selection made is to represent the process rate as a distribution. Specifically, the process rate will be represented as an exponential distribution since many processes can be represented as an exponential distribution [162].

The operating time was found to be an important variable from the importance calculations, especially for  $\text{Impact}_1^E$ . However, like the number of facilities impact, the operating time acts as an input to the model instead of something that must be modeled. Unlike the number of facilities impact, the operating time is a continuous variable; therefore, it can be modeled as distribution or as a single value. It is decided that the operating time will be represented as a simple variable. This is because the importance of the operation time is based on the time that is defined for the model, e.g. 8hr vs. 16hr, not the variability of the operating time.

**Table 83:** Model Representation of Wait Time Impacts

<b>Impact</b>	<b>Options</b>			
Facilities	N/A			
Process Rate	Function of lower level variables	<b>Distribution</b>	Simple value	Not modeled
Operating Time	Distribution	<b>Simple variable</b>		

The remaining impact variables of the logistic sites are the storage capacity, the number of dropoff locations, and the number of docking locations. The storage capacity of the logistic sites was found to contribute about 1% 0.2% to the system for  $\text{Impact}_1^E$  and  $\text{Impact}_3^E$ , respectively, and it does not contribute to the  $\text{Impact}_2^E$  system. Three model representations were identified for the logistic site storage. The first is to track the space available for humanitarian aid. This would involve tracking the resources that are present as the site can calculating the remaining space available. The second option is to represent the storage capacity as a simple variable based on the weight of the aid present. Finally, the storage

capacity of the logics sites could be ignored. This would allow each logistic site to store as much aid as possible. The storage capacity will be represented by a simple variable. This is due to the fact that it contributes to a small but noticeable portion of the system.

The importance values of the Dropoff Locations and the Docking Locations were found to be insignificant, often contributing to half a percentage or less. Two model representations were identified for these impact variables. The first is to track the available docking/dropoff locations and a queue for the locations. The second is not to model the locations. The later was selected because of the insignificance of the variable on the system. The means that the connectors can unload their cargo at the logistic site, no matter the number of other connectors present at the site.

**Table 84:** Model Representation of Remaining Logistic Site Impacts

Impact	Options		
	Storage	Tracking space for storage	<b>Simple variable</b>
Dropoff Locations	Queue	<b>Not Modeled</b>	
Dock Locations	Queue	<b>Not Modeled</b>	

The variables that make up the Civilian system are frequency of visit and the travel time required to reach the logistic site from both the FLS and FLSS. It was found that the frequency of visit contributes to about a third of  $\text{Impact}_2^E$  and 9% of  $\text{Impact}_3^E$ . The variable does not impact  $\text{Impact}_1^E$ . Three model representations were identified for this variable. The first is to represent it as a function of the lower level variables. The second would be to represent the frequency of visit as a distribution. The third would represent the frequency of visit as a simple deterministic variable. The selection of the last two would remove the need to represent the aid given. This variable, the amount of aid that is distributed per visitor, would be of interest in the analysis; therefore the function of lower level variables is selected.

The frequency of visit is impacted by the amount of aid given and the payload limit of the civilian. These variables impact the system significantly, where the aid given contributes

twice as much as the payload. Three model representations were identified for aid given. The first is to represent it as a function of the needs of the individuals that visit the facility. The second would be to represent the variable as a distribution. The third would represent the frequency of visit as a simple deterministic variable. The importance of the aid given is based on the effect of distributing more aid per visitor than it is based on the difference in aid given to different individuals. For this reason and the fact that this variable is of interest in the analysis it will be represented as a simple value.

Three model representations were identified for the maximum payload of the civilian. The first is to represent it as a deterministic value that is uniquely assigned to each individual, which is represented in the system. This would require the use of civilian objects that exist throughout the execution of the model. If this were selected, most likely many more variables would be tracked as well. The second would be to represent the variable as a distribution that is assigned when it is needed. The third would be to represent the variable as a simple deterministic value that is assigned to all people. The last two options would not require the use of civilian objects. The general distribution was selected because the variable was found to be important; however, the tracking of individuals in a simulation is not required and would over complicate the model. It is argued that this variable can be sufficiently represented with a proper distribution. The distribution used will be a Gaussian Distribution.

The Civilian Travel Time contributes between 0.2% and 5% to the system. Four model representations are identified for this variable. The first is to determine the travel time for the individual based on the length of the path that the individual would take and the weather. The path would be a detailed determination of the distance traveled. The weather would be a detailed determination of the velocity traveled based on the weather. The second representation would be a calculation of the travel time based on a distance and velocity estimate. The third representation would represent the travel time with a distribution that would be sampled when it was needed within the simulation. Finally, the fourth option is to assign each travel distance with one deterministic value for all individuals. It is decided that the travel time would be represented with a distance estimate and a velocity estimate.

This is a moderate fidelity representation, which is a reasonable selection based on the importance value of the impact variable. The distance estimate will be addressed in Section 6.5.3.2.

The Civilian Travel Velocity accounts for 3.6% and 1% of the response to  $\text{Impact}_2^E$  and  $\text{Impact}_3^E$ , respectively. Four model representations were identified for this variable. The first would represent the travel velocity based on the environment. The environment includes both the weather and the terrain. Because of the selection made on the civilian travel time, this option is not a possibility. The second option would assign a value to the individual based on a distribution. This value would be owned by the individual. The third option is to assign a value from a distribution as the value is needed in the simulation. The value would not be owned by the individual. Finally, the fourth option would be to assign a fixed deterministic value to every individual. The third option is selected, general distribution. This best aligns with the selections made previously for the civilian. It is argued that the fidelity level of the representation best corresponds with the importance value.

**Table 85:** Model Representation of Civilian Variables

<b>Impact</b>	<b>Options</b>			
Frequency of Visit	Function of lower level variables	Distribution	<b>Simple value</b>	
Aid Given	Individual simple value	General distribution	General simple value	
Civilian Payload	Individually tracked	<b>Distribution</b>	Simple value	Not modeled
Civilian Travel Time	Individual/Path/Weather	<b>Individual /Distance /Velocity</b>	Distribution	<b>Simple value</b>
Civilian Travel Velocity	Based on individual and environment	Individual distribution	<b>General distribution</b>	General simple value

The Security system represents  $4/7$  of the response to  $\text{Impact}_3^E$ . The number of personnel at the logistic sites and the effectiveness of the personnel represent  $2/7$  of the response each. Based on these importance values and the level of decomposition seen in the system definition and impact decomposition, it would be suggested that this portion of the system is further explored. The exact mechanism for how the security personnel aid in reducing the criminal events should be explored, defined in a system definition, and impacts decomposed. Given that this is a proof of concept these steps will not be taken in this example; however, it is important to note how this methodology helps to identify shortcomings of the initial system definition and impact decomposition.

For this proof of concept, four model representations were identified for the number of security personnel. The first is to track each security person and their shift rotations. The second would be to dynamically assign a number of security personnel to each logistic site based on observed criminal events in the area. The third option would be to assign a number

of personnel to each site based on the site type. Finally, it is possible not to model the number of security personnel present. The third option is selected. The number of personnel is very important; therefore, it must be modeled, i.e. the fourth option is not reasonable. The selection of the first two options would require that addition of many programming tasks. To simplify the modeling effort, the simple number is selected. Note that in a project used for real world application, this section of the system should be expanded upon further.

The Security Effectiveness has three model representation options. The first would be to represent it as a function of lower level variables. It is known that based on the variable's importance that this should be used in an actual application; however, this proof of concept it will not be selected. The second option would be to include the security effectiveness as a weighting in the to probability of a criminal event to occur. The final option would be to not model it. The second option is selection because the third option is not feasible based on the variable's importance. The second option is the only one that remains.

**Table 86:** Model Representation of Security Variables

<b>Impact</b>	<b>Options</b>			
Number Personnel	Track Shift Rotations	Unique Number for Site	<b>Simple Number</b>	Not Modeled
Security Effectiveness	Function of lower level variables	<b>Weighting on crime event</b>	Not modeled	

Because the LCAC and LCU are so similar, they will both be represented within the programmed model as a generalization of a larger object. This is similar to what is shown in Figure 120. For this reason, the variables that make up the LCAC and LCU will be represented in the same form. Table 87 shows the morphological matrix for the surface connector variables. Because the importance of the LCAC on the system was found to be much more than the LCU for each of the decompositions, only the LCAC importance values will be referenced.

Many of the options for representing the variables of the surface connectors follow the form: function of lower level variables, distribution, simple variable, and not modeled. These are the options for the variable, unless otherwise noted. The first variable addressed is the unload time, which contributes about 0.3% to 1.6% of the system depending on the top impact measure. The unload time will be represented as a distribution, because it does not contribute to a large portion of the system; however, it is still significant. The simple variable was not selected, because it was deemed to have too low of fidelity. The representation of the unload time as a Gaussian Distribution will provide much more information within the analysis than a simple variable, with little more programming time requirement. Due to the similarity between the load time and unload time the same decision is made for the load time.

The operating time contributes about 0.3% to 1.1% of the system depending on the top impact measure. The operating time will be represented as a simple variable. This is because the operating time acts as an input to the model instead of something that must be modeled. The importance of the operation time is based on the time that is defined for the model, e.g. 8hr vs. 16hr, not the variability of the operating time.

The travel time contributes about 0.3% to 1.1% of the system depending on the top impact measure. The travel time will be represented as a function of lower level variables. Despite the travel time not having a significant impact on the system, the lower level variables are of interest in the analysis. The lower variables considered are the travel distance and the cruise velocity. The travel distance will be calculated as a simple distance between the sea base and the specific location they are traveling to. The cruise velocity will be represented as a simple variable. This is because the importance of the variable is below 0.3% at its highest.

The range, endurance, and refuel rate will not be modeled. This is because the combination of the importance of the three variables is 0.15% at the highest value. These variables are insignificant to the simulation of the system. The surface connectors will travel until their operation time expires for the day. No refueling activities will be modeled.

The payload of the two vessels contribute about 0.3% to 1.4% of the response of the



system. Additionally, this is a variable that is largely controlled. The payload is represented as a simple variable because of these reasons.

**Table 87:** Model Representation of SC Variables

<b>Impact</b>	<b>Options</b>			
Unload Time	Function of lower level variables	<b>Distribu- tion</b>	Simple variable	Not modeled
Load Time	Function of lower level variables	<b>Distribu- tion</b>	Simple variable	Not modeled
Operating Time	Distribution	<b>Simple variable</b>		
Number	<b>N/A</b>			
Travel Time	<b>Function of lower level variables</b>	Distribution	Simple variable	Not modeled
Velocity	Function of lower level variables	Distribution	<b>Simple variable</b>	Not modeled
Range	Function of lower level variables	Distribution	Simple variable	<b>Not modeled</b>
Endurance	Function of lower level variables	Distribution	Simple variable	<b>Not modeled</b>
Refuel Rate	Function of lower level variables	Distribution	Simple variable	<b>Not modeled</b>
Payload	Function of weight and dimension	Distribution	<b>Simple variable</b>	Not modeled

Because the air connectors are so similar, they will both be represented within the programmed model as a generalization of a larger object. This is similar to the treatment of the surface connectors. For this reason, the variables that make up the MH-53E, S-60B,

and MV-22 will be represented in the same form. Table 88 shows the morphological matrix for the air connector variables. Because the importance of the MV-22 on the system was found to be much more than the other two for each of the decompositions, only the MV-22 importance values will be referenced. Additionally, it is observed from the importance values that the S-60B contributes very little to the system. For this reason, the S-60B will not be modeled.

The load and unload time in combination contribute to 0.3% of the system response for Impact<sub>1</sub><sup>E</sup> and less for the other two top level impact measures. It is also known that the time to load and unload the cargo is on the order of a minute. For these reasons the load and unload times will not be modeled. The transfer of cargo will be instant.

The operating time for both platforms contribute about 2% to 5% of the system response depending on the top impact measure. The operating time will be represented as a simple variable. This is because the operating time acts as an input to the model instead of something that must be modeled. The importance of the operation time is based on the time that is defined for the model, e.g. 8hr vs. 16hr, not the variability of the operating time.

The travel time of the air connectors will be represented in the same manner as the travel time of the surface connectors. The travel time will be a function of the distance to the specific FLSS and the velocity. The distance is calculated as the Euclidean distance and the velocity is represented as a simple variable.

The endurance and the range for both platforms contribute about 0.7% to 2% to the response of the system. These variables will be represented as a simple variable. This variable would not be represented well as a distribution, because the range and endurance is a metric that is better controlled than some other variables, e.g. processing time.

The three variables refuel rate, landing time, and takeoff time are all related in the same activity of refueling the air connector. The landing time and takeoff time contribute about 0.6% to 1.6% of the system response. The contribution of the refuel rates are insignificant. Because these three variables are so closely related and they contribute to a small portion of the system response they will all be represented by a single variable, refuel time. This

variable will be modeled as an Exponential Distribution because it is a process.

The payload of the aircraft contribute about 2% to 5.2% of the response of the system. Additionally, this is a variable that is largely controlled. The aircraft simply picks up the cargo so sizing in the compartment is not an issue. The payload is represented as a simple variable because of these reasons.

**Table 88:** Model Representation of AC Variables

<b>Impact</b>	<b>Options</b>			
Unload Time	Function of lower level variables	Distribution	Simple variable	<b>Not modeled</b>
Load Time	Function of lower level variables	Distribution	Simple variable	<b>Not modeled</b>
Operating Time	Distribution	<b>Simple variable</b>		
Number	<b>N/A</b>			
Travel Time	<b>Function of lower level variables</b>	Distribution	Simple variable	Not modeled
Velocity	Function of lower level variables	Distribution	<b>Simple variable</b>	Not modeled
Range	Function of lower level variables	Distribution	<b>Simple variable</b>	Not modeled
Endurance	Function of lower level variables	Distribution	<b>Simple variable</b>	Not modeled
Refuel Rate	Function of lower level variables	Distribution	Simple variable	<b>Not modeled</b>
Landing Time	Function of lower level variables	Distribution	Simple variable	<b>Not modeled</b>
Takeoff Time	Function of lower level variables	Distribution	Simple variable	<b>Not modeled</b>
Payload	Function of lower level variables	Distribution	<b>Simple variable</b>	Not modeled

The amphibious ships contribute about 2.5% to 7% to the response of the overall system.

Currently, the amphibious ships were not further decomposed in the system definition nor in the impact decomposition. For this proof of concept the variables of payload and the time to reload will be modeled. Similar to the other payload representations, this variable will be tracked with a simple deterministic variable. The reload time, however, will be represented with a distribution. The distance the amphibious ships must travel to reload with cargo is not known. Therefore it will be estimated as the mean of the distribution. The variance will be set larger than for the previous distribution because little is known about this process. The reload time will be represented with a Gaussian Distribution, because the primary contributor to the time is assumed to be the distance traveled.

**Table 89:** Model Representation of Amphibious Ship Variables

<b>Impact</b>	<b>Options</b>			
Payload	Function of weight and dimension	Distribution	<b>Simple variable</b>	Not modeled
Reload Time	Function of weight and dimension	<b>Distribution</b>	Simple variable	Not modeled

### 6.5.3.2 Model Representation of System Interactions

The civilian population interacts with the logistic sites and the security personnel. Based on the system definition the civilian performs the actions of ‘gather materials’, ‘consume materials’, and ‘perform criminal activities’. The first activity, ‘gather materials’, involves an interaction with the logistic sites. Based on the model representation of the civilian three model representations are identified for how the logistic sites and civilian. The first model representation would be to model every civilian that would visit a logistic site. Each civilian would have an associated payload, travel velocity, home, and a variable of how many others they support. The travel distance would be calculated from their home to the nearest logistic site. Each civilian would consume resources based on their consumption rate. These civilians would then travel to the nearest logistic site when they need more aid and

would enter the queue at the logistic site. The second representation would be to aggregate the first option. Instead of individuals being modeled, civilians may be represented by an aggregated civilian, e.g. a civilian object for a square nautical mile. The payload and aid carried would be modified according to how many people are aggregated within the nautical mile. This would reduce the computational requirements over the previous option; however, the programmatic requirements are similar. The final option would be to calculate the number of civilians that would visit a logistic site and use that number as an input to the queue in the logistic site. This option would reduce the computational requirements of the model and the programmatic requirements; however, the dispersion of times that the civilians arrive at the logistic site would be lost. Of these options the aggregated civilians was selected as the best representation of the system interactions.

Another issues remains on the interaction of the logistic sites and the civilians. Once the logistic site has finished operating does it process the remaining people, or does it dispatch all those currently in line? In order for the operating time of the logistic site to be measured on its impact on the output of the simulation, the latter of the options must be selected.

**Table 90:** Model Representation of Civilian and Logistic Site

System	Options		
Civilian	Model of individuals	Aggregated civilians	Civilians as inputs
Logistic Site	Process remaining people in queue	Stop all processes	

The next interaction to be investigated is between the civilians and the security personnel. The civilian may perform the activity ‘perform criminal event’. Based on the selections made in Table 86, it is known that the action of this criminal event is impacted by the security personnel. The suggested model for this event to occur is as follows. The civilian has a probability of performing a criminal event. This probability increases as the number of days without aid increases. At zero days the probability is zero. At four days the probability

is one hundred percent. This probability is reduced by the presence of security personnel. The probability of performing a criminal event is then divided by the multiplication of the number of security personnel at the closest logistic site and their effectiveness weighting. An equation is defined in Equation 55, where  $D$  is the number of days without aid,  $Wt_{SP}$  is the security personnel weighting factor, and  $SP$  is the number of security personnel at the nearest logistic facility.

$$P(CE) = \frac{\min(D/4, 1)}{Wt_{SP}SP} \quad (55)$$

The logistic sites interact with the civilians, security personnel, and the connectors. The first two interactions were covered above. The interactions between the logistic site and the connectors involve the dropoff locations and the docking locations. It was determined above that these would not be modeled. The only interaction now is the passing of aid material, which will be a simple variable update within the model.

The connectors interact with the amphibious ships. This interaction involves three aspects. The first is refueling; however, the refueling of the connectors are not being modeled. The second is the exchange of aid material. This involves a simple updating of variable values and a process time for the surface connector, since a queue is not being modeled. Finally, the connectors are stationed on the amphibious ships. If one of the ships leave to reload with cargo, then the connector must connect and leave with them. This requirement must be met before the amphibious vessel leaves, and will be included in the model.

Finally, the command and control interacts with the logistic sites, the connectors, and the amphibious ships. This system was not discussed within the impact decomposition, because it is not the focus of a simulation question. The function of command and control is to direct where the connectors go based on the status of the logistic sites and the amphibious ships. Two questions arise in regards to the command and control interactions. The first is, how does the command and control select which logistic site the connectors travel to? The second is, how does the command and control select which amphibious ship the connector travels to?

The connectors will be sent to the logistic site that has the lowest amount of aid remaining. If there are multiple sites with the same level of aid, the closest will be sent the next shipment of aid. On their return, the connectors will be sent to the amphibious ship that can connect with them and has the highest percentage of aid remaining. If there is a tie then one will be selected at random.

### *6.5.3.3 Model Representation of System Environment Interaction*

There are a few interactions of the environment and the systems defined. The model representation of the civilian does not include an interaction with the environment, terrain nor weather. The air connectors do contain an impact from the weather which contributes about 1% to 2.7% of the response of the system. However, based on the model representation of the air connectors this interaction is best represented by the range of input variables on the velocity of the air connector.

The Sea State impact variable has an impact on the surface connector cruise velocity and the load time at the seabase. The Sea State contributes about 1.7% to 5.7% of the response of the system. However, like the air connectors, based on the model representation this interaction is best represented by the range of input variables of the cruise velocities and the load times.

## **6.6 Experimental Model**

Model representation is followed by the development of communicative models. Based on the survey of the literature and the comparison analysis in Section 4.2, the Unified Modeling Language (UML) is used for this step. The communicative models can be found in Appendix B.5. Following the development of communicative modes the computerized model is programmed. Screen-shots of the computerized model and the code is available in Appendix B.6.

The next step is to develop an experimental model. This is a model which enables the execution of consecutive cases and the recording of output measures. In this application 46 variables were identified as inputs. This list of variables was developed based on the impact decompositions. In order to evenly sample the output space a Latin Hypercube



Sampling (LHS) Design of Experiment (DOE) was selected to form the experimental cases. A DOE was selected because it provides a method for efficiently sampling the variable input space that results in the greatest information of the variable output space. The LHS was selected because it is a common selection when very little is assumed about design space behavior. A total of 500 cases were developed using the mathematical program MatLab. The *lhsdesign* function was used with a minimization of correlation. Due to the limitation of the calculation of Brownian Correlation only four replications were performed, resulting in 2,000 data points used to estimate the normalized Brownian Correlations.

An initial set of 16 output variables were identified shown in Table 91. This set was developed from the impact decompositions. The goal was to enable the comparison of estimated impacts and the observed impacts. In the process of creating the executable model two output measures were not able to be captured. These variables are ‘waitTimeFLS’ and ‘waitTimeFLSS’. When a civilian visits a logistic site there is an associated wait time. A civilian may visit a logistic site more than once a day for a total of 15 days. Each logistic site will be visited by hundreds or thousands of civilians. Finally, there are numerous logistic sites. The computational storage requirement to track these variable were found to be too great; therefore they were not tracked.

**Table 91:** Identified Output Variables

aidDeliveredFLS	avgTravelTimeFLSS	aidDeliveredMH- 53E	aidDeliveredLSD
avgTravelTimeFLS	avgFrequencyOfVisit- FLS	aidDeliveredMV- 22	sumCrimi- nalEvents
avgFrequencyOfVisit- FLS	aidDeliveredLCAC	aidDeliveredLHD	waitTimeFLS
aidDeliveredFLSS	aidDeliveredLCU	aidDeliveredLPD	waitTimeFLSS

## 6.7 Comparison of Results to Impact Decomposition Weightings

The direct impact weightings between the flow rate of aid from the FLS and FLSS to the civilians we estimated to be 0.27 and 0.73, respectively. The Brownian Correlation between the FLS flow rate and the flow rate to the civilians was found to be 0.3757. The Brownian Correlation between the FLSS flow rate and the flow rate to the civilians was found to be 0.8561. This results in normalized Brownian Correlations of 0.3050 and 0.6950, respectively. These results are very close to what was predicted by the impact decomposition.

Comparisons are further made on the impacts to the FLS flow rate. Figure 142 shows the estimated impact values and the calculated normalized Brownian Correlation for a variety of impacts that affect the FLS flow rate. Each of the estimated or measured values are with respect to the FLS flow rate, i.e. the value associated with the number FLS facilities is based on its impact on the FLS flow rate. For each impact variable two numbers are listed. The first number is the estimated impact value found in the impact decomposition. The second number is the calculated Brownian Correlation. The impacts that have the box and text greyed out are impacts that were not modeled, e.g. FLS dock locations. The greyed out arrows indicate that no measurement was made for this impact, e.g. FLS wait time.

Observing the estimated impact values and the measured normalized Brownian Correlations one notes that the two are very similar. The flow rate from the surface connectors to the FLS and the FLS Operations were estimated to contribute to 0.5000 of the overall impact, each. The impact of the surface connector to the FLS was calculated to be 0.5729, a very close result. Correspondingly, the FLS Operations impact the FLS flow rate by 0.4271. The impact variables of the surface connector flow rate are the LCAC flow rate and the LCU flow rate. For these impacts the estimates are very similar to the observed values.

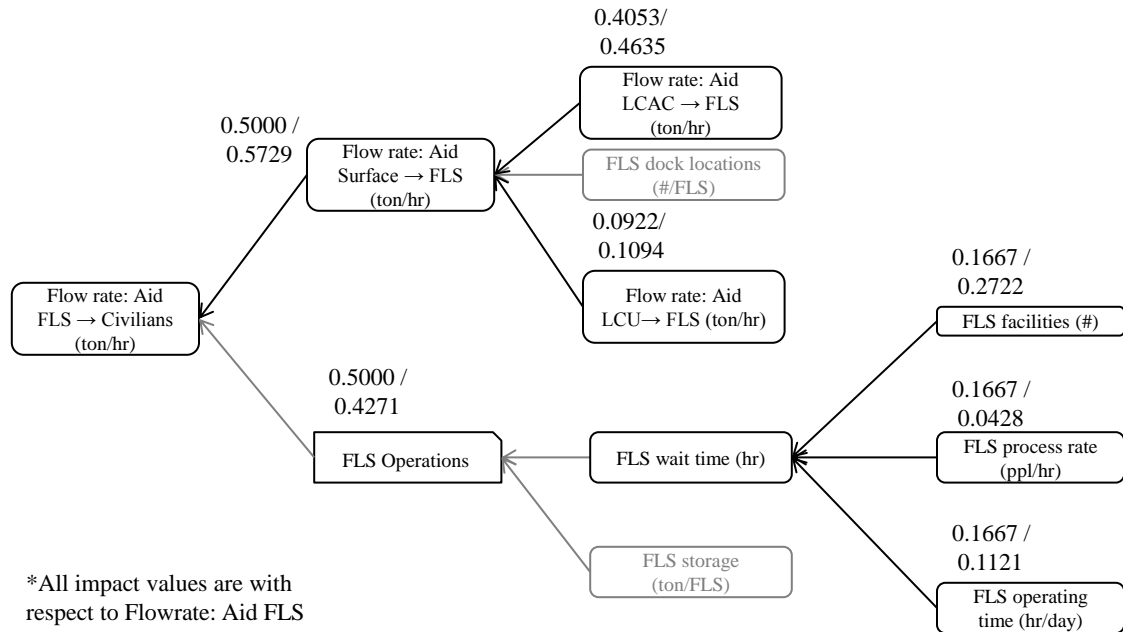
The variables that compose the FLS Operations do not track as closely as the other impacts that were observed. The calculated impact of the number of FLS facilities is larger than was estimated. When one is presented with a disagreement between the estimated impact values and the observed impact values, one must investigate the reason for the disagreement. There are multiple possibilities. The first is a question of verification. The disagreement could be a result of a programming error or a failure in the model architecture.

This possibility should be investigated first. This would involve review the code and the communicative models.

The second possibility for disagreement is a question of validation. There are multiple validation issues to address. The first validation issue is that of data validity. A question that may be investigated is, where the input value ranges accurate for the scenario defined? Different ranges for the input variables would yield very different observed impact values. The next validation issues is that of model representation. A question that may be investigated is, was the selected model representation adequate for this impact?

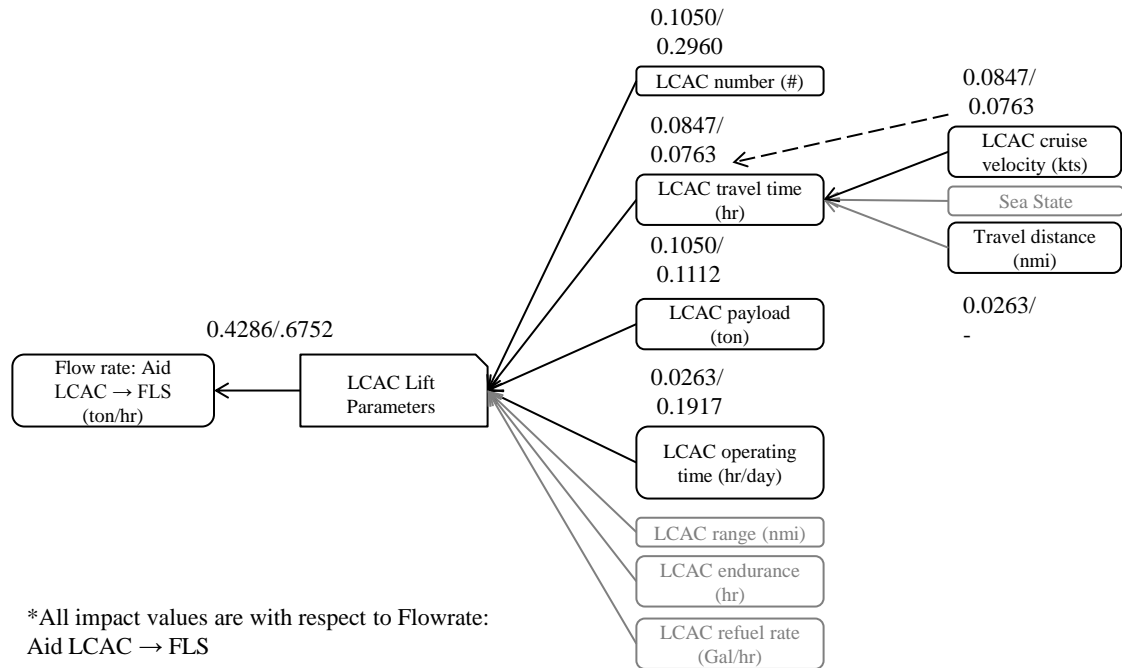
The final possibility for disagreement that should be addressed is that the estimates were incorrect. This would indicate that the system theories from which the impact weightings were developed are flawed. This is an expected occurrence, for if one knew perfectly how the system behaved, there would be no need for a model.

These possibilities were investigated and it was concluded that the initial system theories were incorrect for the variable ranges given. The impact of the number of FLS facilities was under estimated. The impact of the FLS Process Rate was over estimated. This was because the range of values given to the FLS process rate did not result in a choke point in the supply chain of aid to the civilians. If the variable range were reduced then its importance would increase. An example of this is shown below in Section 6.8. Finally, the impact of the FLS Operating Time was slightly under estimated.



**Figure 142:** Comparison of FLS Impact Decomposition to Observed Values

Comparisons are further made on the impacts to the LCAC flow rate. Figure 143 shows the estimated impact values and the calculated normalized Brownian Correlation for the impacts of the Lift Parameters variable set. Each of the estimated or measured values are with respect to the LCAC flow rate. The LCAC Travel Time and LCAC Travel Distance was not tracked for the outputs; however, the LCAC Cruise Velocity was tracked. Since the travel distances were fixed the velocity will act as the contributing impact for the LCAC Travel Time. Comparing the estimates to the observation, it is seen that there are two agreements: LCAC Cruise Velocity and LCAC Payload. The impact of the number of LCACs and the LCAC Payload were found to be under estimated.

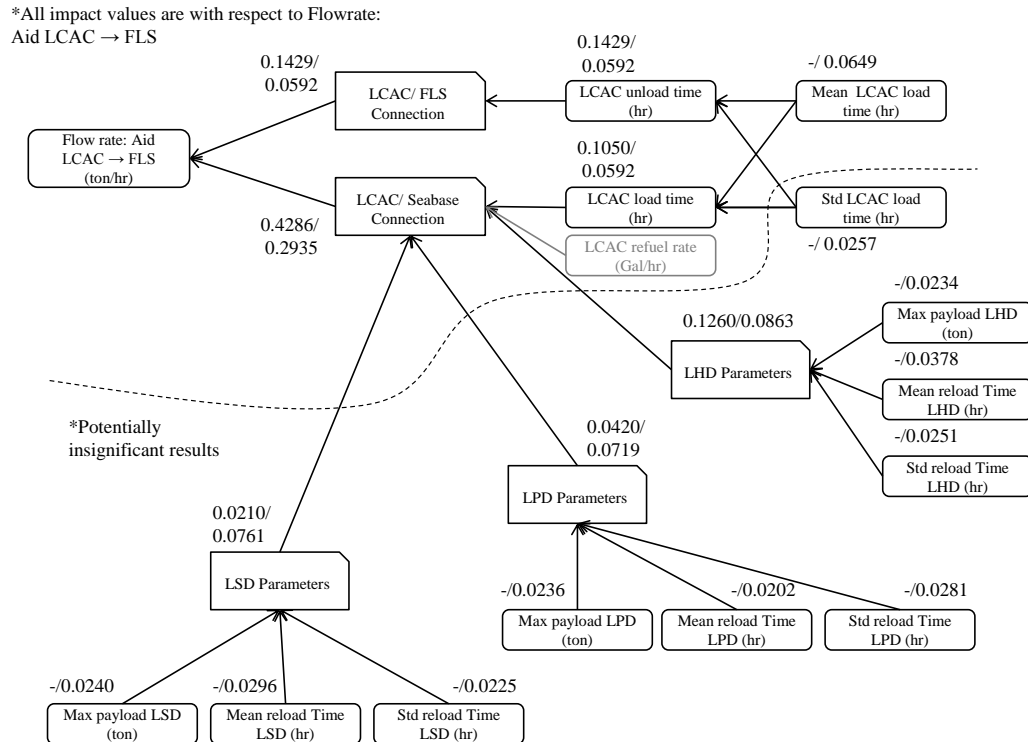


**Figure 143:** Comparison of LCAC Lift Parameters Impact Decomposition to Observed Values

Figure 144 shows the estimated impact values and the calculated normalized Brownian Correlation for the impacts of the FLS Connection and Seabase Connection variable set. Each of the estimated or measured values are with respect to the LCAC flow rate. The estimates in this set were found to be over estimating the observed value. The impact of the amphibious vessels were consistently over estimated. The reason for this overestimation is rooted in the variable ranges of the amphibious ships. It was found that based on these variable ranges, the amphibious ships did not provide a choke point of the flow rate of aid.

Additionally, the reason was discovered as to why the load and unload time did not affect the flow rate as much as initially estimated. Though the loading and unloading time affected the time required to deliver aid and return, the LCAC still performed the same number of deliveries in a day. For example, if the LCAC had a maximum operating time of 16 hours and it took six hours to load deliver, unload, and return, then a small variation on the load unload time would not impact the number of deliveries performed in a day.

As noted in Figure 144, the calculated impacts of many of the variables may be insignificant. A problem arises however when numerous variables that have no impact are included in the normalized correlation calculations. Because the Brownian Correlation overestimates the correlation of non-correlated variables, the effluence of these non-correlated can skew the observed values. For example, in Figure 144 the amphibious ships contribute 0.2935 to the result of the LCAC flow rate; however, each of variables are small. This necessitates a procedure for removing variables that are not impacts.



**Figure 144:** Comparison of LCAC FLS & Seabase Connector Impact Decomposition to Observed Values

The comparison of the estimated impacts to the calculated impacts for a part of the flow rate decomposition was presented here. The remainder of the impacts are displayed in Appendix B.7.

### 6.7.1 Comparison of Results to Same Level Correlations

Below the comparison of the estimated same level correlations are listed with the observed correlations. In the upper right of each matrix the estimated values are listed. In the lower left of each matrix the calculated values are listed. The '-' elements in the lower left of the matrix indicate that a correlation cannot be calculated. This is due to either one of the variables were not modeled or one of the variables is a category of variables.

As can be seen there is great disagreement between the estimated correlations and the observed correlations. These great disagreements occur while the direct impact estimates were show to be mostly accurate. The correlation estimated in the Level 2 Correlations between the flow rate of aid to the civilians from the FLS and FLSS is 0.13, whereas the observed value is 0.08. The correlation estimated in the Level 3 Correlations between the flow rate of aid to the logistic sites from the air connectors and the sea connectors is 0.26, whereas the observed value is 0.18. The differences observed in the Level 4 Correlations are even more drastic. These results suggest that the same level correlations do not act as good indicators of validation.

**Impact<sub>1</sub><sup>E</sup> Level 2 Correlations**

$$\begin{bmatrix} 1.00 & 0.13 \\ 0.08 & 1.00 \end{bmatrix}$$

**Impact<sub>1</sub><sup>E</sup> Level 3 Correlations**

$$\begin{bmatrix} 1.00 & 0.00 & 0.26 & 0.00 \\ - & 1.00 & 0.00 & 0.00 \\ 0.18 & - & 1.00 & 0.00 \\ - & - & - & 1.00 \end{bmatrix}$$

(56)

**Impact<sub>1</sub><sup>E</sup> Level 4 Correlations**

1.00	0.39	0.00	0.00	0.00	0.26	0.26	0.26	0.00	0.00	0.00
0.06	1.00	0.00	0.00	0.00	0.26	0.26	0.26	0.00	0.00	0.00
—	—	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
—	—	—	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
—	—	—	—	1.00	0.00	0.00	0.00	0.00	0.00	0.00
0.07	0.06	—	—	—	1.00	0.34	0.34	0.00	0.00	0.00
—	—	—	—	—	—	1.00	0.34	0.00	0.00	0.00
0.09	0.05	—	—	—	0.09	—	1.00	0.00	0.00	0.00
—	—	—	—	—	—	—	—	1.00	0.00	0.00
—	—	—	—	—	—	—	—	—	1.00	0.00
—	—	—	—	—	—	—	—	—	—	1.00

### 6.8 Iteration on Model Development

An important benefit of this methodology is the ability of the model developers and SMEs to iterate between the predicted impact importance values and the observed simulation results. In the sections above, even though the initial estimates were very similar to observations several differences were observed. For each discrepancy the modelers and SMEs can either trust the simulation output or modify the simulation.

Figure 142 showed that there were several disagreements between the estimated impacts of FLS facility number, FLS process rate, and FLS operating time. One of these impact variables will be iterated on to bring the estimates and the results into better agreement. As stated above the disagreement could be a result of a programming error, inaccurate input data, improper model representation, or inaccurate estimates.

The estimate for the FLS facility number was considered to be an inaccurate estimate. This is because no programming error was discovered, the input data is based on the operational scenario and therefore accurate, and the model representation of a quantity can not be modified. The number of FLS facilities contributed more to the simulation output than initially estimated; however this does not require an update to the computer model.

The estimate for the FLS operating time was considered to be a slightly inaccurate



estimate. This is because no programming error was discovered. The input data is based on the operational scenario and therefore accurate. The range of the operational scenario data could be modified, but the difference between the estimate and the observed values is very small and is not required. Finally, the model representation fidelity cannot be greatly increased. The operating time is a controllable factor in this simulation; therefore, detail would not yield greater understanding.

The estimate for the FLS process rate was significantly smaller than estimated. It was concluded that this is a result of the selection of the variable value range. Upon further investigation the values for the FLS process rate never resulted in the buildup of aid stores at the logistic sites, i.e. the FLS was never a choke point. This would not be the case if the values were lower. The change in the variable range could be achieved through a greater fidelity model that indirectly calculates the processing rate or smaller input variable range.

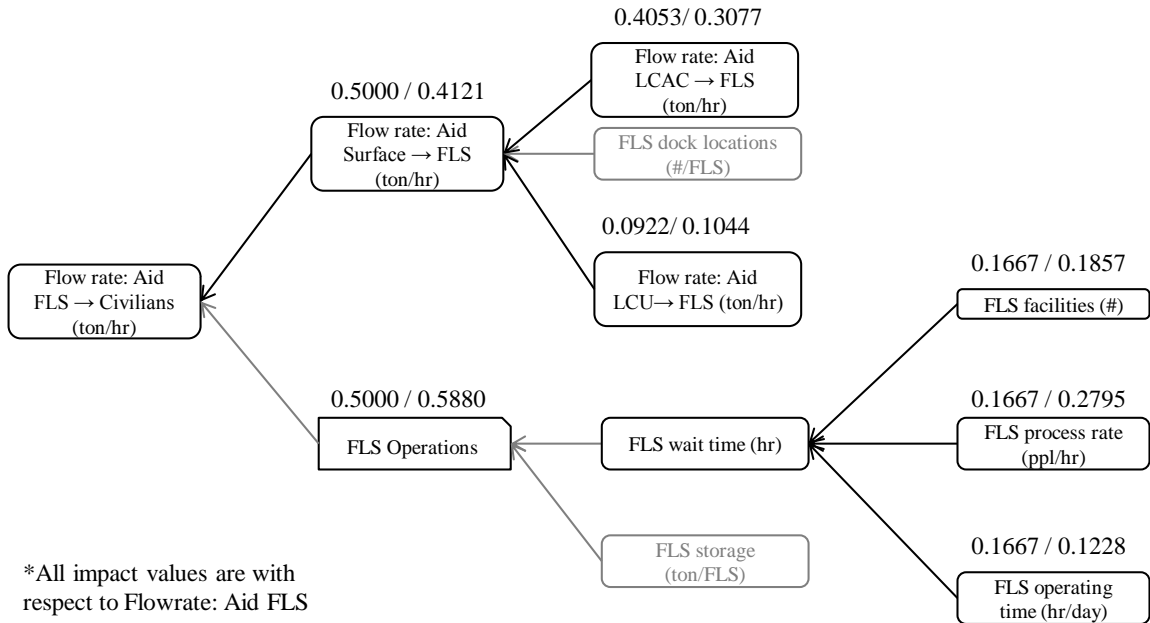
For the purposes of this proof of concept it is assumed that the SMEs were confident in their estimate on the process rate impact values for the FLS and these impacts must be modified. This requires either a change in the variable range or a change in the model. It is decided that the variable range will be modified. This will demonstrate the ability of iteration of the methodology.

The original process rate for the FLS was 175 civ/hr to 525 civ/hr. The new variable ranges for the FLS process rate is defined as 5 civ/hr to 175 civ/hr. This new range will guarantee that the FLS will behave as a choke point for the flow of aid to the civilians.

Figure 145 shows the new results for the flow rate of aid from the FLS to the civilians. This figure is similar to Figure 142 shown above. The first noticeable change is the observed importance of the flow rate of aid from the surface connectors to the FLS and the FLS operations. The importance values in Figure 145 are the reverse of the importance values observed in Figure 142. The FLS is now more important because it will be have as a choke point of aid for certain values of the process rate.

Under the decomposition of the surface connector flow rate the LCU estimated impact matches the observed impact. However, the LCAC estimated impact is greater than was observed. Under the FLS wait time decomposition the estimated impact of the number of

FLS facilities is close to the observed impact. This is a noticeable change to the former simulation observations that resulted in an impact value of 0.2722. The process rate is observed to contribute 0.2795 for the new simulations. This value is much greater than was estimated. This is most likely because the values of the process rate were able to be so low, beyond what is likely. Finally, the FLS operating time estimates are slightly above the importance value observation.



**Figure 145:** Comparison of FLS Impact Decomposition to Observed Values for Model Iteration

This iteration on the model development methodology shows how the disagreement between the estimated importance values and observed importance values can lead to a change in the model representation, aiding conceptual model validation, or in the input value ranges, aiding in data validation. Additionally, this example shows the importance in defining the impact variable ranges while the SMEs assign direct impact values. The iteration between the predicted impact importance values and the observed simulation results is an important benefit of this methodology .

## 6.9 Conclusions on the Proof of Concept

Several observations are made on the application of the proposed methodology to a HA/DR scenario. The first observation is on the nature of assumptions. One of the goals of this methodology was to provide a traceable and dependable process for developing models. An implied requirement of this goal is to list the assumptions made about the system under study. During the system definition step a SysML model was developed of the HA/DR scenario. However, the detail of the SysML model is finite; therefore, assumptions were made about the representation of the scenario in SysML. These assumptions were not directly specified. The conclusion of this is that some assumptions will always be made and undocumented.

The fidelity assumptions made on the system definition are later scrutinized by the calculation of the importance value of the impact variables. If it is determined that a specific impact variable has a high importance value, then the system definition should have that variable decomposed. If it does not, then one should return to the system definition step, understand the impacts and processes that influence the important impact variable, and apply additional detail.

**Observed Benefit of Methodology:** The impact variable decomposition challenges the initial system fidelity assumptions in the system definition, providing a greater understanding of the system before programming efforts begin.

A shortcoming identified of the proposed methodology occurs in the transition from the system definition to the impact variable decomposition within the conceptual modeling step. Significant effort may go into the creation of the system definition; however, very little of the effort translates directly into the impact variable decomposition. Some of the system definition may influence the creation of the impact variable decomposition, but the majority of the impact variable decomposition is developed from scratch. This cumbersome transition results in a waste of effort and a loss of traceability. The traceability can be maintained through the use of descriptive documents. Future research requires the investigation into developing a better transition between these two tasks.

**Observed Limitation of Methodology:** The transition from the system definition to the impact variable decomposition is cumbersome and lacks traceability.

**Research Question:** How could the system definition better support the creation of the impact variable decomposition?

The process of developing an impact variable decomposition of the HA/DR scenario provided many observations and suggestions for future work. Two observations are made on the structure of the impact variable decomposition. The first is creation of the structure of the decomposition. In any given system the impact variable decomposition can take many forms. For the HA/DR scenario presented above numerous structures were developed before the final one was selected. An example of an alternative structure would be to place the flow rates of aid for each system on the same level, and then decompose the systems into their impacting variables. A research objective is defined below which is highly related to the previous research question. It is hypothesized that greater reliance of the impact variable decomposition on the system definition will provide the structure of the decomposition.

**Research Objective:** Determine some guiding principles for the proper decomposition of impact variables.

The second observation on the structure of the impact variable decomposition is based on the decomposition approach used. This approach requires that a specific impact variable must exist on one and only one level. When developing the impact variable decomposition it was found numerous times that a variable could impact multiple systems at different levels in the decomposition. An example of this is an environmental variable that may affect a system on every level. Since this decomposition approach was based on QFD and AHP, these two methods face the same issue. Future work should investigate how the impact variable decomposition can be modified to reduce the hierarchical requirement.

**Research Objective:** How can the analysis of the impact variable decomposition be modified to reduce the hierarchical requirement of the current approach?

Once the model was programmed and executed, the estimated importance values of the impact variables were compared to the observed importance values of the impact variables. Two observations were made. The first is that the agreement between the estimated impact values and the observed impact values was surprisingly high. Additionally, for the situations where disagreement exists, investigation into the nature of the disagreement often leads to greater understanding of the system. An example was giving on the iteration of the model where a impact variable range was modified. This resulted in new observed importance values that supported the initial importance value estimates. These modifications to the system theories and the model resulted in a better understanding of the true system.

**Observed Benefit of Methodology:** The estimated impact values were found to agree with the observed impact values at a surprising level. The disagreements lead to revisions of the system theories or the model and resulted in a better understanding of the system.

In addition to comparing the importance values of the impact variables the same level correlation estimates were compared to the calculated Brownian Correlation. The results showed strong disagreement despite strong agreement between the estimated and observed importance values of the impact variables. It is concluded that the same level correlation estimates are not considered a reliable method for validation at the methodology's current state. Research question 3.F remains unanswered, having every hypothesis proposed in this thesis falsified. The research question is reposed below.

**Observed Limitation of Methodology:** The estimated same level correlation values were found to strongly disagree with the observed values. The same level correlation estimates are not considered a reliable method for validation at the methodology's current state.

**Research Objective:** How should impact matrix correlations be calculated to enable their use in validation activities?

One observed shortcoming of the use of normalized Brownian Correlations is that the

correlation does not estimate zero well. It tends to over estimate the true correlation. This resulted in the accumulation of importance impact values for some classes of variables that may have been nil.

During the selection of model representations for the impact variables the vast combinatorial space of model designs was observed. For every impact variable one may identify a set of possible means to represent it. Then upon the selection of a representation, said selection would also contain multiple possibilities. For example, in the scenario modeled above a civilian was required to travel from their home to a logistic site. This could have been represented in numerous ways. One possible way is to calculate the specific path they may take. If this were selected, then there are multiple algorithms that could be used to find a path. In addition to these combinations, every selection opened up a new set of decisions to make. For example, assuming one were modeling the civilians path, then a terrain must be modeled. Decisions must now be made on how to represent the terrain. Every decision made on model representation has wide ranging impacts throughout the rest of the model representation of the system.

One final criticism can be made about the application of the method. The suggestion that a morphological matrix should be used to define how each relationship in the impact variable decomposition should be modeled was met with some difficulty. The root of this difficulty is that the impact relationships of a system are not independent of each other. The decision on how to model one variable would influence the modeling of another. The hypothesis of addressing each relationship independently in the morphological matrix was flawed. Instead, the modeler should identify boundaries within the system that are easily represented as a whole. Several alternatives would be presented for this system boundary and a selection would be made in the same fashion as the current methodology suggests.

Despite the criticisms presented in this section on the application of the methodology, the methodology did accomplish its goal of providing traceability and thus dependability to the model development process. It was also shown to be beneficial to the model validation process and for developing system theories.

## CHAPTER VII

### CONCLUSIONS

#### *7.1 Summary of Methodology Development*

This thesis began by defining a model as an abstraction of reality or an idea that is used as an aid in answering a set of questions or to aid in communication. A simulation was defined as the execution of the model. From there a good model was defined as one that represents the system that is to be emulated to the appropriate level of fidelity based on the questions presented for the study. This led to a review of verification and validation. A deficiency was found in validation efforts for non-observable systems, i.e. a system where controlled experimentation of the real system for data collection is not feasible or viable. The only means by which a non-observable system can be validated is through subjective operational validation or subjective conceptual model validation. It was observed that subjective operational validation in isolation is not sufficient for models of non-observable systems. This was shown with a military operation simulation of a patrol mission. The simulation was subjectively validated for three different representations of behavior; however, the results for the three representations were very different. Since subjective operational validation in isolation is not sufficient for non-observable system, it was concluded that the primary form of model validation of non-observable systems had to come from the model development. This led to the research objective: identify a methodology to develop a model in a traceable and defensible manner for a non-observable system and has a limited or non-existent history of modeling.

The first research question that arose from the research objective was to identify a model development procedure that is best suited for a non-observable system. Seven model development procedures available from the literature were investigated and rated against a set of criteria. It was then concluded that a procedure presented by Balci in 1986 should be used for model development of non-observable systems. This resulted in the hypothesis to

research question one and set the foundation for the remainder of the thesis. The selection of Balci's 1986 procedure then required the investigation into a procedure for developing the 'system and objective definition', 'conceptual model', 'communicative model', 'experimental model', and the analysis of the 'simulation results'.

From Balci's procedure the step 'system and objective definition' was investigated to determine the possible techniques that could be used to complete this step. Research question two inquired into which technique should be used for this methodology. A set of criteria was defined to aid in the selection of a technique. Five possible techniques were identified from the literature that have previously been applied to system and objective definition. Based on the evaluation of the five identified techniques against the set of criteria it was hypothesized that the Systems Modeling Language (SysML) is best suited for system and objective definition withing this methodology.

The next step was to develop a conceptual model. A review of the literature of conceptual modeling found an increased interest on the subject within the past decade. Despite the increased efforts by the community to address conceptual modeling several shortcomings were identified. This lead to research question three which inquired how a conceptual model could be developed in a traceable and defensible manner that assigns the greatest model fidelity to the most critical components of the system. This lead to a methodology hypothesis and resulted in six sub research questions. Each sub research question was presented with a hypothesis that was supported with either experimentation or a logical argument that compared multiple options.

The proposed methodology for developing a conceptual model starts with a decomposition of impact variables. For a given output measure of the model, numerous variables affect its result. These variables are identified and placed in a hierarchical structure. Impact weightings are then applied to the decomposition. The impact weightings are subjective estimates that attempt to estimate the portion of the response that impact variable effects of the response. Based on these subjective weightings, model representation decisions are made. The subjective weightings help to determine what should be included within the model and to what fidelity it should be represented. Once the computerized model is



developed, these impact weightings will be compared to the output to act as a means of validation.

Numerous studies were conducted, and the best found mathematical measure of the impact variable weighting is the normalized Brownian Correlation. It was found that the normalized Brownian Correlation enabled the calculation and comparison of direct and indirect impacts between the initial subjective impact decomposition and the observed results.

The communicative model step was investigated to determine the possible techniques that could be used to complete this step. This formed research question four. A set of criteria was defined to aid in the selection of a technique. Several possible techniques were identified from the literature that have been applied to this step. Based on the analysis of the literature the Unified Modeling Language was selected for use in this step. This resulted in the hypothesis to research question four.

The final four research questions arise from performing the steps 'experimental model' and 'simulation results'. It was found that the literature was rich in techniques that can be used for developing experimental models and for analyzing simulation results. Despite this, two gaps were identified. The first recognized that many simulation studies of non-observable systems were stochastic in nature. It was observed that for many studies found within the literature that were not models of industrial systems the primary stochastic measure used was the mean. At times, the mean was the only measure that was used. This led to research question five that inquired into which stochastic measures should be used to analyze stochastic simulations. Four common stochastic measures were identified: mean, variance, binomial proportion, and quantiles. These four measures were taken of a stochastic simulation of a mine counter measures mission. It was concluded that the quantile measures perform best as measures of stochastic simulations.

While investigating research question five it was observed from the literature that the number of replications required for accurate estimation of stochastic measures was lacking. In order to determine the number of replications required a confidence interval width must be assumed; however, in the absence of defining a confidence interval width there was no

means to estimate how many replications would be needed. Research question six was posed to address this issue. This research question requires the following activities—for the four identified stochastic measures a large numerical study was performed to determine under what conditions the stochastic measure's confidence intervals are accurate. Then, numerous heuristics were presented for these stochastic measures.

Research question seven asks, which least squares method should be used in the regression of a stochastic response? The least squares methods of ordinary least squares, weighted least squares, and ordinary least squares with constant confidence intervals were investigated for the mean, binomial proportion, and quantile. It was concluded that the ordinary least squares with constant confidence intervals performed best.

Finally, research question eight asks the question, how many replications are required for an accurate regression of a stochastic measure? Investigation into this question resulted in the observation of the relationship between the ratio of the confidence interval width to the range of the measured sample values to the coefficient of determination to the true underlying model. It was found that a ratio value of 0.5 resulting in a coefficient of determination of 0.8. Therefore, if one were regressing a stochastic output and maintained a ratio of 0.5, then a coefficient of determination of 0.8 would be the highest that can be expected.

The research objective of this thesis was to identify a methodology to develop a model in a traceable and defensible manner for a non-observable system and has a limited or non-existent history of modeling. This was accomplished by addressing numerous research questions that arose from the procedure to develop a model. The final presented methodology provides a method for developing models of non-observable systems in a more traceable and defensible manner than methods.

The primary contribution of this thesis is to the field of M&S. Contributions are made to the practice of conceptual model development, a growing discussion within the literature over the past several years. The contribution to conceptual model development will aid in the development of models for non-observable systems. Additional contributions are made to the analysis of stochastic simulations. The methodology presented in this thesis will provide a new and robust method to develop and validate models in a traceable and

defensible manner.

## ***7.2 Summary of Methodology Proof of Concept***

A proof of concept is provided on the methodology developed within this thesis. Numerous scenarios were considered for the application of the methodology. The selected scenario was a Humanitarian Aid/Disaster Relief Mission, where the U.S. Navy has been tasked with distributing aid in an effective manner to the affected population. This scenario was selected because it provided structural, operational, and behavioral aspects.

Upon application of the proposed methodology, it was observed that subjective decomposition and weighting of the scenario proved to be a useful tool for guiding and justifying the form of the eventual model. A few shortcomings of the methodology were identified. The primary shortcomings identified were the linking of information between the steps of the model development procedure, and the difficulty in correctly identifying the structure of the system impacts decomposition. For further details on the conclusion of the proof of concept see the conclusion located in Chapter 6.

## ***7.3 Future Work***

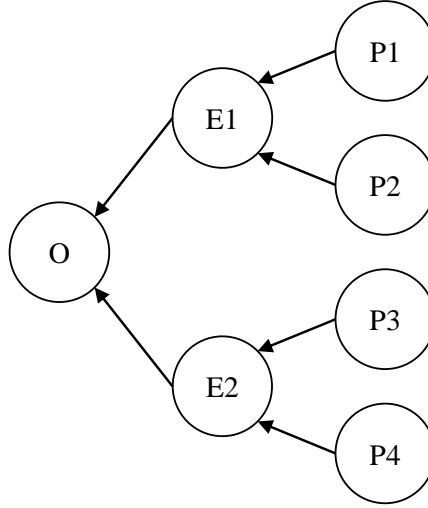
Research is never complete, and several research opportunities exist that can further expand and improve the methodology presented in this thesis. First, continued research into a mathematical measure that best represents the impact weighting. This thesis addressed a set of potential measures and selected the normalized Brownian Correlation. In retrospect the field of sensitivity analysis appears to provide several promising candidates that may outperform the normalized Brownian Correlation. Some potential candidates are standardized coefficients and variance based sensitivity analysis.

As identified in the conclusion of Chapter 6, there were some difficulties in transiting from system definition to impact variable decomposition. It was concluded that system definition should play a larger role in the development of the impact variable decomposition. Continued research would investigate a process to enable a better transition from system definition to impact variable decomposition.

The impact variable decomposition requires more research into the proper development

of its structure. Guiding principles are suggested, for no one approach could be applied to all modeling applications. Additionally, the hierarchical requirement of the impact variable decomposition was found to be limiting. Research is required to determine an analysis approach that would reduce the reliance of a structured hierarchy.

## APPENDIX A



**Figure 146:** Canonical Example for Indirect Impact Testing

$$\text{Corr}(O, E_i) = \frac{E[OE_i] - E[O]E[E_i]}{\sqrt{E[O^2] - E[O]^2 + \sqrt{E[E_i^2] - E[E_i]^2}}} \quad (57)$$

$$\text{Corr}(E_i P_i) = \frac{E[E_i P_i] - E[E_i]E[P_i]}{\sqrt{E[E_i^2] - E[E_i]^2 + \sqrt{E[P_i^2] - E[P_i]^2}}} \quad (58)$$

$$\text{Corr}(OP_i) = \frac{E[OP_i] - E[O]E[P_i]}{\sqrt{E[O^2] - E[O]^2 + \sqrt{E[P_i^2] - E[P_i]^2}}} \quad (59)$$

### A.1 Level 1

$$\begin{aligned}
P_i &= a_i U(0, 1) + b_i \forall i \in [1, 2, 3, 4] \\
E[U^k(0, 1)] &= 1/1 + k \\
E[P_i] &= a_i E[U(0, 1)] + b_i \\
E[P_i^2] &= a_i^2 E[U^2(0, 1)] + 2a_i b_i E[U(0, 1)] + b_i^2 \\
E[E_1] &= p_1 E[P_1] + p_2 E[P_2] \\
E[E_2] &= p_3 E[P_3] + p_4 E[P_4] \\
E[O] &= e_1 E[E_1] + e_2 E[E_2] \\
E[E_1 P_1] &= p_1 E[P_1^2] + p_2 E[P_1] E[P_2] \\
E[E_1 P_2] &= p_2 E[P_2^2] + p_1 E[P_1] E[P_2] \\
E[E_2 P_3] &= p_3 E[P_3^2] + p_4 E[P_3] E[P_4] \\
E[E_2 P_4] &= p_4 E[P_4^2] + p_3 E[P_3] E[P_4] \\
E[E_1^2] &= p_1^2 E[P_1^2] + 2p_1 p_2 E[P_1] E[P_2] + p_2^2 E[P_2^2] \\
E[E_2^2] &= p_3^2 E[P_3^2] + 2p_3 p_4 E[P_3] E[P_4] + p_4^2 E[P_4^2] \\
E[OE_1] &= e_1 E[E_1^2] + e_2 E[E_1] E[E_2] \\
E[OE_2] &= e_2 E[E_2^2] + e_1 E[E_1] E[E_2] \\
E[O^2] &= e_1^2 E[E_1^2] + 2e_1 e_2 E[E_1] E[E_2] + e_2^2 E[E_2^2]
\end{aligned} \tag{60}$$

## A.2 Level 2

$$\begin{aligned}
P_i &= a_i U(0, 1) + b_i \forall i \in [1, 2, 3, 4] \\
E[U^k(0, 1)] &= 1/1 + k \\
E[P_i] &= a_i E[U(0, 1)] + b_i \\
E[P_i^2] &= a_i^2 E[U^2(0, 1)] + 2a_i b_i E[U(0, 1)] + b_i^2 \\
E[E_1] &= p_1 E[P_1] + p_2 E[P_2] + p_{12} E[P_1] E[P_2] \\
E[E_2] &= p_3 E[P_3] + p_4 E[P_4] + p_{34} E[P_3] E[P_4] \\
E[O] &= e_1 E[E_1] + e_2 E[E_2] + e_{12} E[E_1] E[E_2] \\
E[E_1 P_1] &= p_1 E[P_1^2] + p_2 E[P_1] E[P_2] + p_{12} E[P_1^2] E[P_2] \\
E[E_1 P_2] &= p_2 E[P_2^2] + p_1 E[P_1] E[P_2] + p_{12} E[P_1] E[P_2^2] \\
E[E_2 P_3] &= p_3 E[P_3^2] + p_4 E[P_3] E[P_4] + p_{34} E[P_3^2] E[P_4] \\
E[E_2 P_4] &= p_4 E[P_4^2] + p_3 E[P_3] E[P_4] + p_{34} E[P_3] E[P_4^2] \\
E[E_1^2] &= p_1^2 E[P_1^2] + 2p_1 p_2 E[P_1] E[P_2] + p_2^2 E[P_2^2] + 2p_1 p_{12} E[P_1^2] E[P_2] \\
&\quad + 2p_2 p_{12} E[P_1] E[P_2^2] + p_{12}^2 E[P_1^2] E[P_2^2] \\
E[E_2^2] &= p_3^2 E[P_3^2] + 2p_3 p_4 E[P_3] E[P_4] + p_4^2 E[P_4^2] + 2p_3 p_{34} E[P_3^2] E[P_4] \\
&\quad + 2p_4 p_{34} E[P_3] E[P_4^2] + p_{34}^2 E[P_3^2] E[P_4^2] \\
E[OE_1] &= e_1 E[E_1^2] + e_2 E[E_1] E[E_2] + e_{12} E[E_1^2] E[E_2] \\
E[OE_2] &= e_2 E[E_2^2] + e_1 E[E_1] E[E_2] + e_{12} E[E_1^2] E[E_2] \\
E[O^2] &= e_1^2 E[E_1^2] + 2e_1 e_2 E[E_1] E[E_2] + e_2^2 E[E_2^2] + 2e_1 e_{12} E[E_1^2] E[E_2] \\
&\quad + 2e_2 e_{12} E[E_1] E[E_2^2] + e_{12}^2 E[E_1^2] E[E_2^2]
\end{aligned} \tag{61}$$

### A.3 Level 3

$$\begin{aligned}
P_i &= a_i U(0, 1) + b_i \forall i \in [1, 2, 3, 4] \\
E[U^k(0, 1)] &= 1/1 + k \\
E[P_i] &= a_i E[U(0, 1)] + b_i \\
E[P_i^2] &= a_i^2 E[U^2(0, 1)] + 2a_i b_i E[U(0, 1)] + b_i^2 \\
E[P_i^3] &= a_i^3 E[U^3(0, 1)] + 3a_i^2 b_i E[U^2(0, 1)] + 3a_i b_i^2 E[U(0, 1)] + b_i^3 \\
E[P_i^4] &= a_i^4 E[E[U^4(0, 1)](0, 1)] + 4a_i^3 b_i E[U^3(0, 1)] + 6a_i^2 b_i^2 E[U^2(0, 1)] \\
&\quad + 4a_i b_i^3 E[U(0, 1)] + b_i^4 \\
E[P_i^5] &= a_i^5 E[U^5(0, 1)] + 5a_i^4 E[U^4(0, 1)] b_i + 10a_i^3 E[U^3(0, 1)] b_i^2 \\
&\quad + 10a_i^2 E[U^2(0, 1)] b_i^3 + 5a_i E[U(0, 1)] b_i^4 + b_i^5 \tag{62} \\
E[P_i^6] &= a_i^6 E[U^6(0, 1)] + 6a_i^5 E[U^5(0, 1)] b_i + 15a_i^4 E[U^4(0, 1)] b_i^2 \\
&\quad + 20a_i^3 E[U^3(0, 1)] b_i^3 + 15a_i^2 E[U^2(0, 1)] b_i^4 + 6a_i E[U(0, 1)] b_i^5 + b_i^6 \\
E[P_i^7] &= a_i^7 E[U^7(0, 1)] + 7a_i^6 E[U^6(0, 1)] b_i + 21a_i^5 E[U^5(0, 1)] b_i^2 \\
&\quad + 35a_i^4 E[U^4(0, 1)] b_i^3 + 35a_i^3 E[U^3(0, 1)] b_i^4 + 21a_i^2 E[U^2(0, 1)] b_i^5 \\
&\quad + 7a_i E[U(0, 1)] b_i^6 + b_i^7 \\
E[P_i^8] &= a_i^8 E[U^8(0, 1)] + 8a_i^7 E[U^7(0, 1)] b_i + 28a_i^6 E[U^6(0, 1)] b_i^2 \\
&\quad + 56a_i^5 E[U^5(0, 1)] b_i^3 + 70a_i^4 E[U^4(0, 1)] b_i^4 + 56a_i^3 E[U^3(0, 1)] b_i^5 \\
&\quad + 28a_i^2 E[U^2(0, 1)] b_i^6 + 8a_i E[U(0, 1)] b_i^7 + b_i^8
\end{aligned}$$



$$\begin{aligned}
E[E_1] &= p_1 E[P_1] + p_2 E[P_2] + p_{12} E[P_1] E[P_2] + p_{11} E[P_1^2] + p_{22} E[P_2^2] \\
E[E_1^2] &= p_1^2 E[P_1^2] + 2p_1 p_2 E[P_1] E[P_2] + p_2^2 E[P_2^2] + 2p_1 p_{12} E[P_1^2] E[P_2] \\
&\quad + 2p_2 p_{12} E[P_1] E[P_2^2] + p_{12}^2 E[P_1^2] E[P_2^2] + p_{11}^2 E[P_1^4] + p_{22}^2 E[P_2^4] \\
&\quad + 2p_1 p_{11} E[P_1^3] + 2p_1 p_{22} E[P_1] E[P_2^2] + 2p_2 p_{11} E[P_1^2] E[P_2] + 2p_2 p_{22} E[P_2^3] \\
&\quad + 2p_{12} p_{11} E[P_1^3] E[P_2] + 2p_{12} p_{22} E[P_1] E[P_2^3] + 2p_{11} p_{22} E[P_1^2] E[P_2^2] \\
E[E_1^3] &= 3p_1^2 E[P_1^4] p_{11} + 3p_2 E[P_2^5] p_{22}^2 + 3p_1 E[P_1^5] p_{11}^2 \\
&\quad + p_{12}^3 E[P_1^3] E[P_2^3] + 3p_2^2 E[P_2^4] p_{22} + 6p_1 E[P_1] p_2 E[P_2^3] p_{22} \\
&\quad + 6p_1 E[P_1^2] p_2 E[P_2^2] p_{12} + 6p_1 E[P_1^3] p_2 E[P_2] p_{11} + 6p_1 E[P_1^4] p_{12} E[P_2] p_{11} \\
&\quad + 6p_1 E[P_1^2] p_{12} E[P_2^3] p_{22} + 6p_1 E[P_1^3] p_{11} p_{22} E[P_2^2] + 6p_2 E[P_2^2] p_{12} E[P_1^3] p_{11} \\
&\quad + 6p_2 E[P_2^4] p_{12} E[P_1] p_{22} + 6p_2 E[P_2^3] p_{11} E[P_1^2] p_{22} + 6p_{12} E[P_1^3] E[P_2^3] p_{11} p_{22} \\
&\quad + p_1^3 E[P_1^3] + p_2^3 E[P_2^3] + p_{11}^3 E[P_1^6] \\
&\quad + p_{22}^3 E[P_2^6] + 3p_1 E[P_1] p_2^2 E[P_2^2] + 3p_1 E[P_1] p_{22}^2 E[P_2^4] \\
&\quad + 3p_1 E[P_1^3] p_{12}^2 E[P_2^2] + 3p_1^2 E[P_1^2] p_2 E[P_2] + 3p_1^2 E[P_1^3] p_{12} E[P_2] \\
&\quad + 3p_1^2 E[P_1^2] p_{22} E[P_2^2] + 3p_2 E[P_2] p_{11}^2 E[P_1^4] + 3p_2 E[P_2^3] p_{12}^2 E[P_1^2] \\
&\quad + 3p_2^2 E[P_2^3] p_{12} E[P_1] + 3p_2^2 E[P_2^2] p_{11} E[P_1^2] + 3p_{12} E[P_1^5] E[P_2] p_{11}^2 \\
&\quad + 3p_{12} E[P_1] E[P_2^5] p_{22}^2 + 3p_{12}^2 E[P_1^4] E[P_2^2] p_{11} + 3p_{12}^2 E[P_1^2] E[P_2^4] p_{22} \\
&\quad + 3p_{11} E[P_1^2] p_{22}^2 E[P_2^4] + 3p_{11}^2 E[P_1^4] p_{22} E[P_2^2]
\end{aligned} \tag{63}$$

$$\begin{aligned}
E[E_1^4] = & 4p_2^3 E[P_2^5] p_{22} + 4p_1 E[P_1^7] p_{11}^3 + 4p_1^3 E[P_1^5] p_{11} \\
& + 4p_2 E[P_2^7] p_{22}^3 + p_{12}^4 E[P_1^4] E[P_2^4] + 6p_1^2 E[P_1^6] p_{11}^2 \\
& + 6p_2^2 E[P_2^6] p_{22}^2 + 12p_1 E[P_1] p_2 E[P_2^5] p_{22}^2 + 12p_1 E[P_1] p_2^2 E[P_2^4] p_{22} \\
& + 12p_1^2 E[P_1^2] p_2 E[P_2^3] p_{22} + 12p_1^2 E[P_1^3] p_2 E[P_2^2] p_{12} + 12p_1^2 E[P_1^4] p_2 E[P_2] p_{11} \\
& + 12p_1^2 E[P_1^5] p_{12} E[P_2] p_{11} + 12p_1^2 E[P_1^3] p_{12} E[P_2^3] p_{22} + 12p_1^2 E[P_1^4] p_{11} p_{22} E[P_2^2] \\
& + 12p_1 E[P_1^5] p_2 E[P_2] p_{11}^2 + 12p_1 E[P_1^3] p_2 E[P_2^3] p_{12}^2 + 12p_1 E[P_1^2] p_2^2 E[P_2^3] p_{12} \\
& + p_1^4 E[P_1^4] + p_2^4 E[P_2^4] + p_{11}^4 E[P_1^8] + p_{22}^4 E[P_2^8] + 24p_1 E[P_1^4] p_2 E[P_2^2] p_{12} p_{11} \\
& + 24p_1 E[P_1^2] p_2 E[P_2^4] p_{12} p_{22} + 24p_1 E[P_1^3] p_2 E[P_2^3] p_{11} p_{22} \\
& + 24p_1 E[P_1^4] p_{12} E[P_2^3] p_{11} p_{22} + 24p_2 E[P_2^4] p_{12} E[P_1^3] p_{11} p_{22} \\
& + 12p_1 E[P_1^3] p_2^2 E[P_2^2] p_{11} + 12p_1 E[P_1^6] p_{12} E[P_2] p_{11}^2 + 12p_1 E[P_1^2] p_{12} E[P_2^5] p_{22}^2 \\
& + 12p_1 E[P_1^5] p_{12}^2 E[P_2^2] p_{11} + 12p_1 E[P_1^3] p_{12}^2 E[P_2^4] p_{22} + 12p_1 E[P_1^3] p_{11} p_{22}^2 E[P_2^4] \\
& + 12p_1 E[P_1^5] p_{11}^2 p_{22} E[P_2^2] + 12p_2^2 E[P_2^3] p_{12} E[P_1^3] p_{11} + 12p_2^2 E[P_2^5] p_{12} E[P_1] p_{22} \\
& + 12p_2^2 E[P_2^4] p_{11} E[P_1^2] p_{22} + 12p_2 E[P_2^2] p_{12} E[P_1^5] p_{11}^2 + 12p_2 E[P_2^6] p_{12} E[P_1] p_{22}^2 \\
& + 12p_2 E[P_2^3] p_{12}^2 E[P_1^4] p_{11} + 12p_2 E[P_2^5] p_{12}^2 E[P_1^2] p_{22} + 12p_2 E[P_2^5] p_{11} E[P_1^2] p_{22}^2 \\
& + 12p_2 E[P_2^3] p_{11}^2 E[P_1^4] p_{22} + 12p_{12}^2 E[P_1^4] E[P_2^4] p_{11} p_{22} + 12p_{12} E[P_1^3] E[P_2^5] p_{11} p_{22}^2 \\
& + 12p_{12} E[P_1^5] E[P_2^3] p_{11}^2 p_{22} + 6p_{11}^2 E[P_1^4] p_{22}^2 E[P_2^4] + 4p_{11}^3 E[P_1^6] p_{22} E[P_2^2] \\
& + 4p_{12}^3 E[P_1^5] E[P_2^3] p_{11} + 4p_{12}^3 E[P_1^3] E[P_2^5] p_{22} + 4p_{11} E[P_1^2] p_{22}^3 E[P_2^6] \\
& + 4p_2^3 E[P_2^3] p_{11} E[P_1^2] + 4p_{12} E[P_1^7] E[P_2] p_{11}^3 + 4p_{12} E[P_1] E[P_2^7] p_{22}^3 \\
& + 6p_{12}^2 E[P_1^6] E[P_2^2] p_{11}^2 + 6p_{12}^2 E[P_1^2] E[P_2^6] p_{22}^2 + 4p_1^3 E[P_1^3] p_{22} E[P_2^2] \\
& + 4p_2 E[P_2^4] p_{12}^3 E[P_1^3] + 4p_2 E[P_2] p_{11}^3 E[P_1^6] + 6p_2^2 E[P_2^2] p_{11}^2 E[P_1^4] \\
& + 6p_2^2 E[P_2^4] p_{12}^2 E[P_1^2] + 4p_2^3 E[P_2^4] p_{12} E[P_1] + 4p_1 E[P_1^4] p_{12}^3 E[P_2^3] \\
& + 4p_1 E[P_1] p_2^3 E[P_2^3] + 4p_1 E[P_1] p_2^3 E[P_2^6] + 6p_1^2 E[P_1^2] p_2^2 E[P_2^2] \\
& + 6p_1^2 E[P_1^2] p_2^2 E[P_2^4] + 6p_1^2 E[P_1^4] p_{12}^2 E[P_2^2] + 4p_1^3 E[P_1^3] p_2 E[P_2] \\
& + 4p_1^3 E[P_1^4] p_{12} E[P_2]
\end{aligned}$$

(64)

$$\begin{aligned}
E[E_1 P_1] &= p_1 E[P_1^2] + p_2 E[P_1] E[P_2] + p_{12} E[P_1^2] E[P_2] + p_{11} E[P_1^3] + p_{22} E[P_1] E[P_2^2] \\
E[E_1 P_2] &= p_2 E[P_2^2] + p_1 E[P_1] E[P_2] + p_{12} E[P_1] E[P_2^2] + p_{11} E[P_1^2] E[P_2] + p_{22} E[P_2^3] \\
E[E_1^2 P_1] &= p_1^2 E[P_1^3] + E[P_1] p_2^2 E[P_2^2] + p_{11}^2 E[P_1^5] \\
&\quad + E[P_1] p_{22}^2 E[P_2^4] + 2p_1 E[P_1^4] p_{11} + p_{12}^2 E[P_1^3] E[P_2^2] \\
&\quad + 2E[P_1] p_2 E[P_2^3] p_{22} + 2p_1 E[P_1^2] p_2 E[P_2] + 2p_1 E[P_1^3] p_{12} E[P_2] \\
&\quad + 2p_1 E[P_1^2] p_{22} E[P_2^2] + 2p_2 E[P_2^2] p_{12} E[P_1^2] + 2p_2 E[P_2] p_{11} E[P_1^3] \\
&\quad + 2p_{12} E[P_1^4] E[P_2] p_{11} + 2p_{12} E[P_1^2] E[P_2^3] p_{22} + 2p_{11} E[P_1^3] p_{22} E[P_2^2] \\
E[E_1^2 P_2] &= E[P_2] p_1^2 E[P_1^2] + p_2^2 E[P_2^3] + E[P_2] p_{11}^2 E[P_1^4] \\
&\quad + p_{22}^2 E[P_2^5] + 2E[P_2] p_1 E[P_1^3] p_{11} + p_{12}^2 E[P_1^2] E[P_2^3] \\
&\quad + 2p_2 E[P_2^4] p_{22} + 2p_1 E[P_1] p_2 E[P_2^2] + 2p_1 E[P_1^2] p_{12} E[P_2^2] \\
&\quad + 2p_1 E[P_1] p_{22} E[P_2^3] + 2p_2 E[P_2^3] p_{12} E[P_1] + 2p_2 E[P_2^2] p_{11} E[P_1^2] \\
&\quad + 2p_{12} E[P_1^3] E[P_2^2] p_{11} + 2p_{12} E[P_1] E[P_2^4] p_{22} + 2p_{11} E[P_1^2] p_{22} E[P_2^3]
\end{aligned}
\tag{65}$$

$$\begin{aligned}
E[E_2] &= p_3E[P_3] + p_4E[P_4] + p_{34}E[P_3]E[P_4] + p_{11}E[P_3^2] + p_{22}E[P_4^2] \\
E[E_2^2] &= p_3^2E[P_3^2] + 2p_3p_4E[P_3]E[P_4] + p_4^2E[P_4^2] + 2p_3p_{34}E[P_3^2]E[P_4] \\
&\quad + 2p_4p_{34}E[P_3]E[P_4^2] + p_{34}^2E[P_3^2]E[P_4^2] + p_{33}^2E[P_3^4] + p_{44}^2E[P_4^4] \\
&\quad + 2p_3p_{33}E[P_3^3] + 2p_3p_{44}E[P_3]E[P_4^2] + 2p_4p_{33}E[P_3^2]E[P_4] + 2p_4p_{44}E[P_4^3] \\
&\quad + 2p_3p_{34}p_{33}E[P_3^3]E[P_4] + 2p_3p_{44}E[P_3]E[P_4^3] + 2p_{33}p_{44}E[P_3^2]E[P_4^2] \\
E[E_2^3] &= 3p_3^2E[P_3^4]p_{33} + 3p_4E[P_4^5]p_{44}^2 + 3p_3E[P_3^5]p_{33}^2 \\
&\quad + p_{34}^3E[P_3^3]E[P_4^3] + 3p_4^2E[P_4^4]p_{44} + 6p_3E[P_3]p_4E[P_4^3]p_{44} \\
&\quad + 6p_3E[P_3^2]p_4E[P_4^2]p_{34} + 6p_3E[P_3^3]p_4E[P_4]p_{33} + 6p_3E[P_3^4]p_{34}E[P_4]p_{33} \\
&\quad + 6p_3E[P_3^2]p_{34}E[P_4^3]p_{44} + 6p_3E[P_3^3]p_{33}p_{44}E[P_4^2] + 6p_4E[P_4^2]p_{34}E[P_3^3]p_{33} \\
&\quad + 6p_4E[P_4^4]p_{34}E[P_3]p_{44} + 6p_4E[P_4^3]p_{33}E[P_3^2]p_{44} + 6p_{34}E[P_3^3]E[P_4^3]p_{33}p_{44} \\
&\quad + p_3^3E[P_3^3] + p_4^3E[P_4^3] + p_{33}^3E[P_3^6] \\
&\quad + p_{44}^3E[P_4^6] + 3p_3E[P_3]p_4^2E[P_4^2] + 3p_3E[P_3]p_{44}^2E[P_4^4] \\
&\quad + 3p_3E[P_3^3]p_{34}^2E[P_4^2] + 3p_3^2E[P_3^2]p_4E[P_4] + 3p_3^2E[P_3^3]p_{34}E[P_4] \\
&\quad + 3p_3^2E[P_3^2]p_{44}E[P_4^2] + 3p_4E[P_4]p_{33}^2E[P_3^4] + 3p_4E[P_4^3]p_{34}^2E[P_3^2] \\
&\quad + 3p_4^2E[P_4^3]p_{34}E[P_3] + 3p_4^2E[P_4^2]p_{33}E[P_3^2] + 3p_{34}E[P_3^5]E[P_4]p_{33}^2 \\
&\quad + 3p_{34}E[P_3]E[P_4^5]p_{44}^2 + 3p_{34}^2E[P_3^4]E[P_4^2]p_{33} + 3p_{34}^2E[P_3^2]E[P_4^4]p_{44} \\
&\quad + 3p_{33}E[P_3^2]p_{44}^2E[P_4^4] + 3p_{33}^2E[P_3^4]p_{44}E[P_4^2]
\end{aligned}$$

(66)

$$\begin{aligned}
E[E_2^4] = & 4p_4^3 E[P_4^5] p_{44} + 4p_3 E[P_3^7] p_{33}^3 + 4p_3^3 E[P_3^5] p_{33} \\
& + 4p_4 E[P_4^7] p_{44}^3 + p_{34}^4 E[P_3^4] E[P_4^4] + 6p_3^2 E[P_3^6] p_{33}^2 \\
& + 6p_4^2 E[P_4^6] p_{44}^2 + 12p_3 E[P_3] p_4 E[P_4^5] p_{44}^2 + 12p_3 E[P_3] p_4^2 E[P_4^4] p_{44} \\
& + 12p_3^2 E[P_3^2] p_4 E[P_4^3] p_{44} + 12p_3^2 E[P_3^3] p_4 E[P_4^2] p_{34} + 12p_3^2 E[P_3^4] p_4 E[P_4] p_{33} \\
& + 12p_3^2 E[P_3^5] p_{34} E[P_4] p_{33} + 12p_3^2 E[P_3^3] p_{34} E[P_4^3] p_{44} + 12p_3^2 E[P_3^4] p_{33} p_{44} E[P_4^2] \\
& + 12p_3 E[P_3^5] p_4 E[P_4] p_{33}^2 + 12p_3 E[P_3^3] p_4 E[P_4^3] p_{34}^2 + 12p_3 E[P_3^2] p_4^2 E[P_4^3] p_{34} \\
& + p_3^4 E[P_3^4] + p_4^4 E[P_4^4] + p_{33}^4 E[P_3^8] + p_{44}^4 E[P_4^8] + 24p_3 E[P_3^4] p_4 E[P_4^2] p_{34} p_{33} \\
& + 24p_3 E[P_3^2] p_4 E[P_4^4] p_{34} p_{44} + 24p_3 E[P_3^3] p_4 E[P_4^3] p_{33} p_{44} \\
& + 24p_3 E[P_3^4] p_{34} E[P_4^3] p_{33} p_{44} + 24p_4 E[P_4^4] p_{34} E[P_3^3] p_{33} p_{44} \\
& + 12p_3 E[P_3^3] p_4^2 E[P_4^2] p_{33} + 12p_3 E[P_3^6] p_{34} E[P_4] p_{33}^2 + 12p_3 E[P_3^2] p_{34} E[P_4^5] p_{44}^2 \\
& + 12p_3 E[P_3^5] p_{34}^2 E[P_4^2] p_{33} + 12p_3 E[P_3^3] p_{34}^2 E[P_4^4] p_{44} + 12p_3 E[P_3^3] p_{33} p_{44}^2 E[P_4^4] \\
& + 12p_3 E[P_3^5] p_{33}^2 p_{44} E[P_4^2] + 12p_4^2 E[P_4^3] p_{34} E[P_3^3] p_{33} + 12p_4^2 E[P_4^5] p_{34} E[P_3] p_{44} \\
& + 12p_4^2 E[P_4^4] p_{33} E[P_3^2] p_{44} + 12p_4 E[P_4^2] p_{34} E[P_3^5] p_{33}^2 + 12p_4 E[P_4^6] p_{34} E[P_3] p_{44}^2 \\
& + 12p_4 E[P_4^3] p_{34}^2 E[P_3^4] p_{33} + 12p_4 E[P_4^5] p_{34}^2 E[P_3^2] p_{44} + 12p_4 E[P_4^5] p_{33} E[P_3^2] p_{44}^2 \\
& + 12p_4 E[P_4^3] p_{33}^2 E[P_3^4] p_{44} + 12p_{34}^2 E[P_3^4] E[P_4^4] p_{33} p_{44} + 12p_{34} E[P_3^3] E[P_4^5] p_{33} p_{44}^2 \\
& + 12p_{34} E[P_3^5] E[P_4^3] p_{33}^2 p_{44} + 6p_{33}^2 E[P_3^4] p_{44}^2 E[P_4^4] + 4p_{33}^3 E[P_3^6] p_{44} E[P_4^2] \\
& + 4p_{34}^3 E[P_3^5] E[P_4^3] p_{33} + 4p_{34}^3 E[P_3^3] E[P_4^5] p_{44} + 4p_{33} E[P_3^2] p_{44}^3 E[P_4^6] \\
& + 4p_3^3 E[P_4^3] p_{33} E[P_3^2] + 4p_{34} E[P_3^7] E[P_4] p_{33}^3 + 4p_{34} E[P_3] E[P_4^7] p_{44}^3 \\
& + 6p_{34}^2 E[P_3^6] E[P_4^2] p_{33}^2 + 6p_{34}^2 E[P_3^2] E[P_4^6] p_{44}^2 + 4p_3^3 E[P_3^3] p_{44} E[P_4^4] \\
& + 4p_4 E[P_4^4] p_{34}^3 E[P_3^3] + 4p_4 E[P_4] p_{33}^3 E[P_3^6] + 6p_4^2 E[P_4^2] p_{33}^2 E[P_3^4] \\
& + 6p_4^2 E[P_4^4] p_{34}^2 E[P_3^2] + 4p_4^3 E[P_4^4] p_{34} E[P_3] + 4p_3 E[P_3^4] p_{34}^3 E[P_4^3] \\
& + 4p_3 E[P_3] p_4^3 E[P_4^3] + 4p_3 E[P_3] p_{44}^3 E[P_4^6] + 6p_3^2 E[P_3^2] p_4^2 E[P_4^2] \\
& + 6p_3^2 E[P_3^2] p_{44}^2 E[P_4^4] + 6p_3^2 E[P_3^4] p_{34}^2 E[P_4^2] + 4p_3^3 E[P_3^3] p_4 E[P_4] \\
& + 4p_3^3 E[P_3^4] p_{34} E[P_4]
\end{aligned}$$

(67)

$$\begin{aligned}
E[E_2P_3] &= p_3E[P_3^2] + p_4E[P_3]E[P_4] + p_{34}E[P_3^2]E[P_4] + p_{33}E[P_3^3] + p_{44}E[P_3]E[P_4^2] \\
E[E_2P_4] &= p_4E[P_4^2] + p_3E[P_3]E[P_4] + p_{34}E[P_3]E[P_4^2] + p_{33}E[P_3^2]E[P_4] + p_{44}E[P_4^3] \\
E[E_2^2P_3] &= p_3^2E[P_3^3] + E[P_3]p_4^2E[P_4^2] + p_{33}^2E[P_3^5] \\
&\quad + E[P_3]p_{44}^2E[P_4^4] + 2p_3E[P_3^4]p_{33} + p_{34}^2E[P_3^3]E[P_4^2] \\
&\quad + 2E[P_3]p_4E[P_4^3]p_{44} + 2p_3E[P_3^2]p_4E[P_4] + 2p_3E[P_3^3]p_{34}E[P_4] \\
&\quad + 2p_3E[P_3^2]p_{44}E[P_4^2] + 2p_4E[P_4^2]p_{34}E[P_3^2] + 2p_4E[P_4]p_{33}E[P_3^3] \\
&\quad + 2p_{34}E[P_3^4]E[P_4]p_{33} + 2p_{34}E[P_3^2]E[P_4^3]p_{44} + 2p_{33}E[P_3^3]p_{44}E[P_4^2] \\
E[E_2^2P_4] &= E[P_4]p_3^2E[P_3^2] + p_4^2E[P_4^3] + E[P_4]p_{33}^2E[P_3^4] \\
&\quad + p_{44}^2E[P_4^5] + 2E[P_4]p_3E[P_3^3]p_{33} + p_{34}^2E[P_3^2]E[P_4^3] \\
&\quad + 2p_4E[P_4^4]p_{44} + 2p_3E[P_3]p_4E[P_4^2] + 2p_3E[P_3^2]p_{34}E[P_4^2] \\
&\quad + 2p_3E[P_3]p_{44}E[P_4^3] + 2p_4E[P_4^3]p_{34}E[P_3] + 2p_4E[P_4^2]p_{33}E[P_3^2] \\
&\quad + 2p_{34}E[P_3^3]E[P_4^2]p_{33} + 2p_{34}E[P_3]E[P_4^4]p_{44} + 2p_{33}E[P_3^2]p_{44}E[P_4^3]
\end{aligned}
\tag{68}$$

$$\begin{aligned}
E[O] &= e_1E[E_1] + e_2E[E_2] + e_{12}E[E_1]E[E_2] + e_{11}E[E_1^2] + e_{22}E[E_2^2] \\
E[O^2] &= e_1^2E[E_1^2] + 2e_1e_2E[E_1]E[E_2] + e_2^2E[E_2^2] + 2e_1e_{12}E[E_1^2]E[E_2] \\
&\quad + 2e_2e_{12}E[E_1]E[E_2^2] + e_{12}^2E[E_1^2]E[E_2^2] + e_{11}^2E[E_1^4] + e_{22}^2E[E_2^4] \\
&\quad + 2e_1e_{11}E[E_1^3] + 2e_1e_{22}E[E_1]E[E_2^2] + 2e_2e_{11}E[E_1^2]E[E_2] \\
&\quad + 2e_2e_{22}E[E_2^3] + 2e_{12}e_{11}E[E_1^3]E[E_2] + 2e_{12}e_{22}E[E_1]E[E_2^3] \\
&\quad + 2e_{11}e_{22}E[E_1^2]E[E_2^2] \\
E[OP_1] &= e_1E[E_1P_1] + e_2E[E_2]E[P_1] + e_{12}E[E_1P_1]E[E_2] \\
&\quad + e_{11}E[E_1^2P_1] + e_{22}E[E_2^2]E[P_1] \\
E[OP_2] &= e_1E[E_1P_2] + e_2E[E_2]E[P_2] + e_{12}E[E_1P_2]E[E_2] \\
&\quad + e_{11}E[E_1^2P_2] + e_{22}E[E_2^2]E[P_2] \\
E[OP_3] &= e_1E[E_1]E[P_3] + e_2E[E_2P_3] + e_{12}E[E_1]E[E_2P_3] \\
&\quad + e_{11}E[E_1^2]E[P_3] + e_{22}E[E_2^2]P_3 \\
E[OP_4] &= e_1E[E_1]E[P_4] + e_2E[E_2P_4] + e_{12}E[E_1]E[E_2P_4] \\
&\quad + e_{11}E[E_1^2]E[P_4] + e_{22}E[E_2^2]P_4 \\
E[OE_1] &= e_1E[E_1^2] + e_2E[E_1]E[E_2] + e_{12}E[E_1^2]E[E_2] \\
&\quad + e_{11}E[E_1^3] + e_{22}E[E_2^2]E[E_1] \\
E[OE_2] &= e_2E[E_2^2] + e_1E[E_1]E[E_2] + e_{12}E[E_1^2]E[E_2] \\
&\quad + e_{11}E[E_1^2]E[E_2] + e_{22}E[E_2^3]
\end{aligned} \tag{69}$$

## APPENDIX B

### B.1 Requirement Decomposition

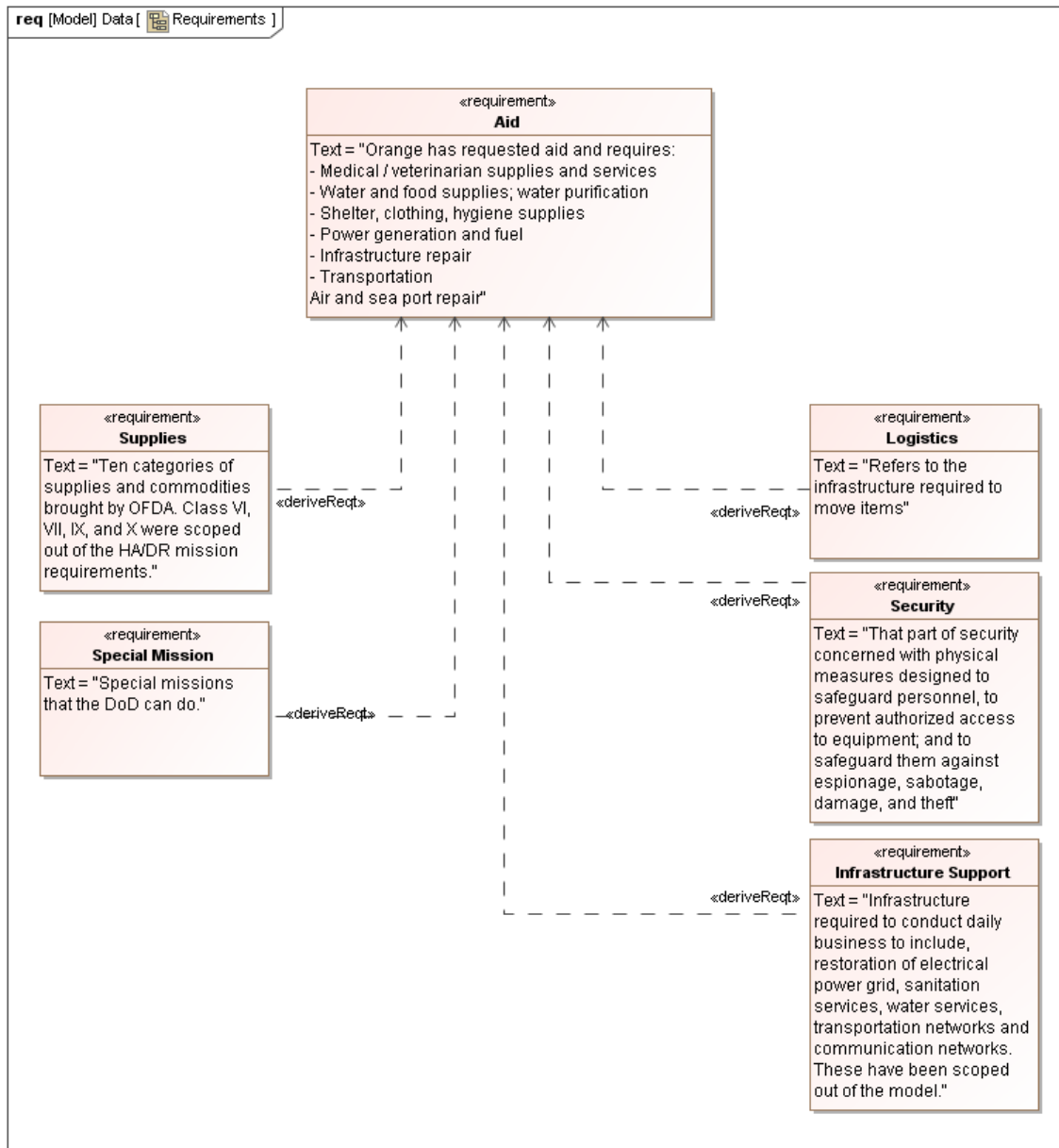


Figure 147: Requirements using Block Definition Diagram



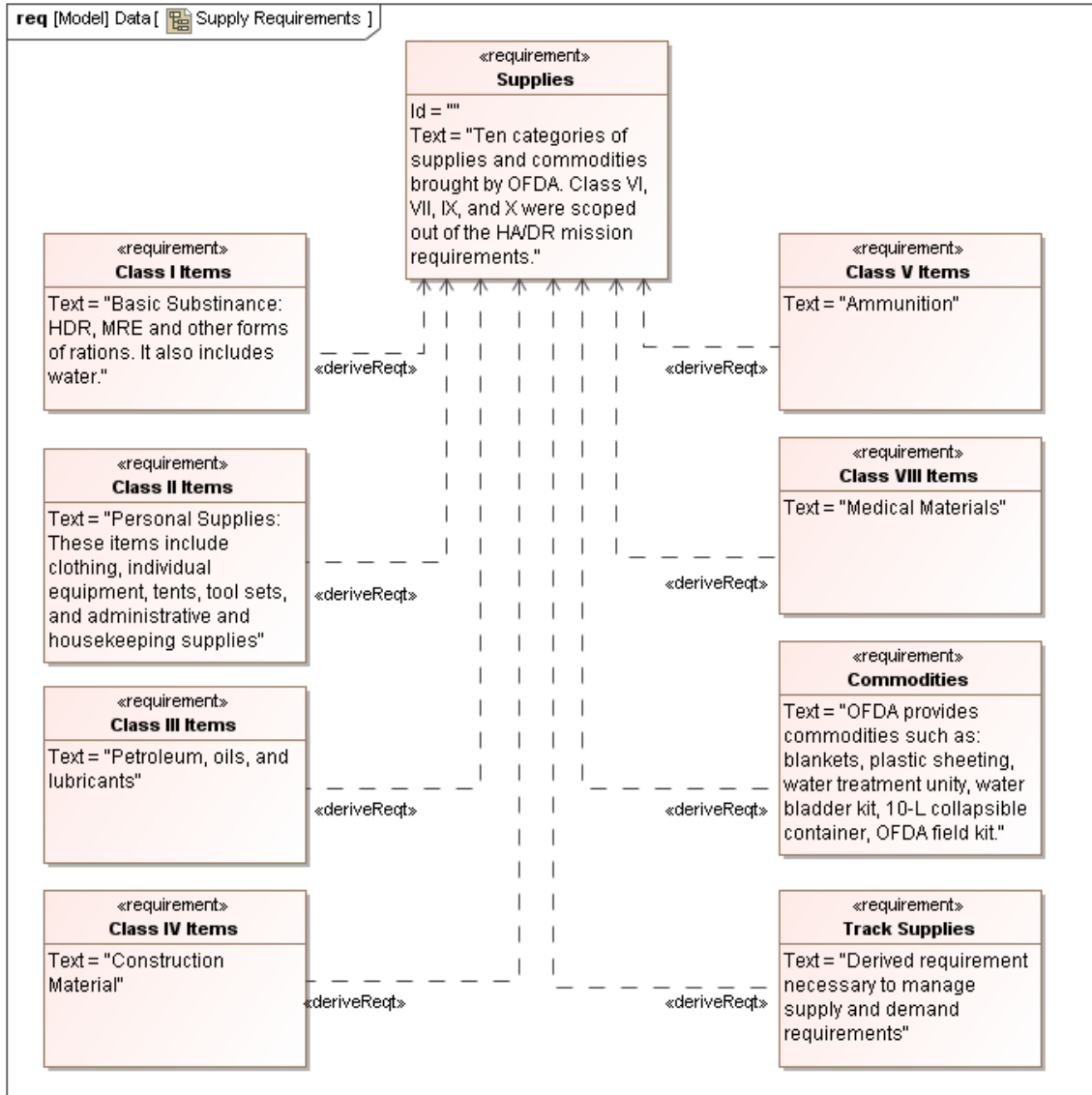


Figure 148: Supply Requirements using Block Definition Diagram

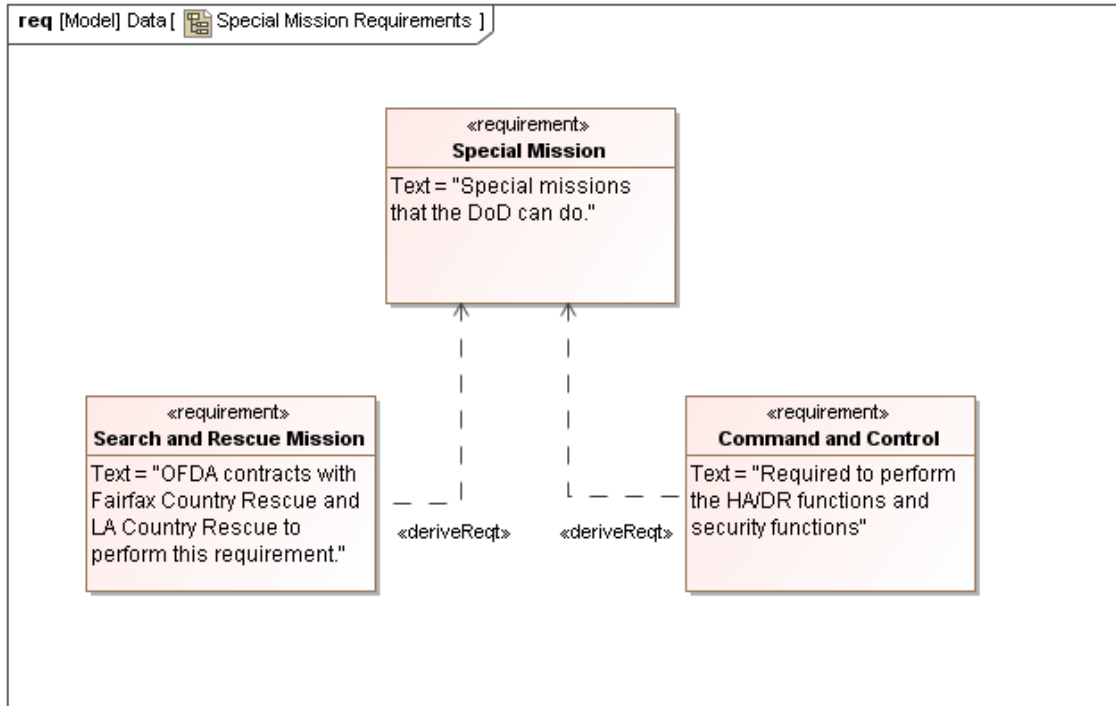


Figure 149: Special Mission Requirements using Block Definition Diagram

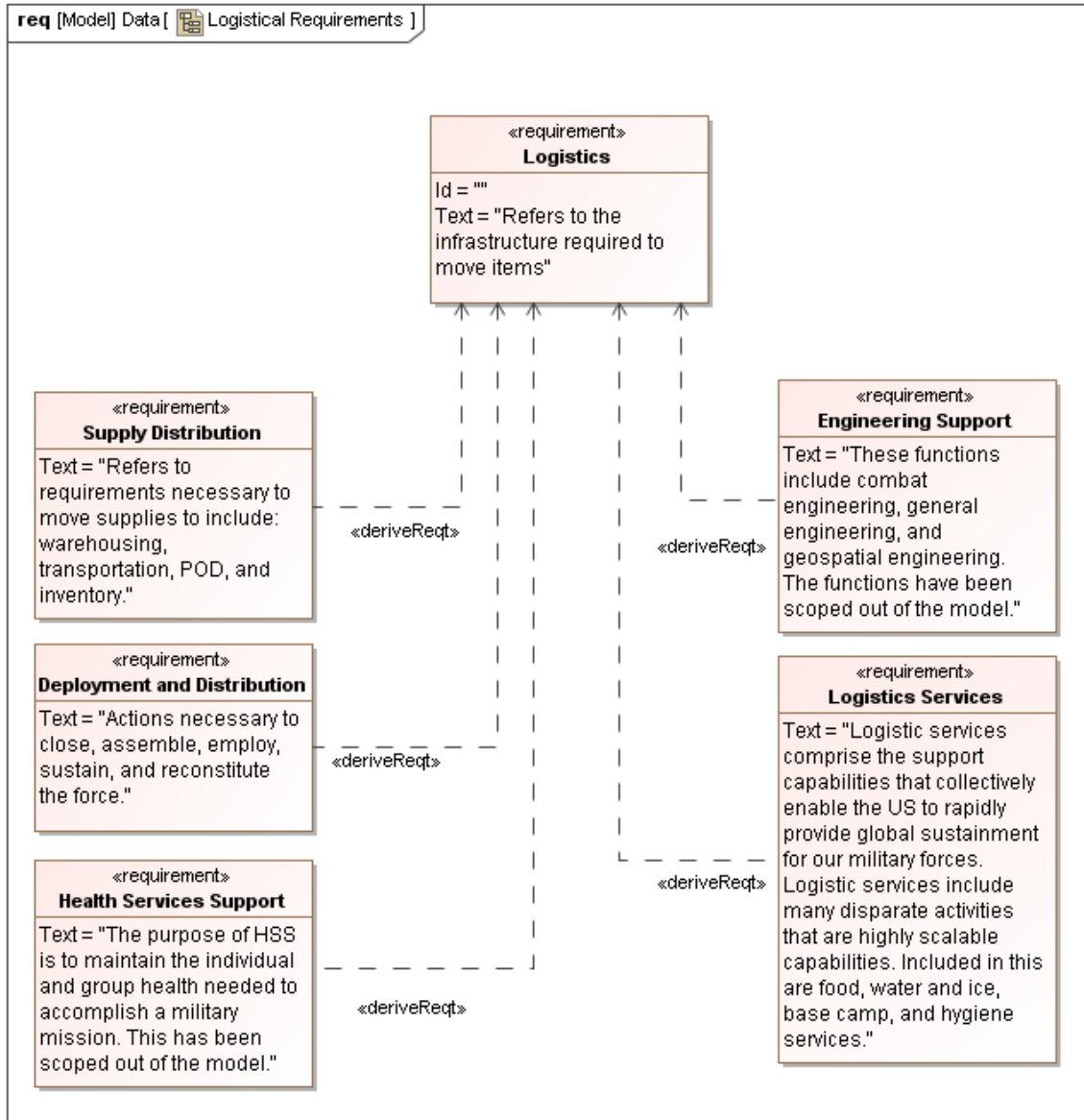


Figure 150: Logistical Requirements using Block Definition Diagram

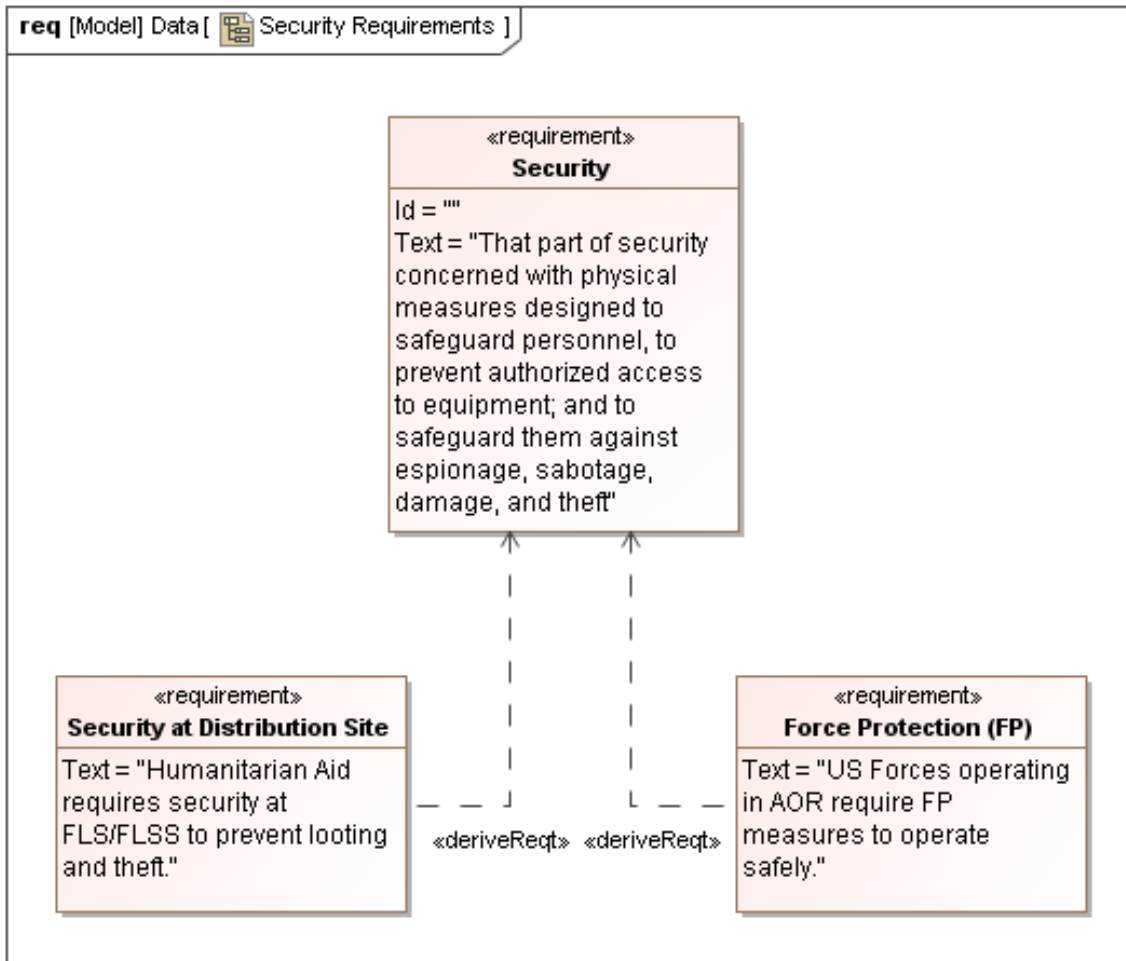


Figure 151: Security Requirements using Block Definition Diagram

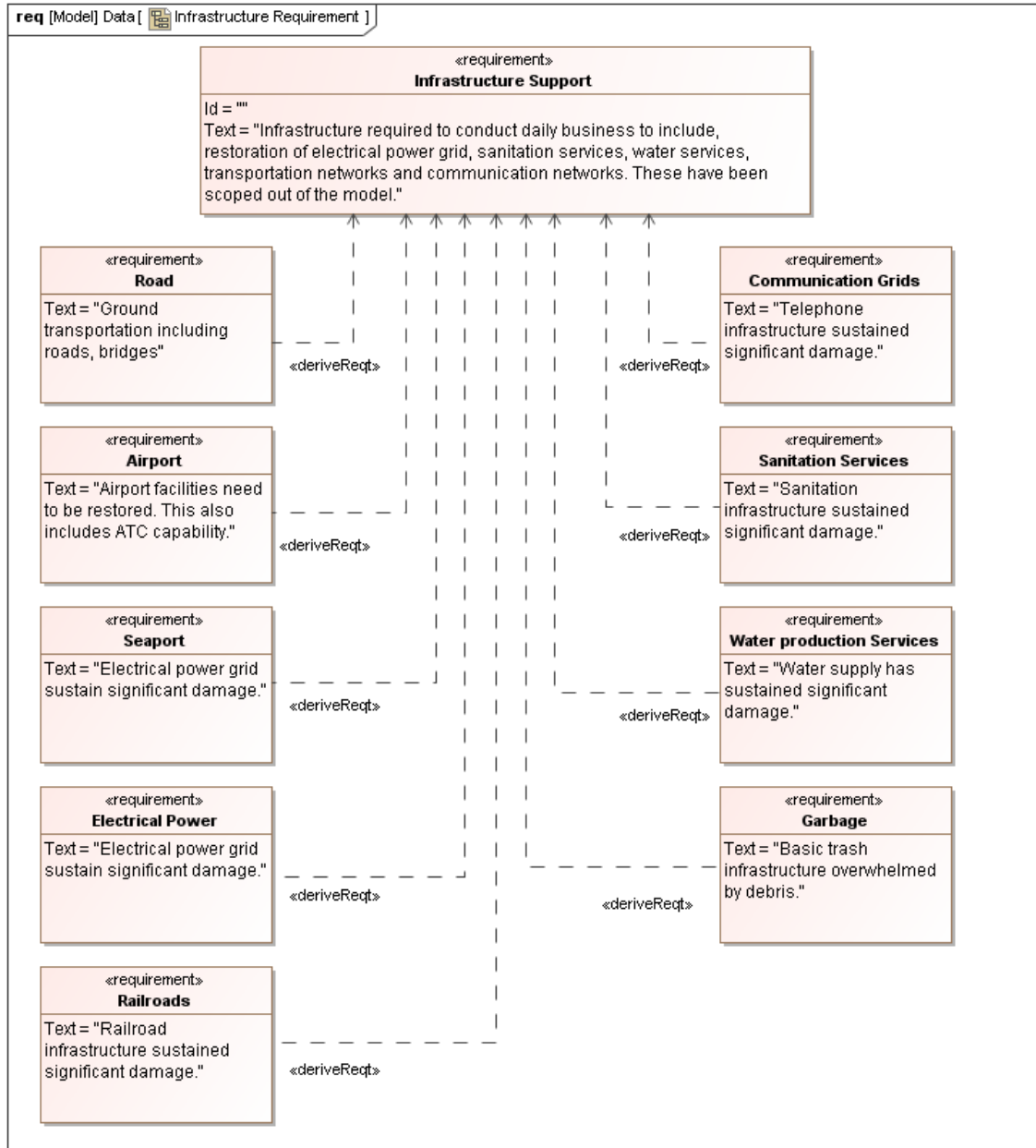


Figure 152: Infrastructure Requirements using Block Definition Diagram

## B.2 Functional Hierarchy

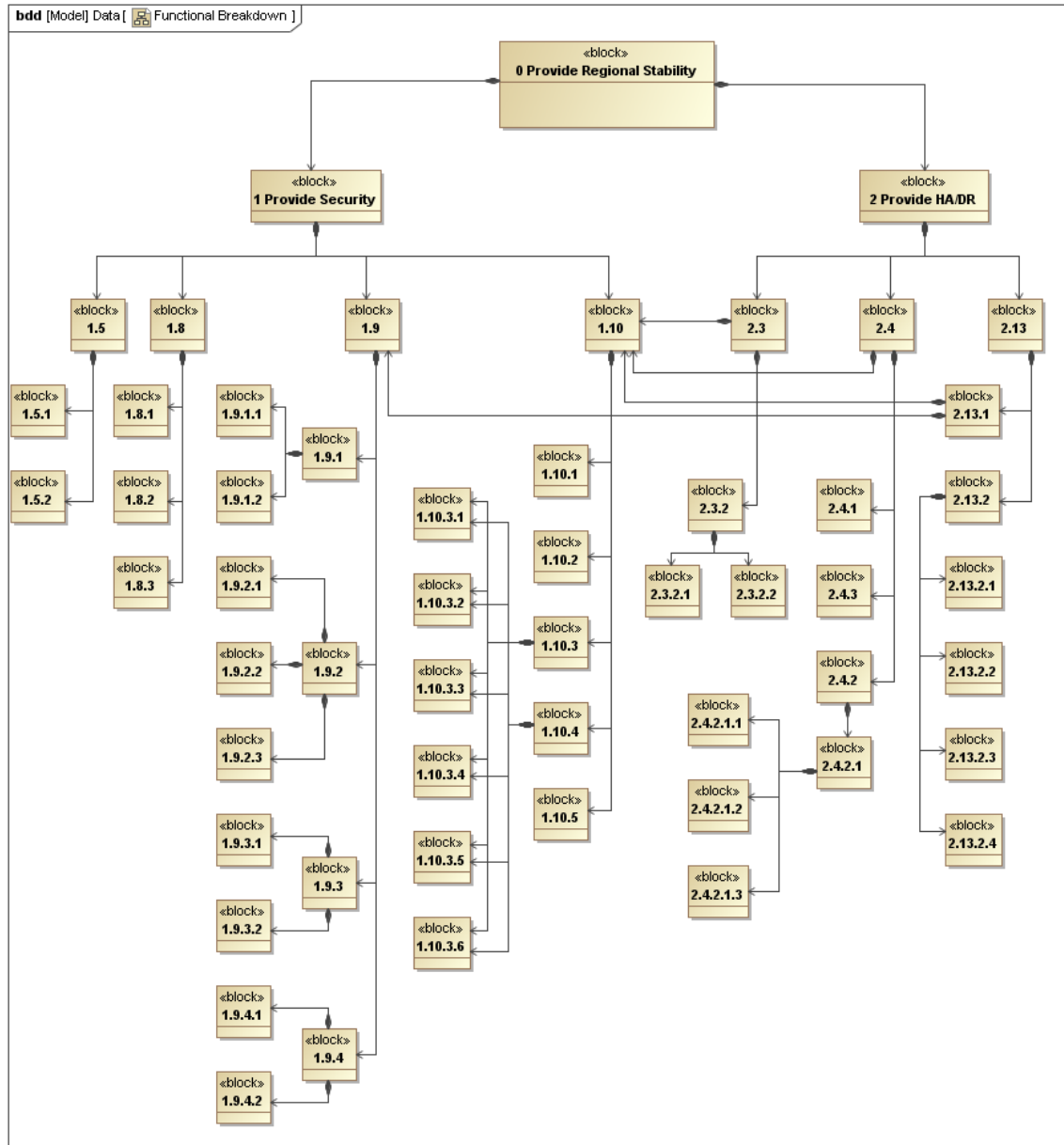


Figure 153: Functional Hierarchy using Block Definition Diagram

**Table 92: HA/DR Functional Hierarchy**

0	Provide Regional Stability
1	Provide Security
1.5	Provide security for HA goods
1.5.1	Provide security at FLS
1.5.2	Provide security at FLSS
1.8	Provide Force Protection
1.8.1	Provide air asset FP
1.8.2	Provide ground FP
1.8.3	Provide maritime FP
1.9	Provide Command and Control
1.9.1	Establish communication network
1.9.1.1	Establish RF network
1.9.1.2	Establish wireless link
1.9.2	Provide for logistics coordination
1.9.2.1	Consolidate supply status
1.9.2.2	Process replenishment request
1.9.2.3	Process redistribution request
1.9.3	Provide for manpower coordination
1.9.3.1	Consolidate manpower status
1.9.3.2	Support manpower requests
1.9.4	Provide for security coordination
1.9.4.1	Coordinate with international military
1.9.4.2	Provide intel support
1.10	Operate Sea Base Connector
1.10.1	Assemble force
1.10.2	Close force
1.10.3	Employ force

	1.10.3.1	Transport Class I
	1.10.3.2	Transport Class III
	1.10.3.3	Transport Class V
	1.10.3.4	Transport Class VI
	1.10.3.5	Transport Class VII
	1.10.3.6	Transport Class VIII
	1.10.4	Sustain force
	Ref	Reference 1.10.3.1
	Ref	Reference 1.10.3.2
	Ref	Reference 1.10.3.3
	Ref	Reference 1.10.3.4
	Ref	Reference 1.10.3.5
	Ref	Reference 1.10.3.6
	1.10.5	Reconstitute force
2		Provide HA/DR
2.3		Provide food security
	Ref	Link to 1.10
	2.3.2	Transport emergency food relief
	2.3.2.1	Transport Class I
	2.3.2.2	Provide tracking of deliverables
2.4		Provide non-food relief
	Ref	Reference to 1.10
	2.4.1	Transport OFDA commodities
	2.4.2	Provide/transport excess equipment
	2.4.2.1	Transport Class IV
	2.4.2.1.1	Transport construction materials
	2.4.2.1.2	Transport storage materials
	2.4.2.1.3	Transport medical supplies
	2.4.3	Transport Class II



2.13	Conduct special missions ISO HA/DR
2.13.1	Provide C4ISR ISO HA/DR
Ref	Reference to 1.9
Ref	Reference to 1.10
2.13.2	Provide logistical support
2.13.2.1	Provide warehousing capacity
2.13.2.2	Provide for liquid storage capability
2.13.2.3	Provide POD
2.13.2.4	Provide inventory tracking capability

### *B.3 Importance Weightings of Impact Variables*

**Table 93:** Flow Rate Level 1: Importance of Impact Values

Variable Name	Impact <sub>1</sub> <sup>E</sup>	Impact <sub>3</sub> <sup>E</sup>
Flow Rate: Aid FLS to Civilians	0.27	0.0386
Flow Rate: Aid FLSS to Civilians	0.73	0.1043

**Table 94:** Flow Rate Level 2: Importance of Impact Values

Variable Name	Impact <sub>1</sub> <sup>E</sup>	Impact <sub>3</sub> <sup>E</sup>
Flow Rate: Aid Surface to FLS	0.1350	0.0193
FLS Operations	0.1350	0.0193
Flow Rate: Aid Surface to FLSS	0.3650	0.0521
FLSS Operations	0.3650	0.0521

**Table 95:** Flow Rate Level 3: Importance of Impact Values

Variable Name	Impact <sub>1</sub> <sup>E</sup>	Impact <sub>3</sub> <sup>E</sup>
Flow Rate: Aid LCAC to FLS	0.1089	0.0156
Flow Rate: Aid LCU to FLS	0.0247	0.0035
FLS Dock Locations	0.0014	0.0002
FLS Wait Time	0.1227	0.0175
FLS Storage	0.0123	0.0018
Flow Rate: Aid MH-53E to FLS	0.1095	0.0156
Flow Rate: Aid S-60B to FLS	0.0110	0.0016
Flow Rate: Aid MV-22 to FLS	0.2446	0.0349
FLSS Dropoff Locations	0.0037	0.0005
FLSS Wait Time	0.3318	0.0474
FLSS Storage	0.0332	0.0047

**Table 96:** Flow Rate Level 4: Importance of Impact Values

Variable Name	Impact <sub>1</sub> <sup>E</sup>	Impact <sub>2</sub> <sup>E</sup>	Impact <sub>3</sub> <sup>E</sup>
LCAC/FLSS Connection	0.0156	0.0035	0.0032
LCAC Lift Parameters	0.0467	0.0106	0.0097
LCAC/Seabase Connection	0.0467	0.0106	0.0097
LCU/FLSS Connection	0.0035	0.0008	0.0007
LCU Lift Parameters	0.0106	0.0024	0.0022
LCU/Seabase Connection	0.0106	0.0024	0.0022
FLS Facilities	0.0409	0.0191	0.0113
FLS Process Rate	0.0409	0.0058	0.0075
FLS Operating Time	0.0409	0.0029	0.0067
Civilian Travel Velocity	0.0000	0.0362	0.0103
MH-53E/FLSS Connection	0.0011	0.0009	0.0004
MH-53E Lift Parameters	0.0723	0.0600	0.0275
MH-53E/Seabase Connection	0.0361	0.0300	0.0137
S-60B/FLSS Connection	0.0001	0.0001	0.0000
S-60B Lift Parameters	0.0072	0.0060	0.0027
S-60B/Seabase Connection	0.0036	0.0030	0.0014
MV-22/FLSS Connection	0.0024	0.0020	0.0009
MV-22 Lift Parameters	0.1614	0.1340	0.0613
MV-22/Seabase Connection	0.0807	0.0670	0.0307
FLSS Facilities	0.1106	0.0800	0.0386
FLSS Process Rate	0.1106	0.1263	0.0519
FLSS Operating Time	0.1106	0.0631	0.0338

**Table 97:** Population Reached Level 1: Importance of Impact Values

Variable Name	Impact <sub>2</sub> <sup>E</sup>	Impact <sub>3</sub> <sup>E</sup>
Population Reached FLS	0.0909	0.0260
Population Reached FLSS	0.9091	0.2597

**Table 98:** Population Reached Level 2: Importance of Impact Values

Variable Name	Impact <sub>2</sub> <sup>E</sup>	Impact <sub>3</sub> <sup>E</sup>
Aid Delivered Surface to FLS	0.0455	0.0130
Access to FLS	0.0455	0.0130
Aid Delivered Air to FLSS	0.4546	0.1299
Access to FLSS	0.4546	0.1299

**Table 99:** Population Reached Level 3: Importance of Impact Values

Variable Name	Impact <sub>2</sub> <sup>E</sup>	Impact <sub>3</sub> <sup>E</sup>
Aid Delivered: from SC to FLS	0.0303	0.0087
FLS Operations	0.0151	0.0043
Time Required to Receive Aid FLS	0.0151	0.0043
Frequency of Visit	0.3334	0.0952
Aid Delivered: from AC to FLSS	0.3030	0.0866
FLSS Operations	0.1515	0.0433
Time Required to Receive Aid FLSS	0.1515	0.0433

**Table 100:** Population Reached Level 4: Importance of Impact Values

Variable Name	Impact <sub>2</sub> <sup>E</sup>	Impact <sub>3</sub> <sup>E</sup>
Aid Delivered LCAC to FLS	0.0247	0.0071
Aid Delivered LCU to FLS	0.0056	0.0016
Wait Time FLS	0.0202	0.0058
Travel Time FLS	0.0101	0.0029
Aid Given	0.2222	0.0635
Civilian Payload	0.1111	0.0317
Aid Delivered MH-53E to FLSS	0.0909	0.0260
Aid Delivered S-60B to FLSS	0.0091	0.0026
Aid Delivered MV-22 to FLSS	0.2030	0.0580
Wait Time FLSS	0.2525	0.0721
Travel Time FLSS	0.0505	0.0144

**Table 101:** Flow Rate Level 5: Importance of Impact Values (a)

Variable Name	Impact <sub>1</sub> <sup>E</sup>	Impact <sub>2</sub> <sup>E</sup>	Impact <sub>3</sub> <sup>E</sup>
LCAC Unload Time	0.0156	0.0035	0.0032
LCAC Operating Time	0.0114	0.0026	0.0024
LCAC Number	0.0114	0.0026	0.0024
LCAC Payload	0.0114	0.0026	0.0024
LCAC Travel Time	0.0114	0.0026	0.0024
LCAC Range	0.0005	0.0001	0.0001
LCAC Endurance	0.0005	0.0001	0.0001
LCAC Refuel Rate	0.0005	0.0001	0.0001
LCAC Load Time	0.0114	0.0026	0.0024
Seabase Operations	0.1265	0.0796	0.0408
Dock Locations	0.0006	0.0001	0.0001
LCU Unload Time	0.0035	0.0008	0.0007
LCU Operating Time	0.0027	0.0006	0.0006
LCU Number	0.0027	0.0006	0.0006
LCU Payload	0.0027	0.0006	0.0006
LCU Travel Time	0.0027	0.0006	0.0006
LCU Load Time	0.0026	0.0006	0.0005
MH-53E Unload Time	0.0011	0.0009	0.0004
MH-53E Operating Time	0.0164	0.0136	0.0062
MH-53E Number	0.0164	0.0136	0.0062
MH-53E Payload	0.0164	0.0136	0.0062
MH-53E Travel Time	0.0164	0.0136	0.0062
MH-53E Range	0.0033	0.0027	0.0012
MH-53E Endurance	0.0033	0.0027	0.0012
MH-53E Refuel Rate	0.0004	0.0003	0.0001
MH-53E Load Time	0.0004	0.0003	0.0001

**Table 102:** Flow Rate Level 5: Importance of Impact Values (b)

Variable Name	Impact <sub>1</sub> <sup>E</sup>	Impact <sub>2</sub> <sup>E</sup>	Impact <sub>3</sub> <sup>E</sup>
Landing Time	0.0084	0.0070	0.0032
Takeoff Time	0.0084	0.0070	0.0032
Pickup Locations	0.0084	0.0070	0.0032
Landing Locations	0.0084	0.0070	0.0032
S-60B Unload Time	0.0001	0.0001	0.0000
S-60B Operating Time	0.0016	0.0014	0.0006
S-60B Number	0.0016	0.0014	0.0006
S-60B Payload	0.0016	0.0014	0.0006
S-60B Travel Time	0.0016	0.0014	0.0006
S-60B Range	0.0003	0.0003	0.0001
S-60B Endurance	0.0003	0.0003	0.0001
S-60B Refuel Rate	0.0000	0.0000	0.0000
S-60B Load Time	0.0000	0.0000	0.0000
MV-22 Unload Time	0.0024	0.0020	0.0009
MV-22 Operating Time	0.0367	0.0305	0.0139
MV-22 Number	0.0367	0.0305	0.0139
MV-22 Payload	0.0367	0.0305	0.0139
MV-22 Travel Time	0.0367	0.0305	0.0139
MV-22 Range	0.0073	0.0061	0.0028
MV-22 Endurance	0.0073	0.0061	0.0028
MV-22 Refuel Rate	0.0008	0.0007	0.0003
MV-22 Load Time	0.0008	0.0007	0.0003

**Table 103:** Flow Rate Level 6: Importance of Impact Values

Variable Name	Impact <sub>1</sub> <sup>E</sup>	Impact <sub>2</sub> <sup>E</sup>	Impact <sub>3</sub> <sup>E</sup>
Travel Distance	0.0172	0.0122	0.0059
Sea State	0.0577	0.0334	0.0178
LCAC Cruise Velocity	0.0029	0.0006	0.0006
LCU Cruise Velocity	0.0007	0.0002	0.0001
MH-53E Cruise Velocity	0.0041	0.0034	0.0016
S-60B Cruise Velocity	0.0004	0.0003	0.0002
MV-22 Cruise Velocity	0.0092	0.0076	0.0035
Weather	0.0274	0.0227	0.0104
LHD Parameters	0.0506	0.0318	0.0163
LPD Parameters	0.0169	0.0106	0.0054
LSD Parameters	0.0084	0.0053	0.0027

#### B.4 Calculated Correlations from Impact Weightings

##### Impact<sub>1</sub><sup>E</sup> Level 2 Correlations

$$\begin{bmatrix} 1.00 & 0.13 \\ 0.00 & 1.00 \end{bmatrix} \quad (70)$$

##### Impact<sub>1</sub><sup>E</sup> Level 3 Correlations

$$\begin{bmatrix} 1.00 & 0.00 & 0.26 & 0.00 \\ 0.00 & 1.00 & 0.00 & 0.00 \\ 0.00 & 0.00 & 1.00 & 0.00 \\ 0.00 & 0.00 & 0.00 & 1.00 \end{bmatrix} \quad (71)$$

##### Impact<sub>1</sub><sup>E</sup> Level 4 Correlations



1.00	0.39	0.00	0.00	0.00	0.26	0.26	0.26	0.00	0.00	0.00
0.00	1.00	0.00	0.00	0.00	0.26	0.26	0.26	0.00	0.00	0.00
0.00	0.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.00	0.00	0.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.00	0.00	0.00	0.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00
0.00	0.00	0.00	0.00	0.00	1.00	0.34	0.34	0.00	0.00	0.00
0.00	0.00	0.00	0.00	0.00	0.00	1.00	0.34	0.00	0.00	0.00
0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.00	0.00	0.00	0.00
0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.00	0.00	0.00
0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.00	0.00
0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.00

**Impact<sub>1</sub><sup>E</sup> Level 6 Correlations**

**Omitted Due to Size**

**Impact<sub>2</sub><sup>E</sup> Level 2 Correlations**

$$\begin{bmatrix} 1.00 & 0.09 \\ 0.00 & 1.00 \end{bmatrix} \quad (72)$$

**Impact<sub>2</sub><sup>E</sup> Level 3 Correlations**

$$\begin{bmatrix} 1.00 & 0.00 & 0.17 & 0.00 \\ 0.00 & 1.00 & 0.00 & 0.00 \\ 0.00 & 0.00 & 1.00 & 0.00 \\ 0.00 & 0.00 & 0.00 & 1.00 \end{bmatrix} \quad (73)$$

**Impact<sub>2</sub><sup>E</sup> Level 4 Correlations**

$$\begin{bmatrix} 1.00 & 0.00 & 0.00 & 0.00 & 0.26 & 0.00 & 0.00 \\ 0.00 & 1.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 \\ 0.00 & 0.00 & 1.00 & 0.00 & 0.00 & 0.00 & 0.00 \\ 0.00 & 0.00 & 0.00 & 1.00 & 0.00 & 0.00 & 0.00 \\ 0.00 & 0.00 & 0.00 & 0.00 & 1.00 & 0.00 & 0.00 \\ 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 1.00 & 0.00 \\ 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 1.00 \end{bmatrix}$$

(74)

**Impact<sub>2</sub><sup>E</sup> Level 5 Correlations**

$$\begin{bmatrix} 1.00 & 0.39 & 0.00 & 0.00 & 0.00 & 0.00 & 0.26 & 0.26 & 0.26 & 0.00 & 0.00 \\ 0.00 & 1.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.26 & 0.26 & 0.26 & 0.00 & 0.00 \\ 0.00 & 0.00 & 1.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 \\ 0.00 & 0.00 & 0.00 & 1.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 \\ 0.00 & 0.00 & 0.00 & 0.00 & 1.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 \\ 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 1.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 \\ 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 1.00 & 0.34 & 0.34 & 0.00 & 0.00 \\ 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 1.00 & 0.34 & 0.00 & 0.00 \\ 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 1.00 & 0.00 & 0.00 \\ 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 1.00 & 0.00 \\ 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 1.00 \end{bmatrix}$$

**Impact<sub>2</sub><sup>E</sup> Level 6 Correlations**

Omitted due to size

## B.5 Communicative Model Development

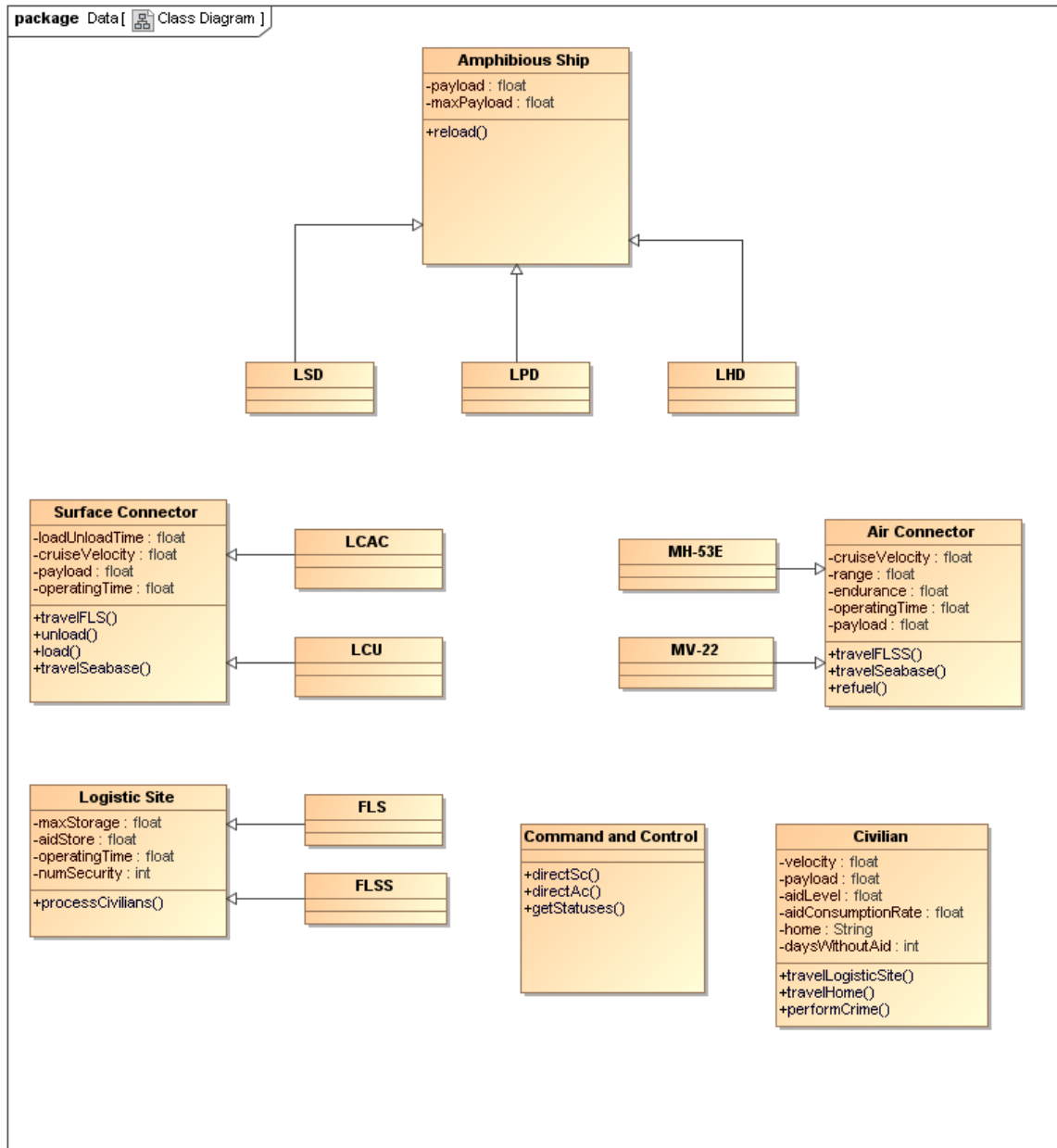
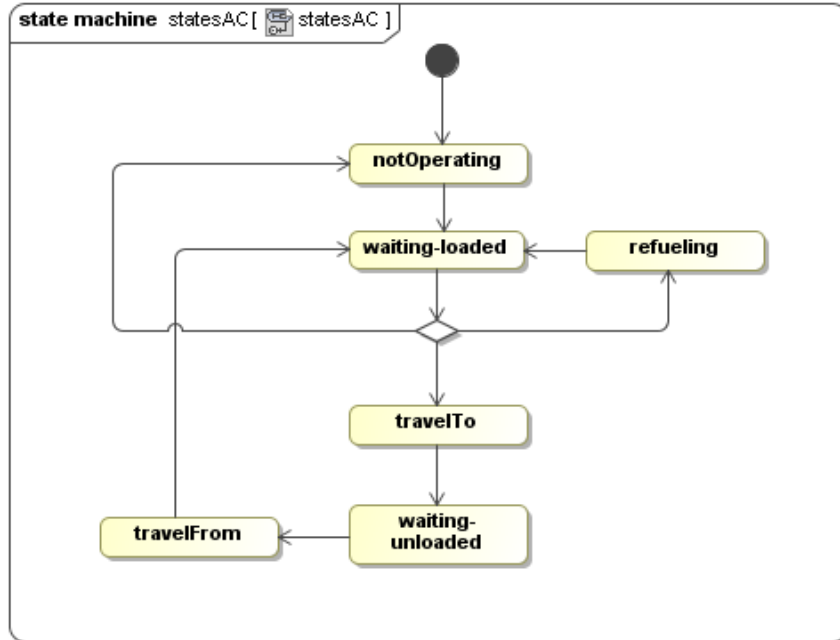


Figure 154: Class Diagram for HA/DR Model



**Figure 155:** Air Connector State Machine Diagram for HA/DR Model

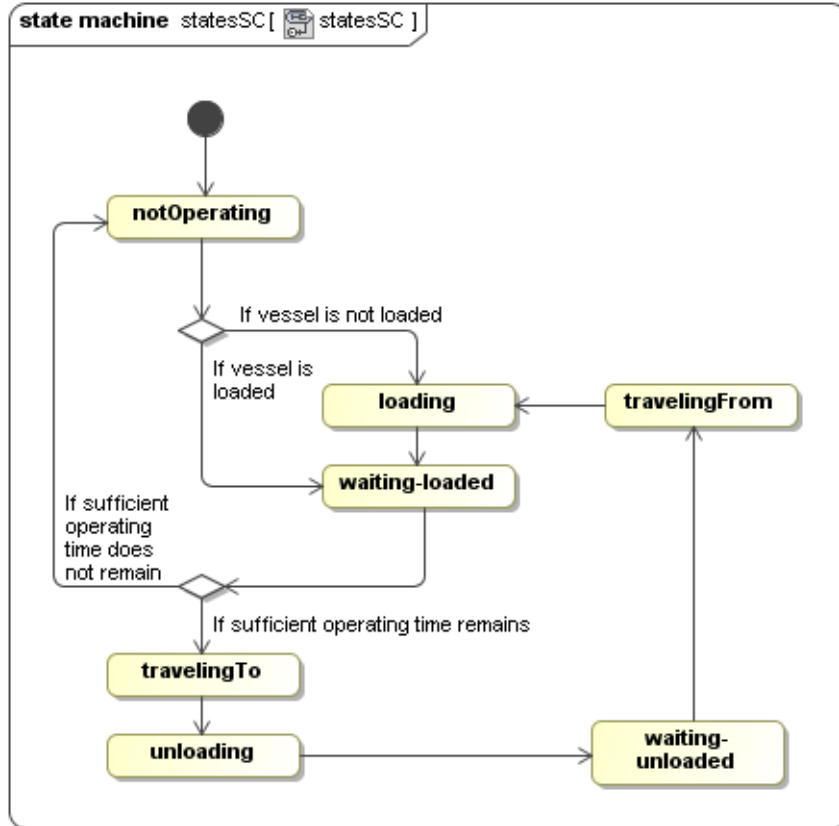


Figure 156: Surface Connector State Machine Diagram for HA/DR Model

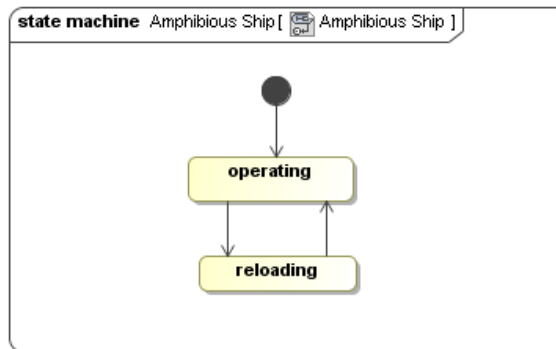
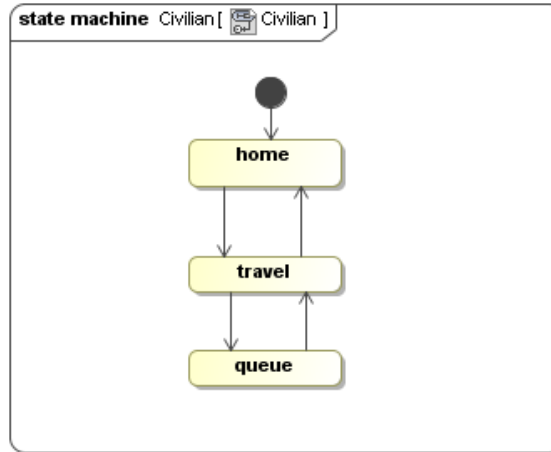


Figure 157: Amphibious Ship State Machine Diagram for HA/DR Model



**Figure 158:** Civilian State Machine Diagram for HA/DR Model

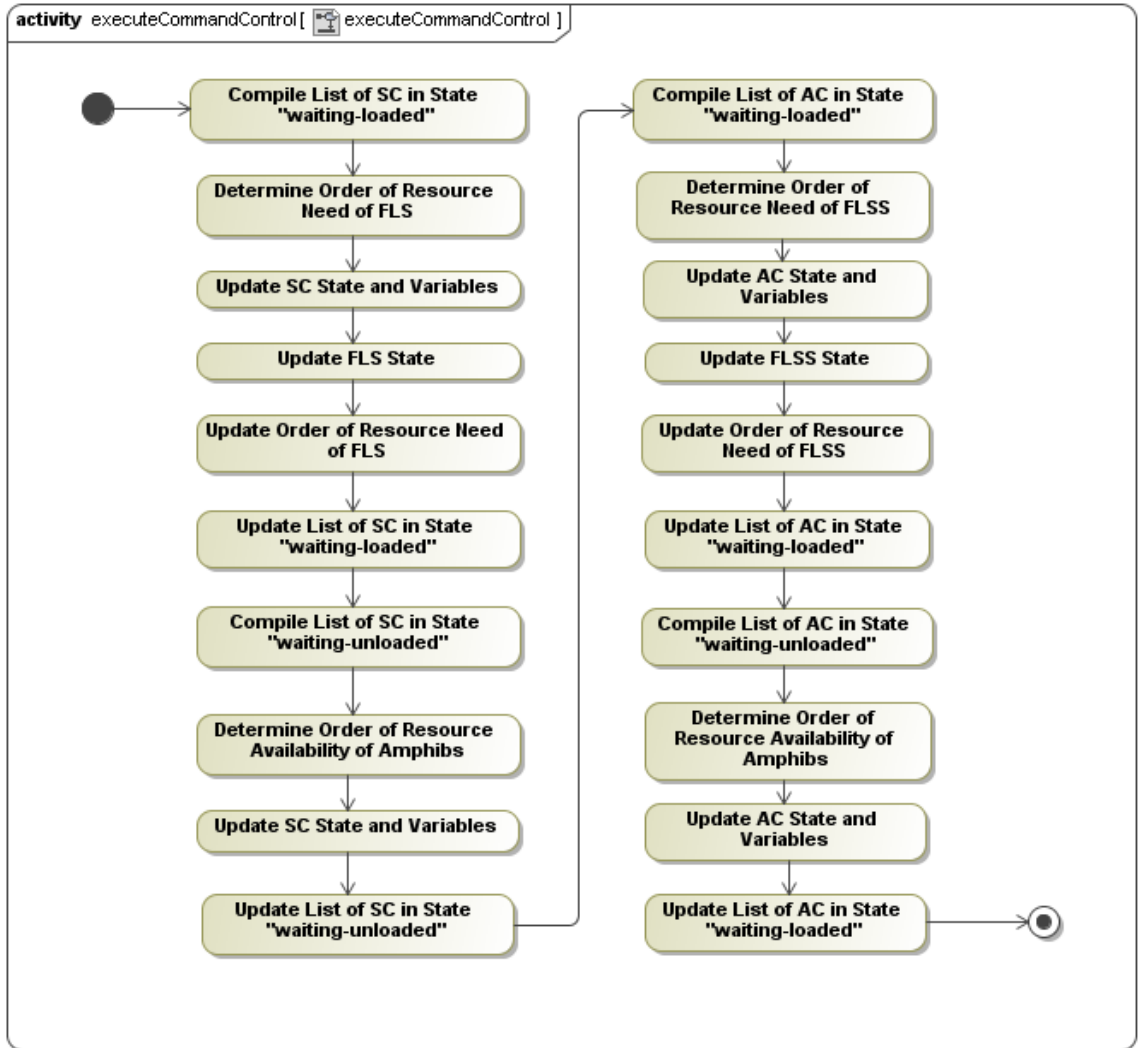


Figure 159: Command and Control Activity Diagram

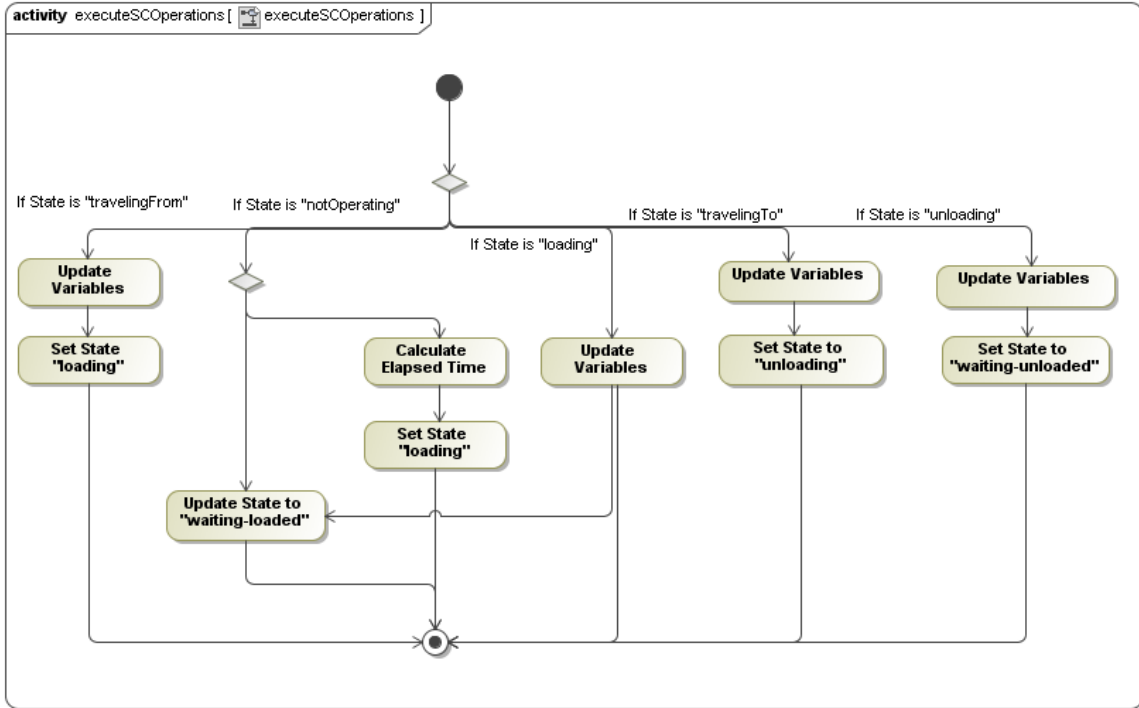


Figure 160: Surface Connector Activity Diagram

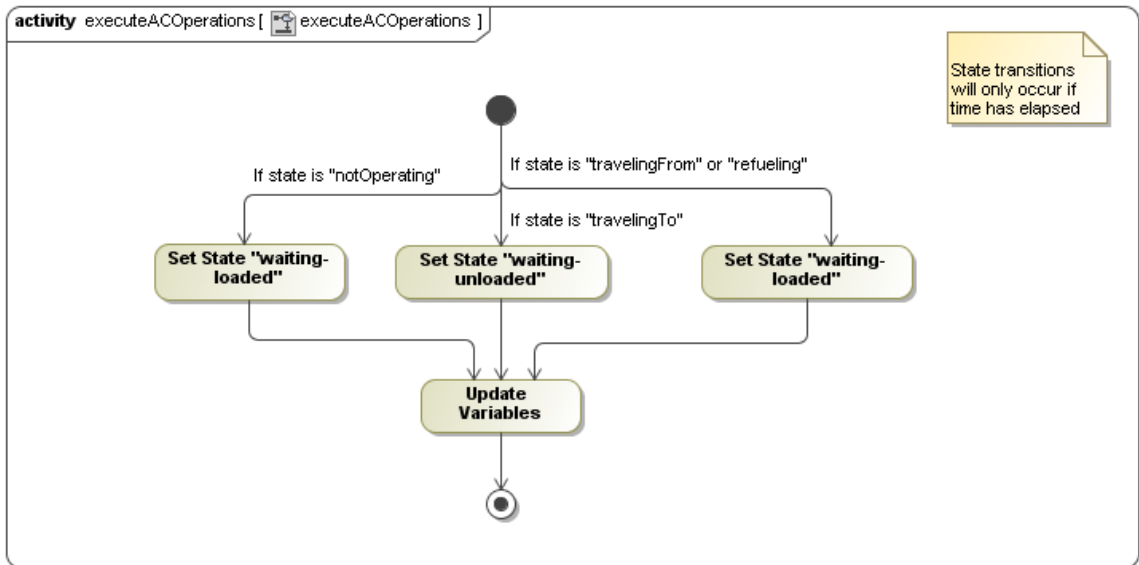


Figure 161: Air Connector Activity Diagram



## B.6 Programmed Model

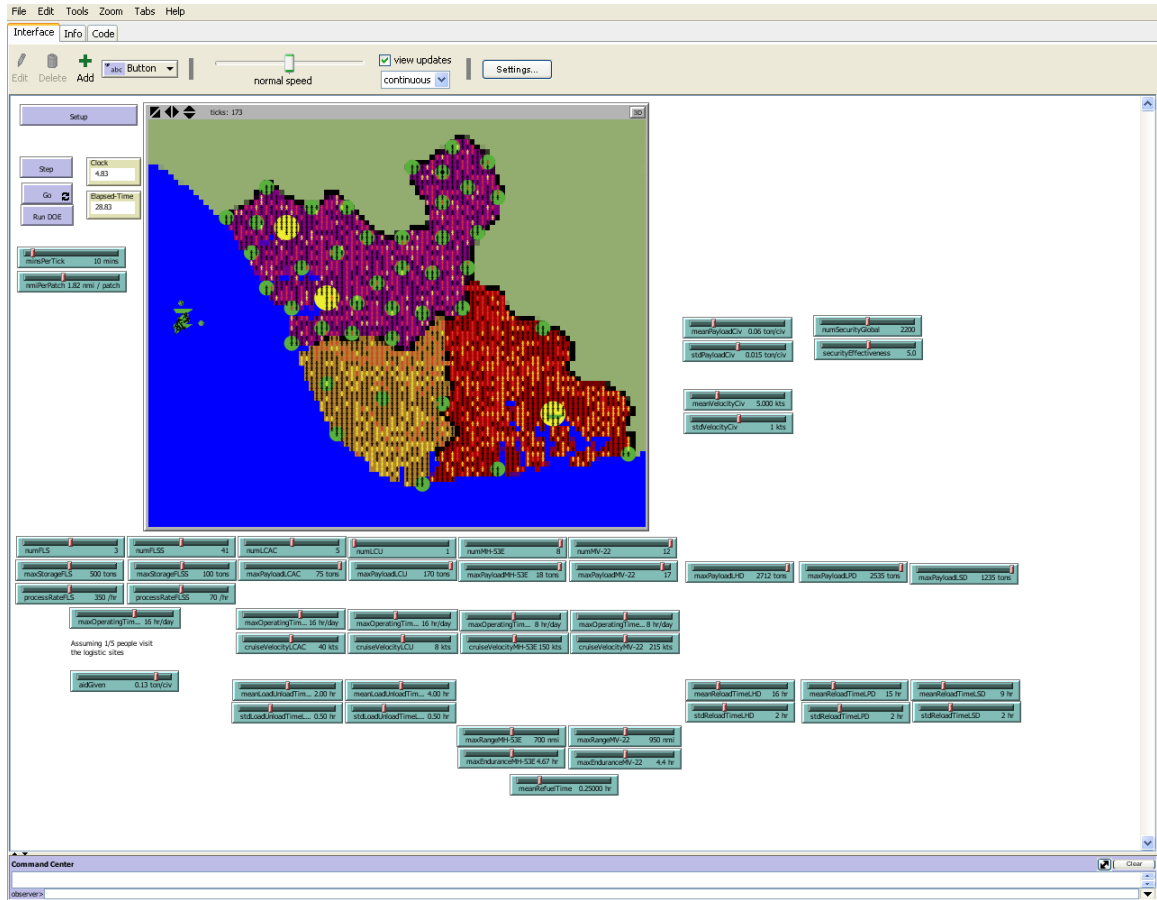
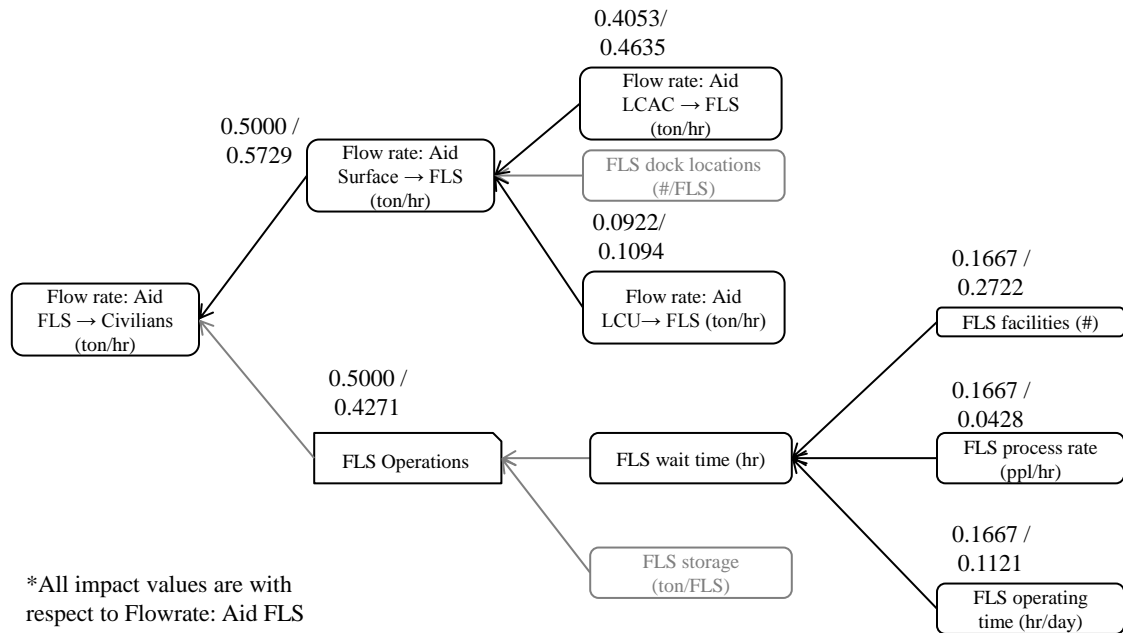
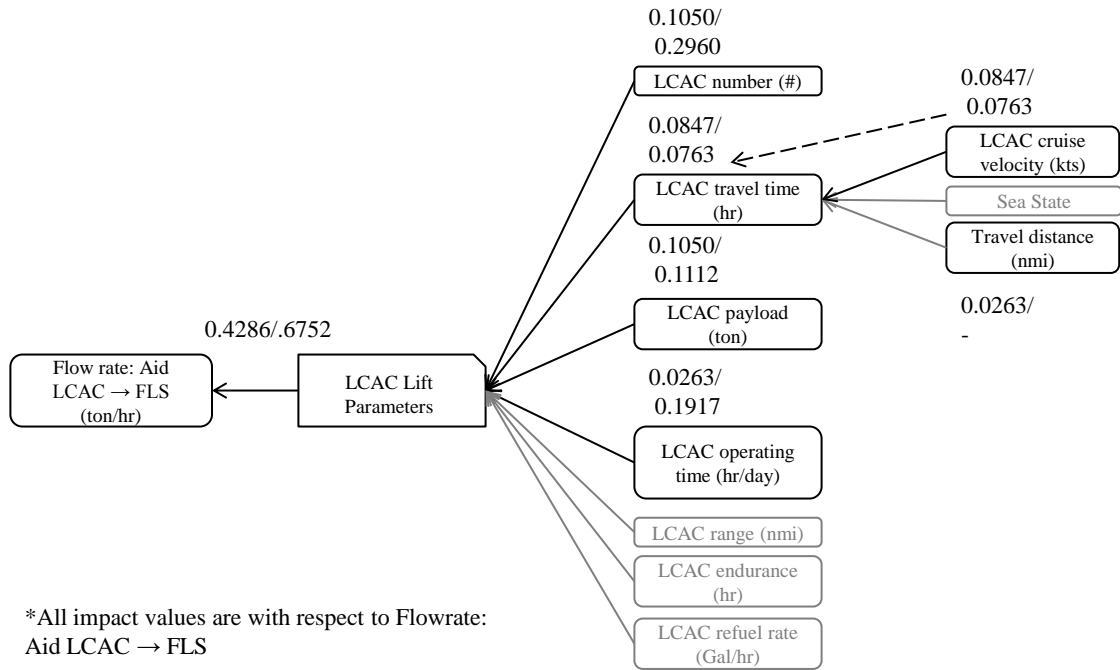


Figure 162: Screen-shot of the HA/DR Model

*B.7 Comparison of Impact Decomposition to Observed Values*

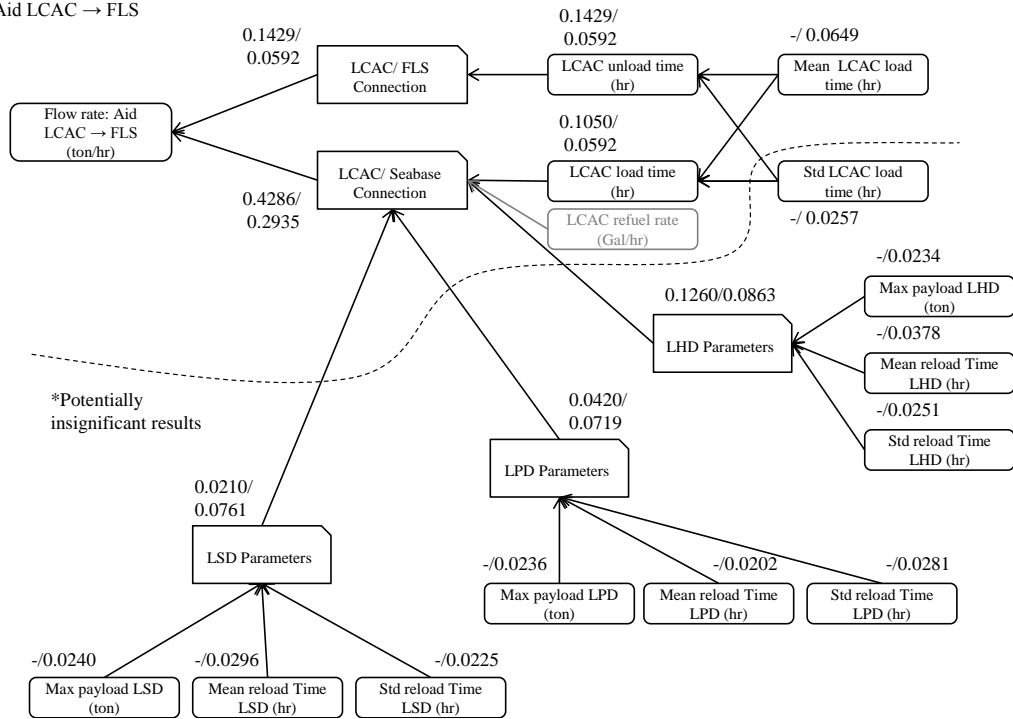


**Figure 163:** Comparison of FLS Impact Decomposition to Observed Values

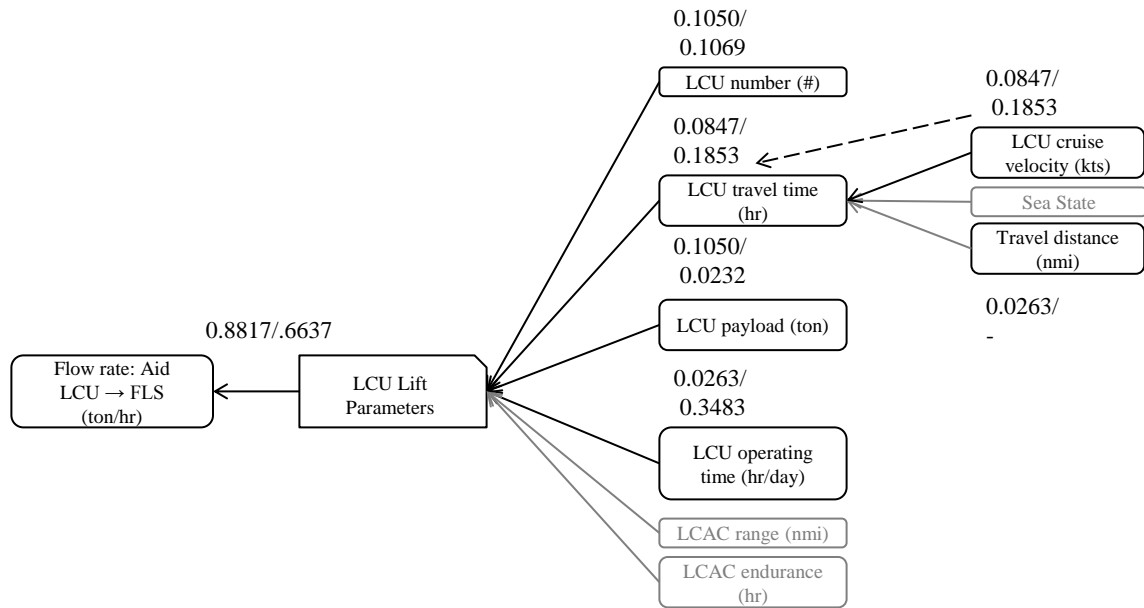


**Figure 164:** Comparison of LCAC Lift Parameters Impact Decomposition to Observed Values

\*All impact values are with respect to Flowrate:  
Aid LCAC → FLS

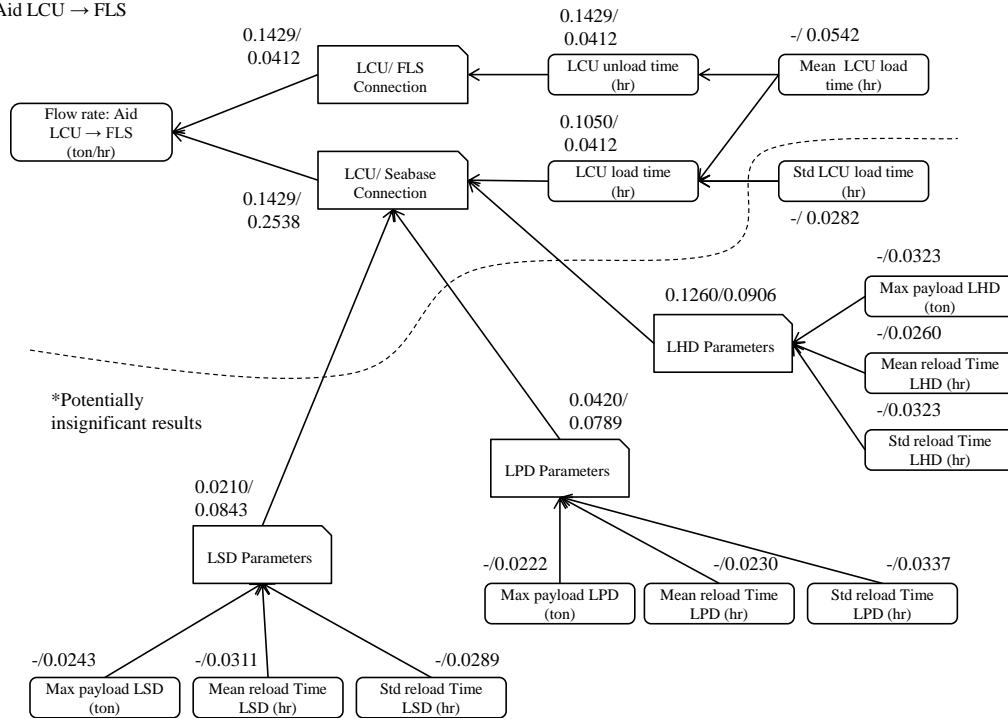


**Figure 165:** Comparison of LCAC FLS & Seabase Connector Impact Decomposition to Observed Values

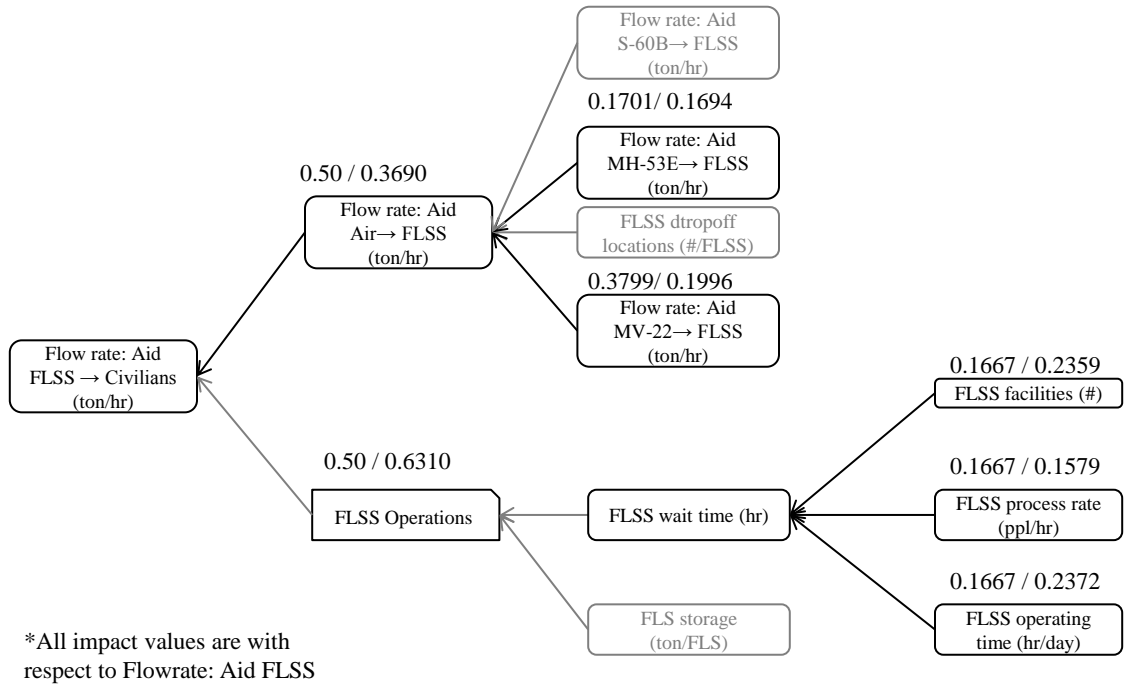


**Figure 166:** Comparison of LCU Lift Parameters Impact Decomposition to Observed Values

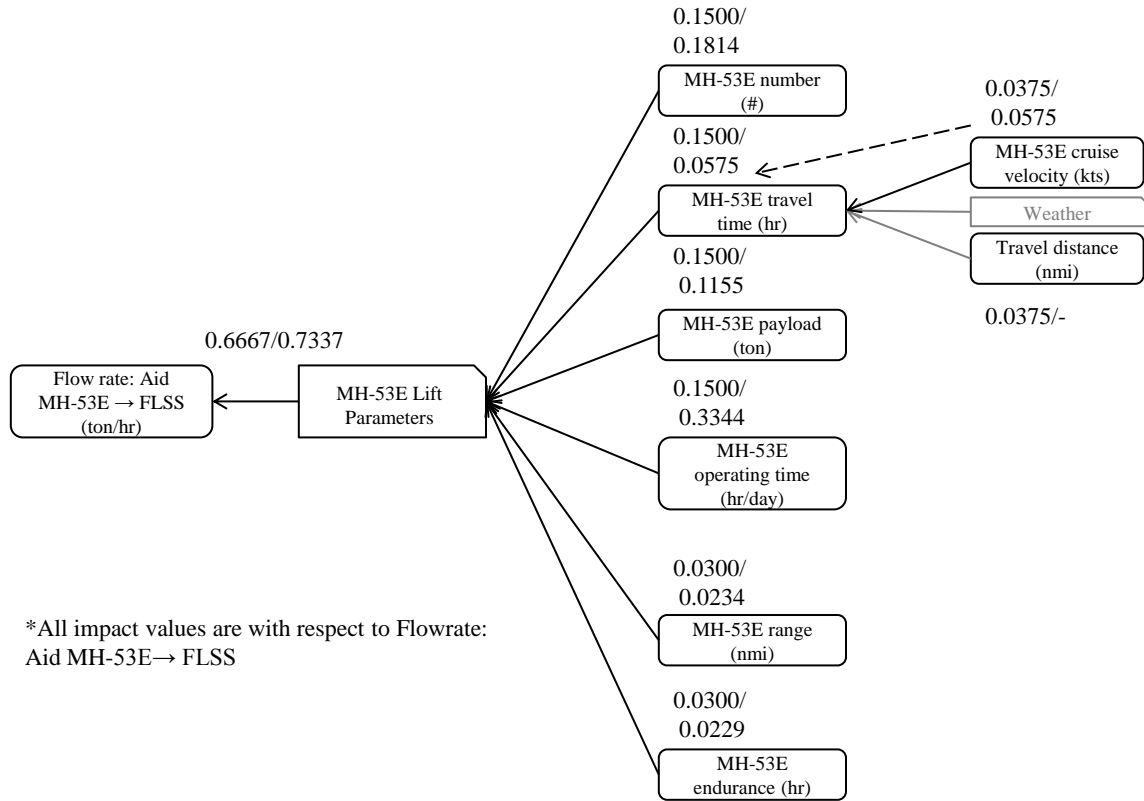
\*All impact values are with respect to Flowrate:  
Aid LCU → FLS



**Figure 167:** Comparison of LCU FLS & Seabase Connector Impact Decomposition to Observed Values

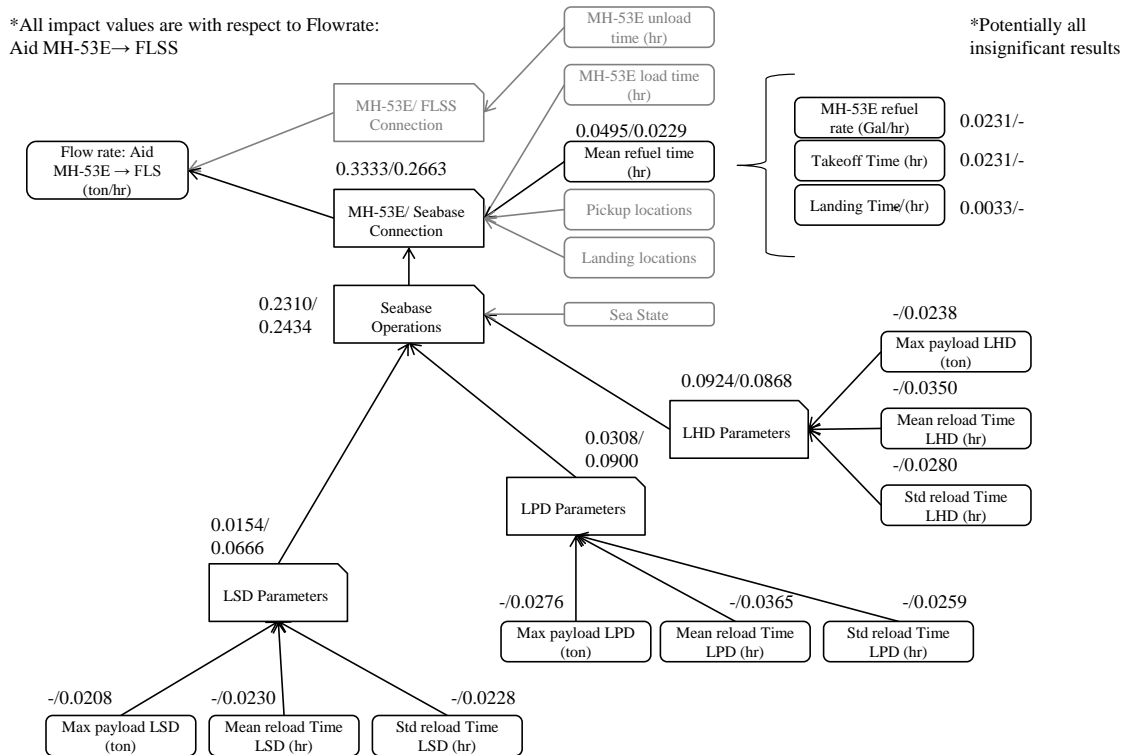


**Figure 168:** Comparison of FLSS Impact Decomposition to Observed Values

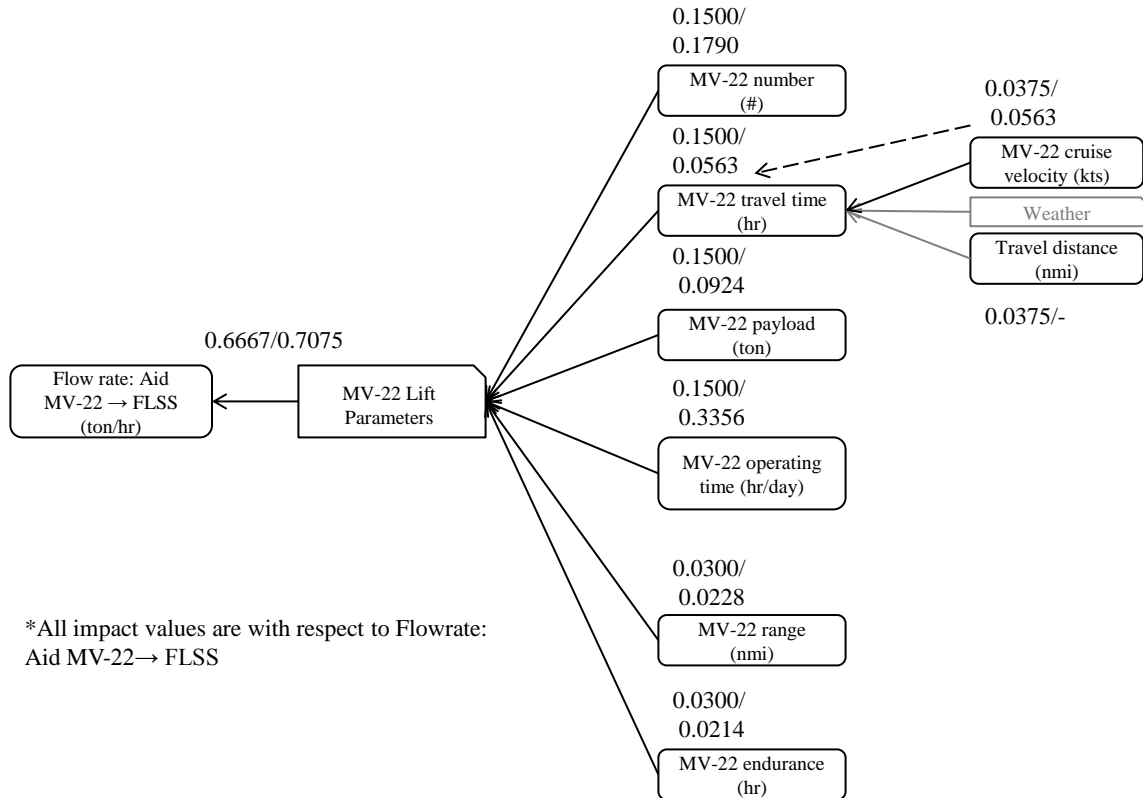


**Figure 169:** Comparison of MH-53E Lift Parameters Impact Decomposition to Observed Values

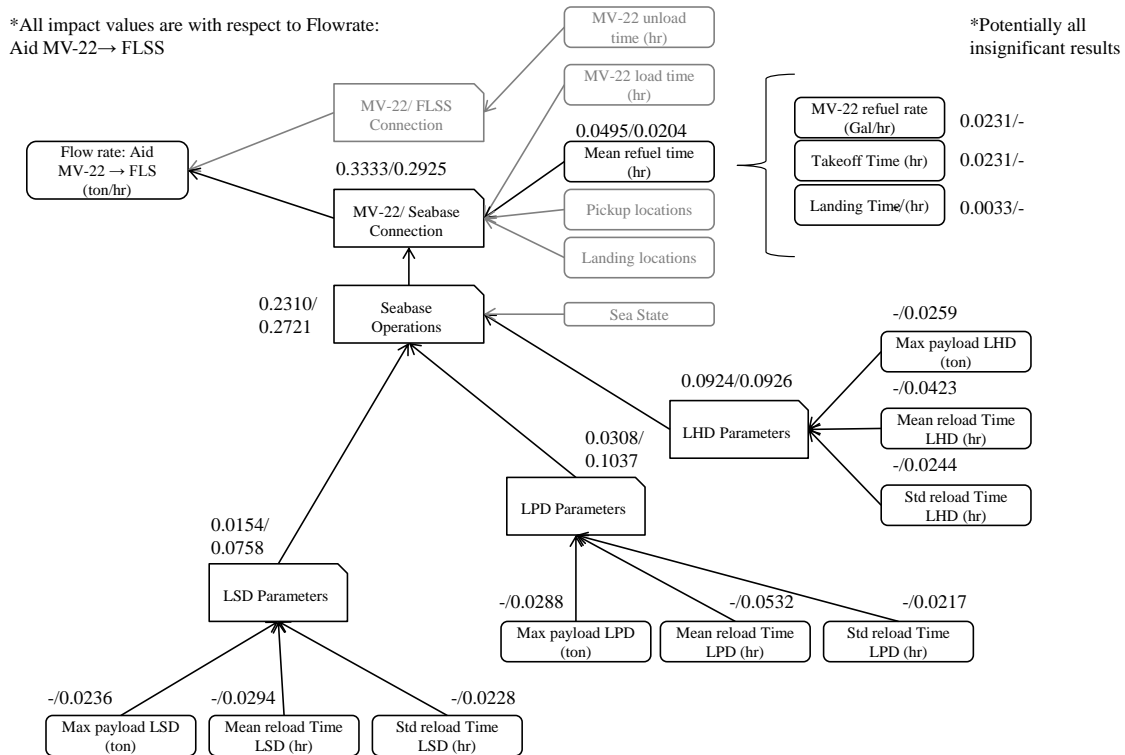




**Figure 170:** Comparison of MH-53E FLSS & Seabase Connector Impact Decomposition to Observed Values



**Figure 171:** Comparison of MV-22 Lift Parameters Impact Decomposition to Observed Values



**Figure 172:** Comparison of MV-22 FLSS & Seabase Connector Impact Decomposition to Observed Values

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